Vietnam Journal of Mechanics, VAST, Vol. 41, No. 2 (2019), pp. 179 – 192
DOI: https://doi.org/10.15625/0866-7136/13306

NONLINEAR DYNAMIC BUCKLING OF FULL-FILLED FLUID SANDWICH FGM CIRCULAR CYLINDER SHELLS

Khuc Van Phu[1], **Le Xuan Doan**[2,*]
[1]*Military Academy of Logistics, Hanoi, Vietnam*
[2]*Academy of Military Science and Technology, Hanoi, Vietnam*
*E-mail: xuandoan1085@gmail.com
Received: 17 November 2018 / Published online: 2 June 2019

Abstract. This paper is concerned with the nonlinear dynamic buckling of sandwich functionally graded circular cylinder shells filled with fluid. Governing equations are derived using the classical shell theory and the geometrical nonlinearity in von Karman–Donnell sense is taken into account. Solutions of the problem are established by using Galerkin's method and Runge–Kutta method. Effects of thermal environment, geometric parameters, volume fraction index k and fluid on dynamic critical loads of shells are investigated.

Keywords: dynamic buckling; dynamic critical loads; FGM-sandwich; full-filled fluid; circular cylinder shell.

1. INTRODUCTION

In recent years, functionally graded material (FGM) have been widely used in many industry due to outstanding characteristics. Plate and shell structures have received considerable attention of scientists in the world. In studies, vibration and dynamic stability of FGM shells are problems interested and achieved encouraging results.

On vibration of shells, Bich and Nguyen [1] studied nonlinear responses of a functionally graded (FG) circular cylinder shell under mechanical loads. Governing equations were based on improved Donnell shell theory. Kim [2] used an analytical method to study natural frequencies of circular cylinder shells made of FGM partially embedded in an elastic medium with an oblique edge based on the first order shear deformation theory (FSDT). In recent times, Duc et al. investigated nonlinear dynamic responses and vibration of imperfect eccentrically stiffened functionally graded thick circular cylindrical shells [3] and the one [4] surrounded on elastic foundation subjected to mechanical and thermal loads. The FSDT and the third order shear deformation theory (TSDT) were employed to solve problems. Bahadori and Najafizadeh [5] analyzed free vibration frequencies of two-dimensional FG axisymmetric circular cylindrical shells resting on Winkler–Pasternak elastic foundations. The Navier-Differential Quadrature solution methods was employed to survey.

Regarding to dynamic buckling problems, Bich et al. [6] based on the classical shell theory and the smeared stiffeners technique to study nonlinear dynamics responses of eccentrically stiffened FG cylindrical panels. The nonlinear static and dynamic buckling problems of imperfect eccentrically stiffened FG thin circular cylinder shells under axial compression load were solved in [7]. Mirzavand et al. [8] studied the post-buckling behavior of FG circular cylinder shells with surface-bonded piezoelectric actuators under the combined action of thermal load and applied actuator voltage. Duc et al. [9, 10] used the TSDT to analyze nonlinear static buckling and post-buckling for imperfect eccentrically stiffened thin and thick FG circular cylinder shells made of S-FGM resting on elastic foundations under thermal-mechanical loads. Lekhnitsky smeared stiffeners technique and Bubnov–Galerkin method were applied in calculation. By using an analytical approach, based on improved Donnell shell theory with von Karman–Donnell geometrical nonlinearity, Bich et al. [11] investigated the buckling and post-buckling of FG circular cylinder shells under mechanical loads including effects of temperature. Nonlinear buckling problems of imperfect eccentrically stiffened FG thin circular cylindrical shells subjected to axial compression load and surrounded by an elastic foundation were solved by Nam et al. [12]. The classical thin shell theory with the von Karman–Donnell geometrical nonlinearity, initial geometrical imperfection and the smeared stiffeners technique were employed to study.

