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HOMOGENIZATION OF VERY ROUGH THREE-DIMENSIONAL INTERFACES FOR THE POROELASTICITY THEORY WITH BIOT'S MODEL

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Abstract. In this paper, we carry out the homogenization of a very rough three-dimensional interface separating two dissimilar generally anisotropic poroelastic solids modeled by the Biot theory. The very rough interface is assumed to be a cylindrical surface that rapidly oscillates between two parallel planes, and the motion is time-harmonic. Using the homogenization method with the matrix formulation of the poroelasicity theory, the explicit homogenized equations have been derived. Since the obtained homogenized equations are totally explicit, they are very convenient for solving various practical problems. As an example proving this, the reflection and transmission of SH waves at a very rough interface of tooth-comb type are considered. The closed-form analytical expressions of the reflection and transmission coefficients have been derived. Based on them, the effect of the incident angle and some material parameters on the reflection and transmission coefficients are examined numerically.

Keywords: homogenization; homogenized equations; very rough interfaces; fluid-saturated porous media.

1. INTRODUCTION

The homogenization of very rough interfaces and boundaries is used to analyze the asymptotic behavior of various theories of the continuum mechanics in domains including a very rough interface or a very rough boundary [1]. It is shown that such an interface and a boundary can be replaced by an equivalent layer within which homogenized equations hold [2]. The main aim of the homogenization of very rough boundaries or very rough interfaces is to determine these homogenized equations.

Nevard and Keller [2] considered the homogenization of three-dimensional interfaces separating two generally anisotropic solids. The homogenized equations have been derived, however, they are still implicit. Gilbert and Ou [3] investigated the homogenization of a very rough three-dimensional interface that separates two dissimilar isotropic poroelastic solids and rapidly oscillates between two parallel planes. The motion of the solids is assumed to be time-harmonic. The homogenized equations have been obtained, but they are also still in implicit form. It should be noted that, for deriving the homogenized equations, Nevard and Keller [2], Gilbert and Ou [3] start from basic equations in component form of the elasticity theory and the poroelasticity theory, respectively.

Using the matrix formulation (not the component formulation) of theories, Vinh and his coworkers carried out the homogenization of two-dimensional very rough interfaces and the explicit homogenized equations have been obtained for the elasticity theory [4–7], for the piezoelectricity theory [8], for the micropolar elasticity [9] and for the poroelasticity with Auriault's model for time-harmonic motions [10].

A cylindrical surface with a very rough right section is a three-dimensional very rough interface (see Fig. 1), and it appears frequently in practical problems. The homogenization of a such interface, called a very rough cylindrical interface, is therefore necessary and significant in practical applications. Recall that, a right section of a cylindrical surface is the intersection of it with a plane perpendicular to its generatrices.

In this paper, we carry out the homogenization of a very rough cylindrical interface that separates two dissimilar generally anisotropic poroelastic solids with time-harmonic motion, and it oscillates between two parallel planes. When the motion of the poroelastic solids is the same along the direction perpendicular to the plane of right section of the very rough cylindrical interface, the problem is reduced to the homogenization of a two-dimensional very rough interface which is the right section (directrix) of the very rough cylindrical interface. Therefore, this paper can be considered as an extension of the investigation by Vinh et al. [10].

There exist two models describing the motion of poroelastic solids: Biot's model [11, 12] and Auriault's model [13, 14]. In Biot's model, the coefficients of equations governing the motion of poroelastic solids are known. Meanwhile, as Auriault's model takes into account the detailed micro-structures of pores including fluid, in order to determine the coefficients of governing equations (homogenized equations) we have to solve numerically the corresponding cell problem, and then apply the homogenization techniques. Therefore, Biot's model is more convenient in use. In this paper, the motion of poroelastic solids is assumed to be governed by the Biot theory [11, 12].

To carry out the homogenization of the very rough cylindrical interface, first, the basic equations and the continuity conditions of the linear theory of anisotropic poroelasticity are written in matrix form. Then, by using an appropriate asymptotic expansion of the solution and following standard techniques of the homogenization method, the explicit homogenized equation and the explicit associate continuity conditions in matrix form are derived.

Since the obtained homogenized equations are totally explicit, i.e. their coefficients are explicit functions of given material and interface parameters, they are of great convenience in solving practical problems. To prove this, the reflection and transmission of SH waves at a very rough interface of tooth-comb type are considered. The closed-form analytical expressions of the reflection and transmission coefficients are obtained. Based on them the dependence of the reflection and transmission coefficients on some parameters is investigated numerically.

2. BASIC EQUATIONS IN MATRIX FORM

Consider an anisotropic poroelastic medium in which the pore fluid is Newtonian and incompressible. According to Biot [11], the basic equations governing the time-harmonic motion of the poroelastic medium are:

$$\operatorname{div}\boldsymbol{\Sigma} + \mathbf{f} = -\omega^2\big[\rho\mathbf{u} + \rho_L\mathbf{w}\big], \tag{1}$$

$$\mathbf{w} = \hat{\mathbf{K}}[-i\omega\rho_L\mathbf{u} + \frac{i}{\omega}\operatorname{grad}p], \tag{2}$$

$$\boldsymbol{\Sigma} = \mathbf{C}\mathbf{e}(\mathbf{u}) - \boldsymbol{\alpha}p, \tag{3}$$

$$\operatorname{div}\mathbf{w} = -\boldsymbol{\alpha} : \mathbf{e}(\mathbf{u}) - \beta p, \tag{4}$$

