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DISPERSION EQUATION OF RAYLEIGH WAVES IN TRANSVERSELY ISOTROPIC NONLOCAL PIEZOELASTIC SOLIDS HALF-SPACE

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Abstract. This study is devoted to investigate the propagation of Rayleigh-type waves in transversely isotropic nonlocal piezoelastic half-space. When the stress-free boundary is maintained at charge-free condition, the dispersion equation for the propagation of Rayleigh waves at the free surface of transversely isotropic piezoelastic solids has been obtained. Based on the obtained dispersion equation, the effect of the nonlocality on the speed of Rayleigh wave is numerically considered. The dependence of velocities of plane waves in transversely isotropic nonlocal piezoelastic medium on the direction of propagation as well as non-dimensional frequency parameter has been also illustrated.

Keywords: dispersion equation, nonlocal, piezoelastic.

1. INTRODUCTION

In recent years piezoelectric materials has drawn much attention towards application in surface acoustic wave (SAW) micro sensors, energy harvesting structure, health monitoring systems, transducers and actuators, etc. Both theoretical and experimental studies on wave propagation in piezoelectric materials have attracted the attention of scientists and engineers during last two decades. The survey of literature can be found in many related texts and books [1, 2]. We mention only a few such as: Zinchuk and Podlipenets [3] obtained dispersion equations for acousto-electric Rayleigh wave in a periodic layer piezoelectric half-space in a study for the 6 mm crystal class. Wave propagation in porous piezoelectric material (PPM), having crystal symmetry 6 mm, is studied analytically by Vashishth et al. [4]. Sharma et al. [1] investigated the propagation of Rayleigh waves in a homogeneous, transversely isotropic, piezothermoelastic half-space subjected to stress free, electrically shorted/charge-free and thermally insulated/isothermal boundary conditions. Secular equations for the half-space in closed form and isolated mathematical conditions in completely separate terms are derived.

Recent development in science and technology requires that the high-performance electromechanical devices must have a higher sensitivity and larger storage capacity but as maller size. Nano scale materials and structures have been introduced and developed ever since. For these materials, the conventional continuum elasticity theory fails to represent the constitutive relationships properly [5]. A non-local model based on Eringens theory of non-local continuum mechanics has been proposed for the effects of the size dependency in very small structures. Particularly, Eringen's nonlocal theory [6] has been extended to study the size dependent mechanical performances of the piezoelectric nanostructures by Ke et al. [7–9]. There are a few research works on the propagation of the guided elastic waves in nanoscale periodic piezoelectric structures. For examples, Chen et al. [10] studied the anti-plane transverse wave propagation in nanoscale periodic layered piezoelectric structures. Yan et al. [11,12] investigated the propagation of guided elastic waves in nanoscale layered periodic piezoelectric composites.

However, only few researchs [11–13] on wave propagation in nanoscale periodic layered piezoelectric structures have been reported in literature due to the complexity of the problem. In addition, many researches [1,14] have been carried out on the propagation of Rayleigh waves in transversely isotropic piezoelastic materials solids. However, to the best of the authors knowledge, there are no researches analyzing the propagation of surface waves in transversely isotropic nonlocal piezoelastic half-space analytically available in the literature. Therefore, the main purpose of this paper is to study the effect of nonlocality on the propagation of Rayleigh waves in transversely isotropic piezoelastic materials. The dispersion equation for the propagation of Rayleigh waves is derived for the boundary is stress-free, maintained at charge-free condition.

2. FORMULATION OF THE PROBLEM

We consider homogeneous transversely isotropic, electrically conducting piezoelectric medium in the undeformed state at initial potential ϕ_0. We assume that the medium is transversely isotropic in such a way that planes of isotropy are perpendicular to x_3-axis. We take the origin of the coordinate system (x_1, x_2, x_3) at any point on the plane surface and x_3-axis pointing vertically downward into the half-space. Thus the considering half-space is represented by $x_3 \geq 0$. For two-dimensional problem in which the plane wave is in the plane x_1x_3), the strains are related to the displacement field u_1, u_3 and the electric potential ϕ. The constitutive equations are given as [1,11]:

- Strain-displacement relations

$$\varepsilon_{11} = u_{1,1}, \quad \varepsilon_{33} = u_{3,3}, \quad \varepsilon_{13} = \varepsilon_{31} = \frac{1}{2}(u_{1,3} + u_{3,1}), \tag{1}$$