For circular cylindrical shells made of FGM filled with fluid, Sheng et al. [13] based on the FSDT to study free vibration characteristics of FG circular cylinder shells with flowing fluid and embedded in an elastic medium subjected to mechanical and thermal loads. This study was expanded to investigate dynamic characteristics of fluid-conveying FGM circular cylinder shells subjected to dynamic mechanical and thermal loads [14]. Zafar Iqbal et al. [15] examined vibration frequencies of FGM circular cylinder shells filled with fluid using wave propagation approach. Vibration frequencies of shell were analyzed for various boundary conditions taking into account the effect of fluid. Shah et al. [16] based on Love's thin-shell theory to investigate natural frequencies of full-filled fluid FG circular cylinder shells resting on Winkler and Pasternak elastic foundations. Wave propagation approach was employed to calculate. Silva et al. [17] studied nonlinear responses of fluid-filled FG circular cylinder shell under mechanical load. Recently, Hong-Liang Dai et al. [18] analyzed thermos electro elastic behaviors of a fluid-filled functionally graded piezoelectric material cylindrical thin-shell under the combination of mechanical, thermal and electrical loads. By using the classical shell theory and Galerkin method, Khuc et al. [19] considered nonlinear vibration of full-filled fluid circular cylinder shells made of sandwich-FGM subjected to mechanical loads in thermal environment.

To best of the authors' knowledge, there is no analytical approach on dynamic buckling of sandwich FGM circular cylinder shells containing fluid. In this paper, nonlinear dynamic buckling of full-filled fluid sandwich FGM circular cylinder shells subjected to mechanical loads in the thermal environment is investigated. Governing equations are derived by using the classical shell theory with the geometrical nonlinearity in von Karman–Donnell sense. Solution of problem is established by using Galerkin's method and Runge–Kutta method. Effects of thermal environment, fluid, structures' geometric parameters and volume fraction index (k) on nonlinear dynamic responses of shell are considered.

2. GOVERNING EQUATIONS

Consider a sandwich FGM circular cylinder shell with geometric parameters: R, h, h_c, and h_m are shown in Fig. 1. Suppose that the full-filled fluid circular cylinder shell made of FGM sandwich subjected to an axial compression load $N_{01} = -p\,(t)\,h$ and a uniformly distributed external pressure $q\,(t)$ varying on time.

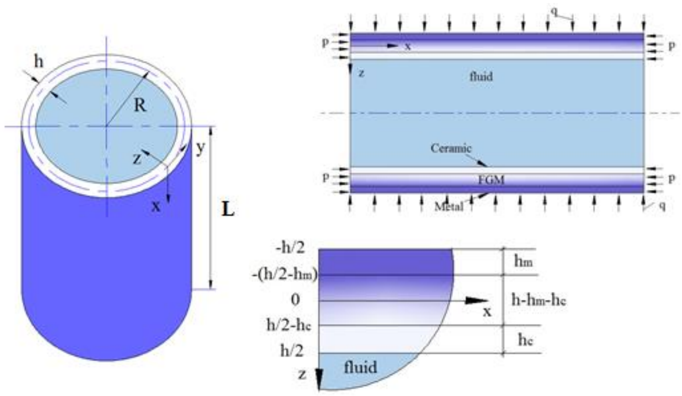

Fig. 1. Model of FGM-sandwich circular cylinder shell

With configuration of sandwich FGM as Fig. 1, suppose that $V_c(z)$ and $V_m(z)$ are the volume fractions of ceramic and metal respectively, the volume fraction of ceramic constituent changes according to the power law and can be expressed as

$$\begin{aligned}
&V_c = 0, -0.5h \le z \le -\left(0.5h - h_m\right),\\
&V_c = \left(\frac{z + 0.5h - h_m}{h - h_c - h_m}\right)^k, -\left(0.5h - h_m\right) \le z \le \left(0.5h - h_c\right), k \ge 0,\\
&V_c = 1, \left(0.5h - h_c\right) \le z \le 0.5h.
\end{aligned} \tag{1}$$

Then the elasticity modulus E, the mass density ρ and the Poisson ratio ν of circular cylinder shell can be evaluated as following

$$\begin{aligned}
&E = E_m V_m + E_c V_c = E_m + \left(E_c - E_m\right) V_c,\\
&\rho = \rho_m V_m + \rho_c V_c = \rho_m + \left(\rho_c - \rho_m\right) V_c,\\
&\nu_m = \nu_c = \text{const}.
\end{aligned} \tag{2}$$

The strain components of the circular cylinder shell are

$$\varepsilon_x = \varepsilon_x^0 - zk_x, \quad \varepsilon_y = \varepsilon_y^0 - zk_y, \quad \gamma_{xy} = \gamma_{xy}^0 - 2zk_{xy}, \tag{3}$$

where

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{4}$$

$$k_x = \frac{\partial^2 w}{\partial x^2}, \quad k_y = \frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = \frac{\partial^2 w}{\partial x \partial y}, \tag{5}$$

in which $\varepsilon_x^0; \varepsilon_y^0; \gamma_{xy}^0$ are the strains at the middle surface; k_x, k_y and k_{xy} are curvatures and the twist.