where $\boldsymbol{\Sigma} = (\sigma_{mn})$ represents the total stress tensor, $\mathbf{C} = (c_{mn})$ is the elasticity tensor of the skeleton, $\boldsymbol{\alpha} = (\alpha_{ij})$ is the Biot effective stress coefficient (tensor), β is the inverse of the Biot modulus reflecting compressibility of the fluid and of the skeleton, p is the fluid pressure (positive for compression), $\mathbf{u} = (u_m)$ is the displacement of the solid part, $\mathbf{w} = f(\mathbf{U}_L - \mathbf{u})$ is the displacement of the fluid relative to the solid skeleton, $\mathbf{w} = (w_m)$, $\mathbf{U}_L$ is the displacement of the fluid part, $\mathbf{e}(\mathbf{u}) = (e_{mn})$ is the strain tensor: $e_{mn} = \dfrac{1}{2}(u_{m,n} + u_{n,m})$, commas indicate differentiation with respect to spatial variables x_m, f is the porosity, $\rho = (1-f)\rho_s + f\rho_L$ is the composite mass density, ρ_L is the mass density of the pore fluid, ρ_s is the mass density of the skeleton, $\hat{\mathbf{K}} = (\hat{k}_{mn}) = [\mathbf{K}^{-1} + i\omega\rho_w\mathbf{I}]^{-1}$, $\rho_w = f^{-1}\rho_L$, $\mathbf{K} = (k_{mn})$ is the generalized Darcy permeability tensor, symmetric and ω-dependent, $\mathbf{f} = (f_m)$ is the volume force acting on the solid part.

From (2), we have

$$w_m = -\hat{\alpha}_{mn}u_n + \frac{i}{\omega}\hat{k}_{mn}p_{,n},\ \ \hat{\alpha}_{mn} = i\omega\rho_L\hat{k}_{mn} = \hat{\alpha}_{nm}. \tag{5}$$

Substitution of Eq. (5) into Eqs. (1) and (4) leads to four equations for unknowns u_1, u_2, u_3 and p, namely

$$\sigma_{mn,n} + \omega^2\hat{\rho}_{mn}u_n + \hat{\alpha}_{mn}p_{,n} + f_m = 0,\ \ m = 1,2,3 \tag{6}$$

$$\big[\hat{k}_{mn}\big(p_{,n} - \omega^2\rho_L u_n\big)\big]_{,m} = i\omega\alpha_{mn}u_{m,n} + i\omega\beta p, \tag{7}$$

where $\hat{\rho}_{mn} = \rho\delta_{mn} - \rho_L\hat{\alpha}_{mn} = \hat{\rho}_{nm}$ and σ_{ij} are expressed in terms of u_1, u_2, u_3 and p by (3).

Four equations $\{(6), (7)\}$ can be written in matrix form as follows

$$\begin{aligned}&\big(\mathbf{A}_{11}\mathbf{v}_{,1} + \mathbf{A}_{12}\mathbf{v}_{,2} + \mathbf{A}_{13}\mathbf{v}_{,3} + \mathbf{A}_{14}\mathbf{v}\big)_{,1} + \big(\mathbf{A}_{21}\mathbf{v}_{,1} + \mathbf{A}_{22}\mathbf{v}_{,2} + \mathbf{A}_{23}\mathbf{v}_{,3} + \mathbf{A}_{24}\mathbf{v}\big)_{,2}\\ &+ \big(\mathbf{A}_{31}\mathbf{v}_{,1} + \mathbf{A}_{32}\mathbf{v}_{,2} + \mathbf{A}_{33}\mathbf{v}_{,3} + \mathbf{A}_{34}\mathbf{v}\big)_{,3} + \mathbf{B}\mathbf{v}_{,1} + \mathbf{G}\mathbf{v}_{,2} + \mathbf{D}\mathbf{v}_{,3} + \mathbf{E}\mathbf{v} + \mathbf{F} = \mathbf{0},\end{aligned} \tag{8}$$

where $\mathbf{v} = [u_1\ u_2\ u_3\ p]^T$, $\mathbf{F} = [f_1\ f_2\ f_3\ 0]^T$, the symbol "$T$" indicates the transpose of a matrix and matrices $\mathbf{A}_{hk}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{D}$ and $\mathbf{E}$ are given by

$$\mathbf{A}_{11} = \begin{bmatrix} c_{11} & c_{16} & c_{15} & 0\\ c_{16} & c_{66} & c_{56} & 0\\ c_{15} & c_{56} & c_{55} & 0\\ 0 & 0 & 0 & \hat{k}_{11}\end{bmatrix},\ \ \mathbf{A}_{12} = \begin{bmatrix} c_{16} & c_{12} & c_{14} & 0\\ c_{66} & c_{26} & c_{46} & 0\\ c_{56} & c_{25} & c_{45} & 0\\ 0 & 0 & 0 & \hat{k}_{12}\end{bmatrix},$$

$$\mathbf{A}_{13} = \begin{bmatrix} c_{15} & c_{14} & c_{13} & 0 \\ c_{56} & c_{46} & c_{36} & 0 \\ c_{55} & c_{45} & c_{35} & 0 \\ 0 & 0 & 0 & \hat{k}_{13} \end{bmatrix}, \ \mathbf{A}_{14} = \begin{bmatrix} 0 & 0 & 0 & -\alpha_{11} \\ 0 & 0 & 0 & -\alpha_{12} \\ 0 & 0 & 0 & -\alpha_{13} \\ i\omega\hat{\alpha}_{11} & i\omega\hat{\alpha}_{12} & i\omega\hat{\alpha}_{13} & 0 \end{bmatrix},$$

$$\mathbf{A}_{21} = \begin{bmatrix} c_{16} & c_{66} & c_{56} & 0 \\ c_{12} & c_{26} & c_{25} & 0 \\ c_{14} & c_{46} & c_{45} & 0 \\ 0 & 0 & 0 & \hat{k}_{12} \end{bmatrix}, \ \mathbf{A}_{22} = \begin{bmatrix} c_{66} & c_{26} & c_{46} & 0 \\ c_{26} & c_{22} & c_{24} & 0 \\ c_{46} & c_{24} & c_{44} & 0 \\ 0 & 0 & 0 & \hat{k}_{22} \end{bmatrix},$$

$$\mathbf{A}_{23} = \begin{bmatrix} c_{56} & c_{46} & c_{36} & 0 \\ c_{25} & c_{24} & c_{23} & 0 \\ c_{45} & c_{44} & c_{34} & 0 \\ 0 & 0 & 0 & \hat{k}_{23} \end{bmatrix}, \ \mathbf{A}_{24} = \begin{bmatrix} 0 & 0 & 0 & -\alpha_{12} \\ 0 & 0 & 0 & -\alpha_{22} \\ 0 & 0 & 0 & -\alpha_{23} \\ i\omega\hat{\alpha}_{12} & i\omega\hat{\alpha}_{22} & i\omega\hat{\alpha}_{23} & 0 \end{bmatrix},$$