- Stress-strain and electric field relations

$$\begin{aligned} &\sigma_{11} = c_{11}\varepsilon_{11} + c_{13}\varepsilon_{33} - e_{31}E_3, \quad \sigma_{33} = c_{13}\varepsilon_{11} + c_{33}\varepsilon_{33} - e_{33}E_3, \\ &\sigma_{13} = \sigma_{31} = 2c_{44}\varepsilon_{13} - e_{15}E_1, \\ &D_1 = 2e_{15}\varepsilon_{13} + \epsilon_{11}E_1, \quad D_3 = e_{13}\varepsilon_{11} + e_{33}\varepsilon_{33} + \epsilon_{33}E_3, \end{aligned} \tag{2}$$

where $E_i = -\phi_{,i}$ is the electric field and D_i the electric displacement, ρ the mass density, σ_{ij} the stress tensor, c_{ij} the elastic parameters tensor, e_{ij} the piezoelectric moduli, ϵ_{ij} the electric permittivity $(i, j = 1, 3)$.

It is well known that in the classical piezoelectricity (CPE) theory, the stresses and the electric displacements at one point only dependend on the local strains and electric fields at the same point. But when the macro size reaches a few nanometers, the CPE continuum theory fails and we have usually to utilize other methods. The essence of the Eringen's nonlocal elasticity theory [6,15] is that the stress at a point x in a body depends not only on the strain at that point but also on the strain at all other points x' in the domain. Recently, Ke et al. [7,8] extended the nonlocal elasticity theory to the piezoelectric nanostructures-the nonlocal continuum theory of piezoelectricity (NLPE). Unlike the CPE continuum theory, the NLPE theory supposes that the stresses and the electrical displacements at one point should be affected by the strains and electrical fields at all points of the whole body. Thus the relationship between the CPE stress and electrical displacement components and the NLPE stress and electrical displacement components can be written as [11,12]

$$t_{mn} = (1 + \epsilon^2\nabla^2)\sigma_{mn}, \quad d_m = (1 + \epsilon^2\nabla^2)D_m, \tag{3}$$

where t_{mn} and d_m are the NLPE stress and electrical displacement components, respectively; σ_{mn} and D_m are the traditional stress and electrical displacement components, respectively. Constant $\epsilon(= e_0 a)$ is the nonlocal parameter (e_0 is the nonlocal constant and a is the internal characteristic length).

For the wave propagation considered in this paper, the body forces, electric charge are ignored. Using the relations (3), the motion equations, Gauss equation are simplified as [11,12]

$$\begin{aligned} &\sigma_{11,1} + \sigma_{13,3} = (1 - \epsilon^2\nabla^2)\rho\ddot{u}_1, \quad \sigma_{13,1} + \sigma_{33,3} = (1 - \epsilon^2\nabla^2)\rho\ddot{u}_3, \\ &D_{1,1} + D_{3,3} = 0. \end{aligned} \tag{4}$$

Substituiting (2) into (4) taking into account (1) we have

$$\begin{aligned} &c_{11}u_{1,11} + c_{44}u_{1,33} + (c_{13} + c_{44})u_{3,13} + (e_{15} + e_{31})\phi_{,13} = (1 - \epsilon^2\nabla^2)\rho\ddot{u}_1, \\ &(c_{13} + c_{44})u_{1,13} + c_{44}u_{3,11} + c_{33}u_{3,33} + e_{15}\phi_{,11} + e_{33}\phi_{,33} = (1 - \epsilon^2\nabla^2)\rho\ddot{u}_3, \\ &(e_{15} + e_{31})u_{1,13} + e_{15}u_{3,11} + e_{33}u_{3,33} - \epsilon_{11}\phi_{,11} - \epsilon_{33}\phi_{,33} = 0. \end{aligned} \tag{5}$$

For the waves propagating in the plane $x_3 = 0$, we take the form of relevant components of displacement and the electric potential ϕ as [1,16]

$$\begin{cases} u_1 = a_1 e^{-\xi y} e^{ik(x_1 - ct)} \\ u_3 = a_3 e^{-\xi y} e^{ik(x_1 - ct)} \\ \phi = A_1 e^{-\xi y} e^{ik(x_1 - ct)} \end{cases} \qquad \text{with} \quad y = kx_3, \tag{6}$$

where a_1, a_3, A_1 are polarization vectors, k is wavenumber, c is speed of wave propagation, ξ is a complex coefficient whose imaginary part should be positive corresponding to the decay condition in the half-space $x_3 > 0$.