By use of Eq. (4), the deformation compatibility equation can be written as

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}. \tag{6}$$

For circular cylindrical shell subjected to mechanical load in temperature environment, the Hooke's law can be defined as

$$\begin{aligned} &\sigma_x = \frac{E(z)}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) - \frac{E(z)\,\alpha(z)\,\Delta T}{1-\nu}, \quad \sigma_y = \frac{E(z)}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y) - \frac{E(z)\,\alpha(z)\,\Delta T}{1-\nu}, \\ &\tau_{xy} = \frac{E(z)}{2(1+\nu)}\gamma_{xy}, \end{aligned} \tag{7}$$

in which $\Delta T = T - T_0$.

Internal forces and moment resultants can be defined by integrating stresses components through the shells' thickness and can be expressed in matrix form as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ -k_x \\ -k_y \\ -2k_{xy} \end{Bmatrix} - \begin{Bmatrix} \Phi_a \\ \Phi_a \\ 0 \\ \Phi_b \\ \Phi_b \\ 0 \end{Bmatrix}, \tag{8}$$

in which $N_x; N_y; N_{xy}$ are internal forces, $M_x; M_y; M_{xy}$ are moment resultants.

Stiffness coefficients and quantities related to thermal load in Eq. (8) are explained in Appendix A. From Eq. (8) the expressions of deformation and moment resultants of sandwich FGM circular cylinder shell can be defined as

$$\begin{aligned} \varepsilon_x^0 &= A_{22}^* N_x - A_{12}^* N_y + B_{11}^* k_x + B_{12}^* k_y + \Phi_a\left(A_{22}^* - A_{12}^*\right), \\ \varepsilon_y^0 &= -A_{12}^* N_x + A_{11}^* N_y + B_{21}^* k_x + B_{22}^* k_y + \Phi_a\left(A_{11}^* - A_{12}^*\right), \\ \gamma_{xy}^0 &= A_{66}^* N_{xy} + 2B_{66}^* k_{xy}, \end{aligned} \tag{9}$$

$$\begin{aligned} M_x &= B_{11}^* N_x + B_{21}^* N_y - D_{11}^* k_x - D_{12}^* k_y + \left[B_{11}\left(A_{22}^* - A_{12}^*\right) + B_{12}\left(A_{11}^* - A_{12}^*\right)\right]\Phi_a - \Phi_b, \\ M_y &= B_{12}^* N_x + B_{22}^* N_y - D_{21}^* k_x - D_{22}^* k_y + \left[B_{12}\left(A_{22}^* - A_{12}^*\right) + B_{22}\left(A_{11}^* - A_{12}^*\right)\right]\Phi_a - \Phi_b, \\ M_{xy} &= B_{66}^* N_{xy} - 2D_{66}^* k_{xy}, \end{aligned} \tag{10}$$

Extended stiffness coefficients in Eq. (9) and Eq. (10) are explained in Appendix B. According to [20], the motion equations of full-filled fluid circular cylinder shell subjected to external pressure $q(t)$ and an axial compression can be given as

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\rho_1\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=\rho_1\frac{\partial^2 v}{\partial t^2},$$
$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+\frac{N_y}{R}+q-p_L=\rho_1\frac{\partial^2 w}{\partial t^2}+2\rho_1\varepsilon\frac{\partial w}{\partial t}, \tag{11}$$

in which ε is the linear damping coefficient and

$$\rho_1=\int_{-h/2}^{h/2}\rho_{(z)}dz=\rho_m h+\rho_{cm}h_c+\frac{\rho_{cm}\left(h-h_c-h_m\right)}{k+1},$$
$$p_L=-\rho_L\frac{\partial\varphi_L}{\partial t}=M_L\frac{\partial^2 w}{\partial t^2}\text{ is the dynamic pressure of fluid acting on the shell,}$$

where $M_L=\dfrac{\rho_L R I_n\left(\lambda_m\right)}{\lambda_m I_n'\left(\lambda_m\right)}$ is the mass of correspondence fluid to the shell vibration and $\lambda_m=\dfrac{m\pi R}{L}$ [19].