$$\mathbf{A}_{31} = \begin{bmatrix} c_{15} & c_{56} & c_{55} & 0 \\ c_{14} & c_{46} & c_{45} & 0 \\ c_{13} & c_{36} & c_{35} & 0 \\ 0 & 0 & 0 & \hat{k}_{13} \end{bmatrix}, \ \mathbf{A}_{32} = \begin{bmatrix} c_{56} & c_{25} & c_{45} & 0 \\ c_{46} & c_{24} & c_{44} & 0 \\ c_{36} & c_{23} & c_{34} & 0 \\ 0 & 0 & 0 & \hat{k}_{23} \end{bmatrix}, \tag{9}$$

$$\mathbf{A}_{33} = \begin{bmatrix} c_{55} & c_{45} & c_{35} & 0 \\ c_{45} & c_{44} & c_{34} & 0 \\ c_{35} & c_{34} & c_{33} & 0 \\ 0 & 0 & 0 & \hat{k}_{33} \end{bmatrix}, \ \mathbf{A}_{34} = \begin{bmatrix} 0 & 0 & 0 & -\alpha_{13} \\ 0 & 0 & 0 & -\alpha_{23} \\ 0 & 0 & 0 & -\alpha_{33} \\ i\omega\hat{\alpha}_{13} & i\omega\hat{\alpha}_{23} & i\omega\hat{\alpha}_{33} & 0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \hat{\alpha}_{11} \\ 0 & 0 & 0 & \hat{\alpha}_{12} \\ 0 & 0 & 0 & \hat{\alpha}_{13} \\ -i\omega\alpha_{11} & -i\omega\alpha_{12} & -i\omega\alpha_{13} & 0 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & \hat{\alpha}_{12} \\ 0 & 0 & 0 & \hat{\alpha}_{22} \\ 0 & 0 & 0 & \hat{\alpha}_{23} \\ -i\omega\alpha_{12} & -i\omega\alpha_{22} & -i\omega\alpha_{23} & 0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & \hat{\alpha}_{13} \\ 0 & 0 & 0 & \hat{\alpha}_{23} \\ 0 & 0 & 0 & \hat{\alpha}_{33} \\ -i\omega\alpha_{13} & -i\omega\alpha_{23} & -i\omega\alpha_{33} & 0 \end{bmatrix}, \ \mathbf{E} = \omega^2 \begin{bmatrix} \hat{\rho}_{11} & \hat{\rho}_{12} & \hat{\rho}_{13} & 0 \\ \hat{\rho}_{12} & \hat{\rho}_{22} & \hat{\rho}_{23} & 0 \\ \hat{\rho}_{13} & \hat{\rho}_{23} & \hat{\rho}_{33} & 0 \\ 0 & 0 & 0 & -i\beta/\omega \end{bmatrix}.$$

3. CONTINUITY CONDITIONS IN MATRIX FORM

Consider a linear poroelastic body that occupies three-dimensional domains Ω^+, Ω^-, their interface is a very rough cylindrical surface, whose generatrices are parallel to $0x_2$ and its right section (directrix) L, belong to the plane $x_2 = 0$, is expressed by equation $x_3 = h(y)$, $y = x_1/\epsilon$ $(\epsilon > 0)$, where $h(y)$ is a periodic function of period 1 (see Fig. 1). Suppose that the interface oscillates between two planes $x_3 = -A$ $(A > 0)$ and $x_3 = 0$, and in the plane $x_2 = 0$: in the domain $0 < x_1 < \epsilon$ (i.e. $0 < y < 1$), any straight

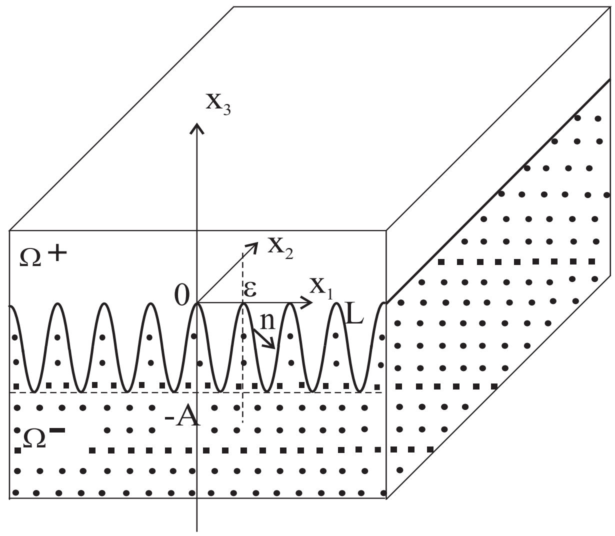

Fig. 1. Three-dimensional domains Ω^+ and Ω^- are separated by a very rough cylindrical surface whose generatrices are parallel to $0x_2$ and its right section (directrix) L (belong to the plane $x_2 = 0$) is expressed by equation $x_3 = h(y), y = x_1/\epsilon$, $h(y)$ is a periodic function of period 1

line $x_3 = x_3^0 = \text{const}$ $(-A < x_3^0 < 0)$ has exactly two intersections with the right section L. Let $0 < \epsilon \ll 1$, then the interface is called very rough interface of Ω^+ and Ω^-. Suppose that the domains Ω^+, Ω^- are occupied by different homogeneous poroelastic materials. In particular, the material parameters are defined as

$$c_{ij},\ k_{ij},\ \alpha,\ \beta,\ f,\ \rho_s,\ \rho_w,\ \rho_L = \begin{cases} c_{ij+},\ k_{ij+},\ \alpha_+,\ \beta_+,\ f_+,\ \rho_{s+},\ \rho_{w+},\ \rho_{L+},\ \ x_3 > h(\dfrac{x_1}{\epsilon}) \\ c_{ij-},\ k_{ij-},\ \alpha_-,\ \beta_-,\ f_-,\ \rho_{s-},\ \rho_{w-},\ \rho_{L-},\ \ x_3 < h(\dfrac{x_1}{\epsilon}) \end{cases} \tag{10}$$