Substituting the expressions for displacement and electric potential from (6) into (5), we obtain the three homogeneous equations in three unknowns a_1, a_3, A_1. For a nontrivial solution of these equations, we must have $\det(\mathbf{M}) = 0$, in which

$$\mathbf{M} = \begin{bmatrix} c_{11} - c_{44}\xi^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1-\xi^2) & (c_{13}+c_{44})i\xi & (e_{15}+e_{31})i\xi \\ (c_{13}+c_{44})i\xi & c_{44} - c_{33}\xi^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1-\xi^2) & e_{15} - e_{33}\xi^2 \\ (e_{15}+e_{31})i\xi & e_{15} - e_{33}\xi^2 & \epsilon_{33}\xi^2 - \epsilon_{11} \end{bmatrix}. \tag{7}$$

This is the characteristics equation and it has the form

$$h_6 p^6 + h_4 p^4 + h_2 p^2 + h_0 = 0, \tag{8}$$

where $p = i\xi$ and the coefficients $h_i, (i = 0, 2, 4, 6)$ are given in the Appendix.

Eq. (8) is a cubic polynomial in p^2. We order p_n, $n = 1, 2, \ldots, 6$, in such a way that p_1, p_2, p_3 correspond to waves traveling in the positive x_3 direction, and p_4, p_5, p_6 correspond to the ones traveling in the negative x_3, respectively. Since, we are interested in surface waves only so it is essential that motion is confined to free surface $x_3 = 0$ of the half-space so that the characteristic roots p_i^2 must satisfy the radiation condition $Im(p_i) \geq 0$. Then the general solution for displacements and electric potential are written as [1]

$$\begin{cases} u_1 = \sum_{j=1}^{3} a_{1j} e^{p_j y} e^{ik(x_1 - ct)} \\ u_3 = \sum_{j=1}^{3} a_{3j} e^{p_j y} e^{ik(x_1 - ct)} \\ \phi = \sum_{j=1}^{3} A_{1j} e^{p_j y} e^{ik(x_1 - ct)} \end{cases} \tag{9}$$

where a_{1j}, a_{3j} and A_{1j} are the amplitudes of the displacements and the electric potentials, respectively.

Remark: For the propagation of plane waves with phase velocity c in the direction making an angle θ with the vertical axis, a surface wave of this form is expressed by

$$\begin{cases} u_1 = a_1 e^{ik(p_1 x_1 + p_3 x_3 - ct)} \\ u_3 = a_3 e^{ik(p_1 x_1 + p_3 x_3 - ct)} \\ \phi = A_1 e^{ik(p_1 x_1 + p_3 x_3 - ct)} \end{cases} \tag{10}$$

where $p_1 = \sin\theta, p_3 = \cos\theta$ are components of propagation unit vector. Substituting (10) into (5) and obtain a matrix similar to the matrix $\mathbf{M}$ in Eq. (7). By letting the determinant of this matrix equal zero, we have a quadratic equation in c^2. Therefore, we obtain two real roots $c_j (j = 1, 2)$ corresponding the speeds of plane waves propagating in the medium.

3. BOUNDARY CONDITIONS AND DISPERSION EQUATIONS

In this section, the Rayleigh wave equation for transversely isotropic nonlocal piezoelastic half-space can be derived using the boundary conditions at the surface of the half-space. In the present study, boundary conditions appropriate for particle motion in the $x_1 x_3$ plane are considered at the plane surface $x_3 = 0$. This surface is considered to be stress-free (mechanical conditions), which requires the normal stress σ_{33} as well as the tangential stress σ_{13} to vanish at the surface $x_3 = 0$. That means

$$\sigma_{13} = \sigma_{33} = 0. \tag{11}$$

Another condition is required to represent that the surface of half-space is maintained at charge free condition (open circuit-surface), namely

$$D_3 = 0. \tag{12}$$

Substituting (9) into the boundary conditions (11), (12) and taking into account (2), we have a system of linear equations

$$\begin{cases} \sum_{j=1}^{3} \left[c_{44}(a_{3j} + a_{1j}p_j) + e_{15}A_{1j} \right] = 0, \\ \sum_{j=1}^{3} \left[c_{13}a_{1j} + c_{33}a_{3j}p_j + e_{33}A_{1j}p_j \right] = 0, \\ \sum_{j=1}^{3} \left[e_{13}a_{1j} + e_{33}a_{3j}p_j - \epsilon_{33}A_{1j}p_j \right] = 0. \end{cases} \tag{13}$$