Applying the Volmir's assumption [21] into Eqs. (11) (because of $u\ll w, v\ll w$), the equations of motion can be rewritten as follows

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0,\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,$$
$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+\frac{N_y}{R}+q=\left(\rho_1+M_L\right)\frac{\partial^2 w}{\partial t^2}+2\rho_1\varepsilon\frac{\partial w}{\partial t}. \tag{12}$$

The first and the second equation of Eqs. (12) are satisfied identically by recommending the stress function:

$$N_x=\frac{\partial^2 F}{\partial y^2},\quad N_y=\frac{\partial^2 F}{\partial x^2},\quad N_{xy}=-\frac{\partial^2 F}{\partial x\partial y}. \tag{13}$$

Substituting Eqs. (9) and (13) into Eq. (6), and Eq. (13) into the third equation of Eqs. (12) we obtain the system of two equations

$$A_{11}^*\frac{\partial^4 F}{\partial x^4}+\left(A_{66}^*-2A_{12}^*\right)\frac{\partial^4 F}{\partial x^2\partial y^2}+A_{22}^*\frac{\partial^4 F}{\partial y^4}$$
$$+B_{21}^*\frac{\partial^4 w}{\partial x^4}+\left(B_{11}^*+B_{22}^*-2B_{66}^*\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+B_{12}^*\frac{\partial^4 w}{\partial y^4}+\frac{1}{R}\frac{\partial^2 w}{\partial x^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2+\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=0, \tag{14}$$

$$(\rho_1 + M_L)\frac{\partial^2 w}{\partial t^2} + 2\rho_1\varepsilon\frac{\partial w}{\partial t} + D_{11}^*\frac{\partial^4 w}{\partial x^4} + (D_{12}^* + D_{21}^* + 4D_{66}^*)\frac{\partial^4 w}{\partial x^2\partial y^2} + D_{22}^*\frac{\partial^4 w}{\partial y^4} - B_{21}^*\frac{\partial^4 F}{\partial x^4}$$
$$- B_{12}^*\frac{\partial^4 F}{\partial y^4} - (B_{11}^* + B_{22}^* - 2B_{66}^*)\frac{\partial^4 F}{\partial x^2\partial y^2} - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} - \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} - \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + q = 0. \quad (15)$$

Eqs. (14) and (15) are governing equations used to investigate nonlinear dynamic buckling of full-filled fluid circular cylinder shell made of sandwich FGM.

3. DYNAMIC BUCKLING SOLUTION

Suppose that the circular cylinder shell under simply supported at both ends and subjected to axial compression load $N_{01} = -ph$. In which p is average axial stress acting on the ends of the shell. Therefor boundary conditions are defined as

$$w = 0, M_x = 0, N_x = N_{01}, N_{xy} = 0 \text{ at } x = 0 \text{ and } x = L.$$

The shells' deflection satisfying above conditions can be written as

$$w = f(t)\sin\frac{m\pi x}{L}\sin\frac{ny}{R}, \quad (16)$$

where m, n are numbers of half waves in generating line direction and circumference direction, respectively.

The solution of stress function F in Eq. (14) can be defined as

$$F = F_1\cos 2\alpha x + F_2\cos 2\beta y - F_3\sin\alpha x\sin\beta y - N_{01}\frac{y^2}{2} - \left(-\frac{\beta^2}{8A_{11}^*}f_{(t)}^2 + \psi f_{(t)}^+ N_{01}\frac{A_{12}^*}{A_{11}^*} + \Gamma\right)\frac{x^2}{2}, \quad (17)$$