where $c_{ij+}, \ldots, \rho_{L+}, c_{ij-}, \ldots, \rho_{L-}$ are constant. Correspondingly, the matrices $\mathbf{A}_{kh}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{D}$, $\mathbf{E}$ are given by

$$\mathbf{A}_{kh}, \mathbf{B}, \mathbf{G}, \mathbf{D}, \mathbf{E} = \begin{cases} \mathbf{A}_{kh}^{(+)}, \mathbf{B}^{(+)}, \mathbf{G}^{(+)}, \mathbf{D}^{(+)}, \mathbf{E}^{(+)} \text{ for } x_3 > h(\dfrac{x_1}{\epsilon}) \\ \mathbf{A}_{kh}^{(-)}, \mathbf{B}^{(-)}, \mathbf{G}^{(-)}, \mathbf{D}^{(-)}, \mathbf{E}^{(-)} \text{ for } x_3 < h(\dfrac{x_1}{\epsilon}) \end{cases} \tag{11}$$

where $\mathbf{A}_{kh}^{(+)}, \ldots, \mathbf{E}^{(+)}$ $\left(\mathbf{A}_{kh}^{(-)}, \ldots, \mathbf{E}^{(-)}\right)$ are expressed by (9) in which $c_{ij}, \ldots, \rho_L$ are replaced by $c_{ij+}, \ldots, \rho_{L+}$ $\left(c_{ij-}, \ldots, \rho_{L-}\right)$, respectively. Note that matrices $\mathbf{A}_{kh}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{D}$, $\mathbf{E}$ do not depend on x_2.

Suppose that Ω^+, Ω^- are perfectly welded to each other along L. Then, the continuity condition is of the form

$$\begin{aligned} &[\,u_i\,]_L = 0,\ i = 1,2,3,\ [\,p\,]_L = 0, \\ &[\sigma_{ik} n_k]_L = 0,\ i = 1,2,3,\ [i\omega w_k n_k]_L = 0, \end{aligned} \tag{12}$$

where n_k is the x_k-component of the unit normal to the curve (right section) L, and we introduce the notation $[\,.\,]_L$, defined such as: $[\,\mathbf{f}\,]_L = \mathbf{f}^+ - \mathbf{f}^-$ on L.

In view of (3) and (5), in matrix form the continuity condition (12) takes the form

$$\begin{aligned}&\big[\mathbf{v}\big]_L = \mathbf{0},\ \Big[\big(\mathbf{A}_{11}\mathbf{v}_{,1} + \mathbf{A}_{12}\mathbf{v}_{,2} + \mathbf{A}_{13}\mathbf{v}_{,3} + \mathbf{A}_{14}\mathbf{v}\big)n_1\\ &+ \big(\mathbf{A}_{31}\mathbf{v}_{,1} + \mathbf{A}_{32}\mathbf{v}_{,2} + \mathbf{A}_{33}\mathbf{v}_{,3} + \mathbf{A}_{34}\mathbf{v}\big)n_3\Big]_L = \mathbf{0}.\end{aligned} \tag{13}$$

4. EXPLICIT HOMOGENIZED EQUATION IN MATRIX FORM

Following Bensoussan et al. [15] we suppose that $\mathbf{v}(x_1, x_2, x_3, \epsilon) = \mathbf{U}(x_1, y, x_2, x_3, \epsilon)$, and we express $\mathbf{U}$ as follows (see Vinh et al. [4–6, 8])

$$\begin{aligned}\mathbf{U} = \mathbf{V} + \epsilon\Big(\mathbf{N}^1\mathbf{V} + \mathbf{N}^{11}\mathbf{V}_{,1} + \mathbf{N}^{12}\mathbf{V}_{,2} + \mathbf{N}^{13}\mathbf{V}_{,3}\Big) + \epsilon^2\big(\mathbf{N}^2\mathbf{V} + \mathbf{N}^{21}\mathbf{V}_{,1} + \mathbf{N}^{22}\mathbf{V}_{,2} + \mathbf{N}^{23}\mathbf{V}_{,3}\\ + \mathbf{N}^{211}\mathbf{V}_{,11} + \mathbf{N}^{212}\mathbf{V}_{,12} + \mathbf{N}^{213}\mathbf{V}_{,13} + \mathbf{N}^{222}\mathbf{V}_{,22} + \mathbf{N}^{223}\mathbf{V}_{,23} + \mathbf{N}^{233}\mathbf{V}_{,33}\big) + O(\epsilon^3),\end{aligned} \tag{14}$$

where $\mathbf{V} = \mathbf{V}(x_1, x_2, x_3)$ (being independent of y), $\mathbf{N}^1$, $\mathbf{N}^{11}$, $\mathbf{N}^{12}$, $\mathbf{N}^{13}$, $\mathbf{N}^2$, $\mathbf{N}^{21}$, $\mathbf{N}^{22}$, $\mathbf{N}^{23}$, $\mathbf{N}^{211}$, $\mathbf{N}^{212}$, $\mathbf{N}^{213}$, $\mathbf{N}^{222}$, $\mathbf{N}^{223}$, $\mathbf{N}^{233}$ are 4×4-matrix valued functions of y and x_3 (not depending on x_1, x_2), and they are y-periodic with period 1. Since $y = x_1/\epsilon$, we have $\mathbf{v}_{,1} = \mathbf{U}_{,1} + \epsilon^{-1}\mathbf{U}_{,y}$.