For each $p_j (j = 1, 2, 3)$, the three corresponding unknowns a_{1j}, a_{3j}, A_{1j} (7) are in a relationship given by matrix $\mathbf{M}$ and we can express them as $a_{1j} = \alpha_j A_{1j}, a_{3j} = \beta_j A_{1j}$ where

$$\alpha_j = \frac{(c_{13} + c_{44})p_j(e_{15} + e_{33}p_j^2) - \left(c_{44} + c_{33}p_j^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1 + p_j^2)\right)(e_{15} + e_{31})p_j}{\delta_j},$$

$$\beta_j = \frac{(c_{13} + c_{44})p_j^2(e_{15} + e_{31}) - \left(c_{11} + c_{44}p_j^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1 + p_j^2)\right)(e_{15} + e_{33}p_j^2)}{\delta_j},$$

$$\begin{aligned} \delta_j = {} & \left[c_{11} + c_{44}p_j^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1 + p_j^2)\right]\left[c_{44} + c_{33}p_j^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2 (1 + p_j^2)\right] \\ & - (c_{13} + c_{44})^2 p_j^2, \quad j = 1, 2, 3. \end{aligned}$$

Then we obtain a system of linear equations in amplitudes A_{11}, A_{12}, A_{13} only and it is in the form

$$\begin{cases} (c_{44}\beta_1 + c_{44}\alpha_1 p_1 + e_{15})A_{11} + (c_{44}\beta_2 + c_{44}\alpha_2 p_2 + e_{15})A_{12} + (c_{44}\beta_3 + c_{44}\alpha_3 p_3 + e_{15})A_{13} = 0, \\ (c_{13}\alpha_1 + c_{33}\beta_1 p_1 + e_{33}p_1)A_{11} + (c_{13}\alpha_2 + c_{33}\beta_2 p_2 + e_{33}p_2)A_{12} + (c_{13}\alpha_3 + c_{33}\beta_3 p_3 + e_{33}p_3)A_{13} = 0, \\ (e_{31}\alpha_1 + e_{33}\beta_1 p_1 - \epsilon_{33}p_1)A_{11} + (e_{31}\alpha_2 + e_{33}\beta_2 p_2 - \epsilon_{33}p_2)A_{12} + (e_{31}\alpha_3 + e_{33}\beta_3 p_3 - \epsilon_{33}p_3)A_{13} = 0. \end{cases} \tag{14}$$

The dispersion equation of Rayleigh waves is obtained from $\det(\mathbf{CO}) = 0$ where matrix $\mathbf{CO}$ is the matrix of coefficients of the system of equation above as

$$\begin{bmatrix} c_{44}\beta_1 + c_{44}\alpha_1 p_1 + e_{15} & c_{44}\beta_2 + c_{44}\alpha_2 p_2 + e_{15} & c_{44}\beta_3 + c_{44}\alpha_3 p_3 + e_{15} \\ c_{13}\alpha_1 + c_{33}\beta_1 p_1 + e_{33}p_1 & c_{13}\alpha_2 + c_{33}\beta_2 p_2 + e_{33}p_2 & c_{13}\alpha_3 + c_{33}\beta_3 p_3 + e_{33}p_3 \\ e_{31}\alpha_1 + e_{33}\beta_1 p_1 - \epsilon_{33}p_1 & e_{31}\alpha_2 + e_{33}\beta_2 p_2 - \epsilon_{33}p_2 & e_{31}\alpha_3 + e_{33}\beta_3 p_3 - \epsilon_{33}p_3 \end{bmatrix}. \tag{15}$$

This dispersion equation is in implicit form and it shows the relation between the phase velocity c and the wave number k of the Rayleigh waves and the parameters of the medium.

4. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate theoretical results obtained in the preceding sections, the material chosen for the numerical calculations is CdSe (6 mm class) of hexagonal symmetry, which is transversely isotropic material. The physical data for a single crystal of CdSe material is given below [1,14]

$$\begin{aligned} &c_{11} = 7.41 \times 10^{10}\ \mathrm{Nm^{-2}}, c_{13} = 3.93 \times 10^{10}\ \mathrm{Nm^{-2}}, c_{33} = 8.36 \times 10^{10}\ \mathrm{Nm^{-2}}, \\ &c_{44} = 1.32 \times 10^{10}\ \mathrm{Nm^{-2}}, \rho = 5504\ \mathrm{kgm^{-3}}, e_{15} = -0.138\ \mathrm{Cm^{-2}}, e_{31} = -0.16\ \mathrm{Cm^{-2}}, \\ &e_{33} = 0.347\ \mathrm{Cm^{-2}}, \epsilon_{11} = 8.26 \times 10^{-11}\ \mathrm{C^2N^{-1}m^{-2}}, \epsilon_{33} = 9.03 \times 10^{-11}\ \mathrm{C^2N^{-1}m^{-2}}, \\ &e_0 = 0.39, a = 0.421 \times 10^{-9}\ \mathrm{m}, \epsilon = e_0 a. \end{aligned} \tag{16}$$

Denote $ep = k^2\epsilon^2$ the dimensionless frequency where k is the wavenumber. This is an important parameter that provides us the information of the wave-length of Rayleigh waves comparing to the nonlocal parameter of the medium.

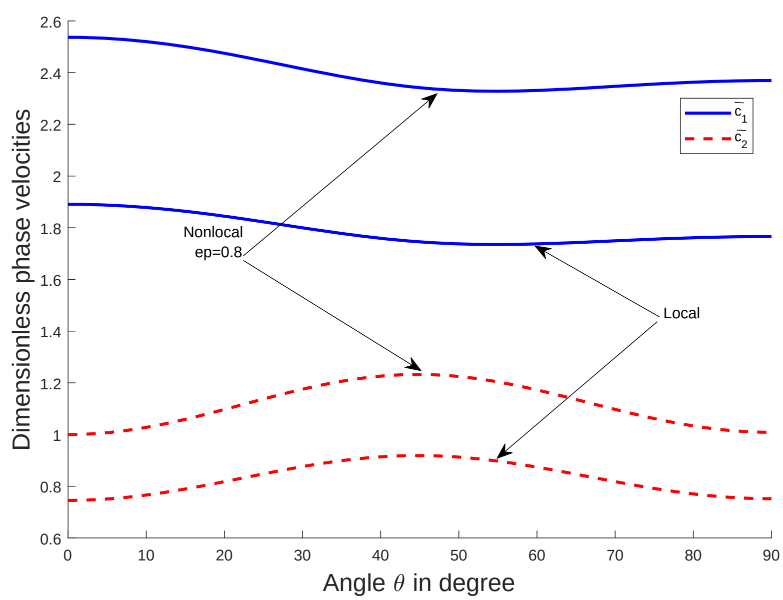

Fig. 1. Dimensionless velocities of plane waves $\bar{c}_1$ and $\bar{c}_2$ depending on the angle directions θ for local and nonlocal case

First, we evaluate the effect of nonlocality to the speed of plane waves. Fig. 1 shows the dimentionless speed of plane waves c/b_S, where $b_S = \sqrt{c_{44}/\rho}$ is the speed of SH-type wave, depending on the direction of propagation θ (angle between the direction of propagation and vertical axis) in the piezoelectric medium for two case of local theory and nonlocal theory with the nonlocal parameter given by $ep = 0.8$. It can be seen that the phase velocities c_1, c_2 in the nonlocal theory case are greater the ones in the local theory case. It can be concluded that the nonlocality has significant effect on the velocities of propagation of plane waves.

Next, the variations of the phase velocities with dimensionless parameter ep for $\theta = \pi/3$ are depicted in Fig. 2. Generally, this figure shows that the phase velocities c_1, c_2 are decreasing when ep is increasing. When $ep < 1$ these velocities decrease rapidly decrease while $ep > 1$ they are quite stable.