Following the same procedure as the one carried out by Vinh et al. [9], one can derive the explicit homogenized equation (equation for $\mathbf{V}$) in matrix form of Eq. (8), namely

- *For* $x_3 > 0$:

$$\begin{aligned}\mathbf{A}_{hk}^{(+)}\mathbf{V}_{,kh} + \big(\mathbf{A}_{14}^{(+)} + \mathbf{B}^{(+)}\big)\mathbf{V}_{,1} + \big(\mathbf{A}_{24}^{(+)} + \mathbf{G}^{(+)}\big)\mathbf{V}_{,2}\\ + \big(\mathbf{A}_{34}^{(+)} + \mathbf{D}^{(+)}\big)\mathbf{V}_{,3} + \mathbf{E}^{(+)}\mathbf{V} + \mathbf{F}^{(+)} = \mathbf{0}.\end{aligned} \tag{15}$$

- *For* $x_3 < -A$:

$$\begin{aligned}\mathbf{A}_{hk}^{(-)}\mathbf{V}_{,kh} + \big(\mathbf{A}_{14}^{(-)} + \mathbf{B}^{(-)}\big)\mathbf{V}_{,1} + \big(\mathbf{A}_{24}^{(-)} + \mathbf{G}^{(-)}\big)\mathbf{V}_{,2}\\ + \big(\mathbf{A}_{34}^{(-)} + \mathbf{D}^{(-)}\big)\mathbf{V}_{,3} + \mathbf{E}^{(-)}\mathbf{V} + \mathbf{F}^{(-)} = \mathbf{0}.\end{aligned} \tag{16}$$

- *For* $-A < x_3 < 0$:

$$\begin{aligned}&\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\mathbf{V}_{,11} + \Big[\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle + \langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\Big]\mathbf{V}_{,12} + \langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle\mathbf{V}_{,13}\\
&+ \Big[\langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\mathbf{V}_{,1}\Big]_{,3} + \Big[\langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle - \langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle + \langle\mathbf{A}_{22}\rangle\Big]\mathbf{V}_{,22}\\
&+ \Big[\langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle - \langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle + \langle\mathbf{A}_{23}\rangle\Big]\mathbf{V}_{,23} + \Big[\Big(\langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle\\
&- \langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle + \langle\mathbf{A}_{32}\rangle\Big)\mathbf{V}_{,2}\Big]_{,3} + \Big[\Big(\langle\mathbf{A}_{33}\rangle + \langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle - \langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle\Big)\mathbf{V}_{,3}\Big]_{,3}\\
&+ \Big[\langle\mathbf{B}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1} + \langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle\Big]\mathbf{V}_{,1} + \Big[\langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle - \langle\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle\\
&+ \langle\mathbf{A}_{24}\rangle + \langle\mathbf{B}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle - \langle\mathbf{B}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle + \langle\mathbf{G}\rangle\Big]\mathbf{V}_{,2} + \Big[\langle\mathbf{D}\rangle + \langle\mathbf{B}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle\\
&- \langle\mathbf{B}\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle\Big]\mathbf{V}_{,3} + \Big[\Big(\langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle - \langle\mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle + \langle\mathbf{A}_{34}\rangle\Big)\mathbf{V}\Big]_{,3}\\
&+ \Big[\langle\mathbf{E}\rangle + \langle\mathbf{B}\mathbf{A}_{11}^{-1}\rangle\langle\mathbf{A}_{11}^{-1}\rangle^{-1}\langle\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle - \langle\mathbf{B}\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle\Big]\mathbf{V} + \langle\mathbf{F}\rangle = 0.\end{aligned} \tag{17}$$

The associate continuity conditions are of the form

$$[\mathbf{V}]_{L^*} = 0,\ [\mathbf{\Sigma}_3^0]_{L^*} = 0,\ L^* : x_3 = 0,\ x_3 = -A, \tag{18}$$

where

$$\begin{aligned}\mathbf{\Sigma}_3^0 = &\Big[\langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle \mathbf{A}_{11}^{-1}\rangle^{-1}\langle \mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle - \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{14}\rangle + \langle \mathbf{A}_{34}\rangle\Big]\mathbf{V}\\ &+ \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle \mathbf{A}_{11}^{-1}\rangle^{-1}\mathbf{V}_{,1} + \Big[\langle \mathbf{A}_{32}\rangle + \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle \mathbf{A}_{11}^{-1}\rangle^{-1}\langle \mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle\\ &- \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\rangle\Big]\mathbf{V}_{,2} + \Big[\langle \mathbf{A}_{33}\rangle + \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\rangle\langle \mathbf{A}_{11}^{-1}\rangle^{-1}\langle \mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle - \langle \mathbf{A}_{31}\mathbf{A}_{11}^{-1}\mathbf{A}_{13}\rangle\Big]\mathbf{V}_{,3},\end{aligned} \tag{19}$$

and

$$\langle \varphi \rangle = \int_0^1 \varphi dy = (y_2 - y_1)\varphi^+ + (1 - y_2 + y_1)\varphi^-. \tag{20}$$

It is readily to verify that, when the motion of the poroelastic solids is the same along the generatrix direction $0x_2$, i.e. $\mathbf{V}$ does not depend on x_2, the homogenized equation (17) is simplified to Eq. (27) in Vinh et al. [10]. It should be noted that the matrices $\mathbf{A}_{ik}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{E}$ in Eq. (17) (corresponding to Biot's model) are not equal to the matrices $\mathbf{A}_{ik}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{E}$, respectively, in Eq. (27) in Vinh et al. [10] (corresponding to Auriault's model), in general.

5. REFLECTION AND REFRACTION OF SH WAVE WITH A VERY ROUGH INTERFACE OF TOOTH-COMB TYPE

In this section we consider the reflection and transmission of SH waves ($u_1 \equiv u_3 \equiv p \equiv 0, u_2 = u_2(x_1, x_3)$) at a very rough interface of tooth-comb type separating two orthotropic poroelastic half-spaces. By the meaning of homogenization, this problem is reduced to the reflection and transmission of SH waves ($V_1 \equiv V_3 \equiv P \equiv 0, V_2 = V_2(x_1, x_3)$) through a homogeneous material layer occupying the domain $-A \leq x_3 \leq 0$ (see Fig. 2). For orthotropic poroelastic materials, we have [16]

$$\begin{aligned}&c_{k4} = c_{k5} = c_{k6} = 0,\ k = 1, 2, 3,\ c_{45} = c_{46} = c_{56} = 0,\\ &\alpha_{12} = \alpha_{13} = \alpha_{23} = 0,\ k_{12} = k_{13} = k_{23} = 0.\end{aligned} \tag{21}$$

In view of (21), from (5) we have

$$\hat{\alpha}_{12} = \hat{\alpha}_{13} = \hat{\alpha}_{23} = 0,\ \hat{k}_{12} = \hat{k}_{13} = \hat{k}_{23} = 0,\ \hat{\rho}_{12} = \hat{\rho}_{13} = \hat{\rho}_{23} = 0. \tag{22}$$

From Eqs. (15)–(17) and taking into account (21), (22) (without the body forces), the motion of SH waves is governed by the equations

$$c_{66+}V_{2,11} + c_{44+}V_{2,33} + (re_+ - i\, im_+)\, V_2 = 0,\ \text{ for } x_3 > 0, \tag{23}$$

$$c_{66-}V_{2,11} + c_{44-}V_{2,33} + (re_- - i\, im_-)\, V_2 = 0,\ \text{ for } x_3 < -A, \tag{24}$$

$$\langle c_{66}^{-1}\rangle^{-1}V_{2,11} + \langle c_{44}\rangle V_{2,33} + \Big[\langle re\rangle - i\langle im\rangle\Big] V_2 = 0,\ \text{ for } -A < x_3 < 0 \tag{25}$$

where

$$
\begin{aligned}
re_{+} &= \omega^2\Big[\rho_{+} - \frac{\omega^2\rho_{L+}^2\rho_{w+}k_{22+}^2}{1+\omega^2\rho_{w+}^2k_{22+}^2}\Big],\ im_{+} = \frac{\omega^3\rho_{L+}^2k_{22+}}{1+\omega^2\rho_{w+}^2k_{22+}^2},\\
re_{-} &= \omega^2\Big[\rho_{-} - \frac{\omega^2\rho_{L-}^2\rho_{w-}k_{22-}^2}{1+\omega^2\rho_{w-}^2k_{22-}^2}\Big],\ im_{-} = \frac{\omega^3\rho_{L-}^2k_{22-}}{1+\omega^2\rho_{w-}^2k_{22-}^2},\\
\langle re\rangle &= \omega^2\Big[\langle\rho\rangle - \frac{\omega^2\langle\rho_L^2\rho_w k_{22}^2\rangle}{1+\omega^2\langle\rho_w^2k_{22}^2\rangle}\Big],\ \langle im\rangle = \frac{\omega^3\langle\rho_L^2k_{22}\rangle}{1+\omega^2\langle\rho_w^2k_{22}^2\rangle}.
\end{aligned} \tag{26}
$$

In addition to Eqs. (23)–(25), are required the continuity conditions on lines L^*: $x_3 = -A, x_3 = 0$, namely

$$
\big[V_2\big]_{L^*} = 0,\ \ \big[\sigma_{23}^0\big]_{L^*} = 0, \tag{27}
$$

where $\sigma_{23}^0 = \langle c_{44}\rangle V_{2,3}$.

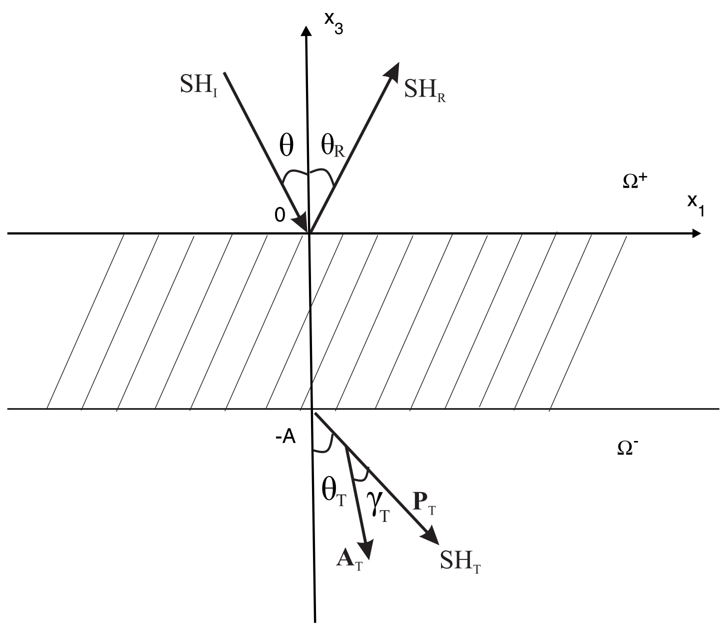

Fig. 2. The reflection and refraction of SH wave with the homogenized layer

Assume that a homogeneous incident SH_I wave with the unit amplitude, the incident angle θ, propagates in the half-space Ω^+ (Fig. 2). When striking at the layer it generates a reflected SH_R wave propagating in the half-space Ω^+ and a refracted SH_T wave traveling in the half-space Ω^-. Following Borcherdt [17], the homogeneous incident SH_I wave, the reflected SH_R wave, the (transmitted) refracted SH_T wave are of the form

$$V_{2I} = e^{-(A_{1I}x_1 + A_{3I}x_3)} e^{-i(P_{1I}x_1 + P_{3I}x_3)}, \tag{28}$$

$$V_{2R} = R\, e^{-(A_{1R}x_1 + A_{3R}x_3)} e^{-i(P_{1R}x_1 + P_{3R}x_3)}, \tag{29}$$

$$V_{2T} = T\, e^{-(A_{1T}x_1 + A_{3T}x_3)} e^{-i(P_{1T}x_1 + P_{3T}x_3)}, \tag{30}$$

where R is the reflection coefficient, T is the refraction coefficient, $\mathbf{P}_I(P_{1I}, P_{3I})$, $\mathbf{P}_R(P_{1R}, P_{3R})$, $\mathbf{P}_T(P_{1T}, P_{3T})$ represent the propagation vectors and $\mathbf{A}_I(A_{1I}, A_{3I})$, $\mathbf{A}_R(A_{1R}, A_{3R})$, $\mathbf{A}_T(A_{1T}, A_{3T})$ represent the attenuation vectors of the homogeneous incident SH_I wave, reflected SH_R wave, refracted SH_T wave, respectively and (see Vinh et al. [10])

$$\begin{aligned} &P_{1I} = P_I \sin\theta, \;\; P_{3I} = -P_I \cos\theta, \;\; P_I = |\mathbf{P}_I|, \\ &A_{1I} = A_I \sin\theta, \;\; A_{3I} = -A_I \cos\theta, \;\; A_I = |\mathbf{A}_I|. \end{aligned} \tag{31}$$