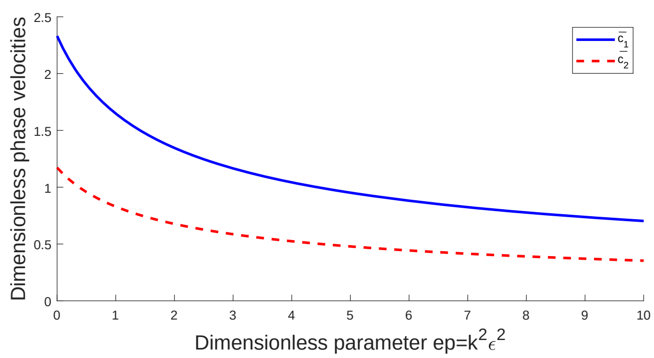

Fig. 2. The comparison of variations of the phase velocities with dimensionless parameter $ep = k^2\epsilon^2$ for $\theta = \pi/3$

Finally, the dimensionless speed of Rayleigh wave $x = X/c_{44}$ with $X = \rho c^2$ depends upon the dimensionless parameter ep is illustrated by Fig. 3 for the boundary condition of open circuit surface (maintained charge free). The speed of Rayleigh wave is decreasing when the parameter ep is increasing.

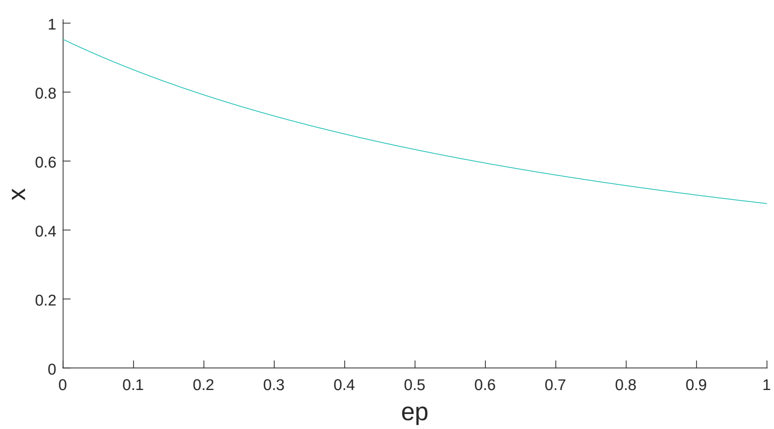

Fig. 3. Effect of parameter ep on the speed of Rayleigh wave x for the open circuit surface

5. CONCLUSIONS

In the present work, we have studied the propagation of Rayleigh waves in transversely isotropic piezoelastic nonlocal materials. Some important features are drawn below:

(i) Under certain type of specific boundary condition: the surface $x_3 = 0$ is considered to be stress-free and maintained at charge-free condition, the dispersion equation of the Rayleigh waves is given. It is numerically concluded that the nonlocality has significant effect on the speed of Rayleigh wave.

(ii) Phase velocities of plane waves are computed numerically and their variation on the incident angle θ, dimensionless frequency parameter ep, are presented graphically. The effect the nonlocality on the velocities of plane waves are also expressed through numerical example and the effect is also significant.

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APPENDIX

The coefficients of characteristic equation

$$h_4 = -b_9 b_3^2 + 2b_3 b_4 b_7 - b_5 b_4^2 - b_2 b_7^2 - 2b_1 b_8 b_7 + b_1 b_5 b_{10} + b_1 b_6 b_9 + b_2 b_5 b_9,$$

$$h_2 = -b_3^2 b_{10} + 2b_3 b_4 b_8 - b_6 b_4^2 - b_1 b_8^2 - 2b_2 b_7 b_8 + b_1 b_6 b_{10} + b_2 b_5 b_{10} + b_2 b_6 b_9,$$

$$h_0 = b_2 b_6 b_{10} - b_2 b_8^2, h_6 = b_1 b_5 b_9 - b_1 b_7^2, b_1 = c_{44} - Xk^2\epsilon^2, b_2 = c_{11} - X - Xk^2\epsilon^2,$$

$$b_3 = c_{13} + c_{44}, b_4 = e_{15} + e_{31}, b_5 = c_{33} - Xk^2\epsilon^2, b_6 = c_{44} - X - Xk^2\epsilon^2,$$

$$b_7 = e_{33}, b_8 = e_{15}, b_9 = -\epsilon_{33}, b_{10} = -\epsilon_{11}, X = \rho c^2.$$