Substituting (28) into Eq. (23) yields

$$A_I = \sqrt{\frac{-re_+ + \sqrt{re_+^2 + im_+^2}}{2(c_{66+}\sin^2\theta + c_{44+}\cos^2\theta)}}, \quad P_I = \sqrt{\frac{re_+ + \sqrt{re_+^2 + im_+^2}}{2(c_{66+}\sin^2\theta + c_{44+}\cos^2\theta)}}. \tag{32}$$

Snell's law gives immediately

$$P_{1I} = P_{1R} = P_{1T}, A_{1I} = A_{1R} = A_{1T}. \tag{33}$$

Substituting Eq. (29) into Eq. (23) and using equalities (33) yield

$$P_{3R} = -P_{3I}, \quad A_{3R} = -A_{3I}. \tag{34}$$

Equalities (31), (33) and (34) say that the refracted SH_R wave is a homogeneous wave with the reflection angle $\theta_R = \theta$ (Fig. 2). Introducing Eq. (30) into Eq. (24) and using equalities (33) lead to

$$\begin{aligned} A_{3T} &= -\sqrt{\frac{-[re_- - c_{66-}(P_{1I}^2 - A_{1I}^2)] + \sqrt{[re_- - c_{66-}(P_{1I}^2 - A_{1I}^2)]^2 + [im_- - 2c_{66-}P_{1I}A_{1I}]^2}}{2c_{44-}}}, \\ P_{3T} &= -\sqrt{\frac{[re_- - c_{66-}(P_{1I}^2 - A_{1I}^2)] + \sqrt{[re_- - c_{66-}(P_{1I}^2 - A_{1I}^2)]^2 + [im_- - 2c_{66-}P_{1I}A_{1I}]^2}}{2c_{44-}}}. \end{aligned} \tag{35}$$

In view of Snell's law, one can see that the general solution of Eq. (25) is given by

$$V_2 = (B_1 \mathrm{e}^{-i\hat{K}_3 x_3} + B_2 e^{i\hat{K}_3 x_3}) e^{-i(P_{1I} - iA_{1I})x_1}, \tag{36}$$

where B_1 and B_2 are constants to be determined and

$$\hat{K}_3 = \sqrt{\frac{\langle re\rangle - \langle c_{66}^{-1}\rangle^{-1}(P_{1I}^2 - A_{1I}^2) - i[\langle im\rangle - 2\langle c_{66}^{-1}\rangle^{-1} P_{1I}A_{1I}]}{\langle c_{44}\rangle}}. \tag{37}$$

It is easy to verify that $\hat{K}_3 = \hat{P}_3 - i\hat{A}_3$ where (real numbers) $\hat{P}_3$, $\hat{A}_3$ are given by

$$\hat{P}_3 = \sqrt{\frac{[\langle re\rangle - \langle c_{66}^{-1}\rangle^{-1}(P_{1I}^2 - A_{1I}^2)] + \sqrt{[\langle re\rangle - \langle c_{66}^{-1}\rangle^{-1}(P_{1I}^2 - A_{1I}^2)]^2 + [\langle im\rangle - 2\langle c_{66}^{-1}\rangle^{-1}P_{1I}A_{1I}]^2}}{2\langle c_{44}\rangle}},$$
$$\hat{A}_3 = \frac{\langle im\rangle - 2\langle c_{66}^{-1}\rangle^{-1}P_{1I}A_{1I}}{2\langle c_{44}\rangle\hat{P}_3}. \tag{38}$$

Using (28)–(30), (36) and the continuity conditions (27) yields a system of four equations for B_1, B_2, R and T, namely

$$\begin{aligned}
&B_1 + B_2 = R + 1,\\
&B_1 - B_2 = -\frac{c_{44+}(A_{3I} + iP_{3I})(1-R)}{\langle c_{44}\rangle(\hat{A}_3 + i\hat{P}_3)},\\
&B_1 e^{-(\hat{A}_3 + i\hat{P}_3)A} + B_2 e^{(\hat{A}_3 + i\hat{P}_3)A} = Te^{(A_{3T}+iP_{3T})A},\\
&B_1 e^{-(\hat{A}_3 + i\hat{P}_3)A} - B_2 e^{(\hat{A}_3 + i\hat{P}_3)A} = -\frac{c_{44-}(A_{3T} + iP_{3T})}{\langle c_{44}\rangle(\hat{A}_3 + i\hat{P}_3)}Te^{(A_{3T}+iP_{3T})A}.
\end{aligned} \tag{39}$$

Solving the system (39) for R and T we obtain closed-form analytical expressions for the reflection and transmission coefficients, namely

$$R = \frac{pr - sn}{mr - qn},\ \ T = \frac{ms - pq}{mr - qn}, \tag{40}$$

where

$$\begin{aligned}
&m = a_1 \mathrm{e}^{-(\hat{A}_3 + i\hat{P}_3)A} + a_2 \mathrm{e}^{(\hat{A}_3 + i\hat{P}_3)A},\ n = -2\mathrm{e}^{(A_{3T}+iP_{3T})A},\\
&p = -\{a_2 \mathrm{e}^{-(\hat{A}_3 + i\hat{P}_3)A} + a_1 \mathrm{e}^{(\hat{A}_3 + i\hat{P}_3)A}\},\ q = a_1 \mathrm{e}^{-(\hat{A}_3 + i\hat{P}_3)A} - a_2 \mathrm{e}^{(\hat{A}_3 + i\hat{P}_3)A},\\
&r = 2\frac{c_{44-}(A_{3T} + iP_{3T})}{\langle c_{44}\rangle(\hat{A}_3 + i\hat{P}_3)}\mathrm{e}^{(A_{3T}+iP_{3T})A},\ s = -\{a_2 \mathrm{e}^{-(\hat{A}_3 + i\hat{P}_3)A} - a_1 \mathrm{e}^{(\hat{A}_3 + i\hat{P}_3)A}\},\\
&a_1 = 1 + \frac{c_{44+}(A_{3I} + iP_{3I})}{\langle c_{44}\rangle(\hat{A}_3 + i\hat{P}_3)},\ a_2 = (2 - a_1).
\end{aligned} \tag{41}$$

From (40) and (41) one can see that R and T depend on 13 dimensionless parameters, namely

$$\begin{aligned}
&\varepsilon_1 = \frac{a}{a+b},\ \varepsilon_2 = \frac{c_{44-}}{c_{44+}},\ \varepsilon_3 = \frac{c_{66+}}{c_{44+}},\ \varepsilon_4 = \frac{\omega^2\rho_+ A^2}{c_{44+}},\ \varepsilon_5 = \omega\rho_+ k_{22+},\ \varepsilon_6 = \frac{\rho_{L+}}{\rho_+},\\
&\varepsilon_7 = \frac{c_{66-}}{c_{44-}},\ \varepsilon_8 = \frac{\omega^2\rho_- A^2}{c_{44-}},\ \varepsilon_9 = \omega\rho_- k_{22-},\ \varepsilon_{10} = \frac{\rho_{L-}}{\rho_-},\ \theta,\ f_1,\ f_2.
\end{aligned} \tag{42}$$

Using formulas (40), (41) we consider the dependence of the moduli $|R|$ and $|T|$ of the reflection and refraction coefficients on some dimensionless parameters.

It can be seen from Fig. 3 that:

(i) When the incident angle θ_0 increases, moduli $|R|$, $|R_0|$ increase and moduli $|T|$, $|T_0|$ decrease, $|R| < |R_0|, |T| > |T_0|$ in which $|R|, |T|$, $(|R_0|, |T_0|)$ are the reflection, refraction coefficients with the rough interface, (without the rough interface) (see Fig. 3(a)).

(ii) The increasing of ε_1, ε_2 makes the reflection coefficient increasing and makes the transmission coefficient decreasing (see Figs. 3(b), 3(c)).

(iii) In contrast, the increasing of ε_4 makes the reflection coefficient decreasing and makes the transmission coefficient increasing (see Fig. 3(d)).

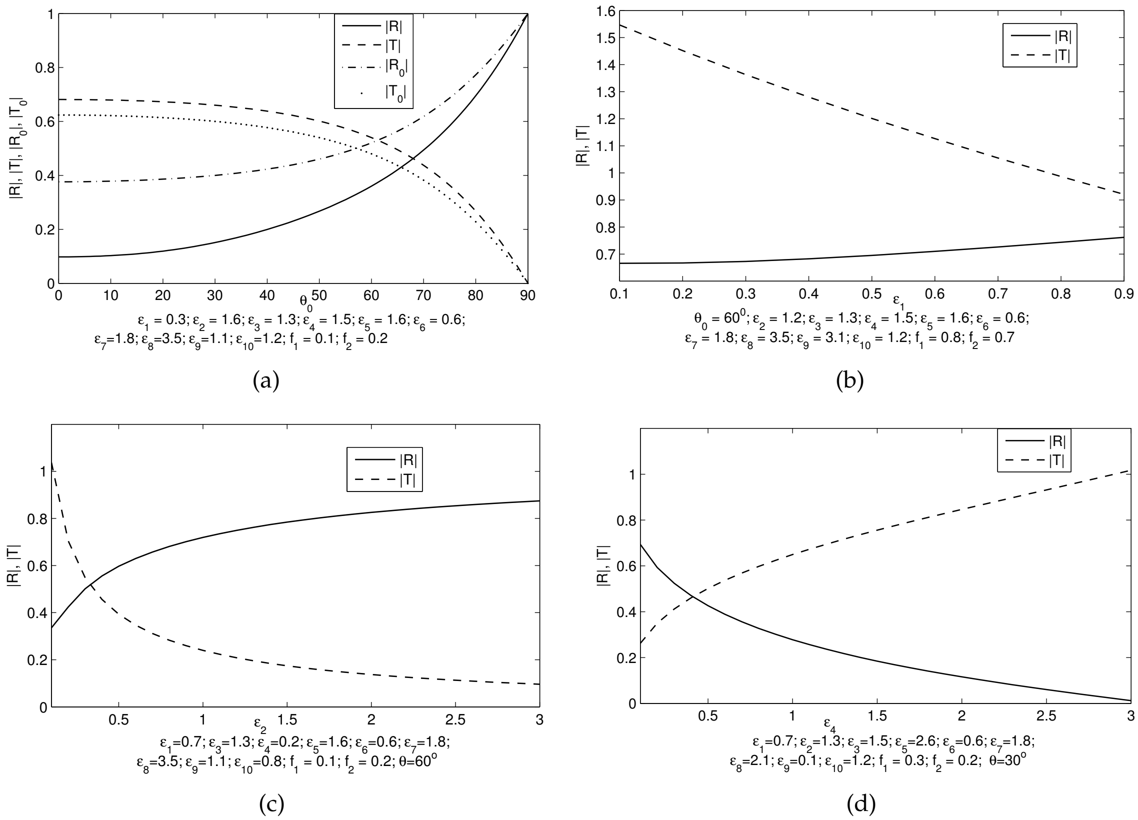

Fig. 3. The dependence of the moduli $|R|$ and $|T|$ of the reflection and transmission coefficients on θ_0 (a), ε_1 (b), ε_2 (c), ε_4 (d)

6. CONCLUSIONS

In this paper the homogenization of a very rough cylindrical interface that separates two dissimilar generally anisotropic poroelastic solids with time-harmonic motion, and oscillates rapidly between two parallel planes is investigated. The explicit homogenized equation in matrix form has been derived by applying the homogenization method. Since the obtained homogenized equations are fully explicit, they are a powerful tool for investigating various practical problems. As an example, the reflection and transmission of

SH waves at a very rough interface of tooth-comb type are considered. The closed-form analytical expressions of the reflection and transmission coefficients have been obtained. Employing them, the effect of the incident angle and the material parameters on the reflection and transmission coefficients is investigated numerically.

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