in which

$$F_1 = F_1^* f_{(t)}^2 = \frac{\beta^2}{32\alpha^2 A_{11}^*}f_{(t)}^2, \quad F_2 = F_2^* f_{(t)}^2 = \frac{\alpha^2}{32\beta^2 A_{22}^*}f_{(t)}^2,$$
$$F_3 = F_3^* f_{(t)} = \frac{\alpha^4 B_{21}^* + (\alpha\beta)^2(B_{11}^* + B_{22}^* - 2B_{66}^*) + \beta^4 B_{12}^* - \frac{\alpha^2}{R}}{\alpha^4 A_{11}^* + (\alpha\beta)^2(A_{66}^* - 2A_{12}^*) + \beta^4 A_{22}^*}f_{(t)},$$
$$\psi = \frac{\gamma\eta}{mn\pi^2 A_{11}^*}\left[(\alpha^2 A_{11}^* - \beta^2 A_{12}^*)F_3^* - (\alpha^2 B_{21}^* + \beta^2 B_{22}^*) + \frac{1}{R}\right],$$
$$\Gamma = \frac{(A_{11}^* - A_{12}^*)\Phi_a}{A_{11}^*}, \gamma = \left[(-1)^m - 1\right], \eta = \left[(-1)^n - 1\right] \alpha = \frac{m\pi}{L}, \beta = \frac{n\pi}{R}.$$

Substituting Eq. (16) and Eq. (17) into Eq. (15), then using the Galerkin method we obtain

$$(\rho_1 + M_L)\frac{d^2 f}{dt^2} + 2\rho_1\varepsilon\frac{df}{dt} + H_1 f_{(t)}^3 + H_2 f_{(t)}^2 + H_3 f_{(t)}^+ \frac{4\gamma\eta}{mn\pi^2 R}\left(\frac{A_{12}^*}{A_{11}^*}N_{01} + \Gamma\right) = \frac{4\gamma\eta}{mn\pi^2}q, \quad (18)$$

in which

$$H_1 = 2\left(\alpha\beta\right)^2\left(F_1^* + F_2^*\right) + \frac{\beta^4}{8A_{11}^*},$$

$$H_2 = -\frac{16\gamma\eta}{3mn\pi^2}\left[\left(4\alpha^4 B_{21}^* - \frac{\alpha^2}{R}\right)F_1^* + 4\beta^4 B_{12}^* F_2^* - \frac{1}{2}\left(\alpha\beta\right)^2 F_3^* + \frac{3}{32}\frac{\beta^2}{RA_{11}^*} + \frac{3mn\pi^2}{16\gamma\eta}\beta^2\psi\right],$$

$$H_3 = \left[\alpha^4 D_{11}^* + \left(\alpha\beta\right)^2\left(D_{12}^* + D_{21}^* + 4D_{66}^*\right) + \beta^4 D_{22}^*\right]$$
$$+\left[\alpha^4 B_{21}^* + \left(\alpha\beta\right)^2\left(B_{11}^* + B_{22}^* - 2B_{66}^*\right) + \beta^4 B_{12}^* - \frac{\alpha^2}{R}\right]F_3^* - \left(\alpha^2 + \frac{A_{12}^*}{A_{11}^*}\beta^2\right)N_{01} - \Gamma\beta^2 + \frac{4\gamma\eta}{mn\pi^2 R}\psi.$$

Eq. (18) is used to investigate the nonlinear dynamic buckling of full-filled fluid FGM sandwich circular cylinder shells under mechanical load in thermal environment.

Nonlinear dynamic buckling analysis

For dynamic buckling analysis, this paper investigates two cases:

- Case 1. Consider a full-filled fluid sandwich FGM circular cylinder shell under linear axial compression load varying on time $N_{01} = -ph$ with $p = c_1 t$ (c_1-loading speed), $q = 0$.

- Case 2. Consider a full-filled fluid sandwich FGM circular cylinder shell under a pre-axial compression load and an external uniformly distributed pressure varying on time: N_{01} = const; $q = ct$ (c_2-loading speed).

In order to analyze the dynamic buckling problem of the considered shells, firstly Eq. (18) is solved for each case respectively to determine the nonlinear dynamic responses; secondarily based on these obtained dynamic responses, the dynamic critical time t_{cr} can be obtained according to Budiansky–Roth criterion [22]. This criterion is based on that for large value of loading speed, the amplitude time curve of obtained displacement response increases sharply depending on time and this curve obtains a maximum by passing from the slope point and at the corresponding time $t = t_{cr}$ the stability loss occurs. Here $t = t_{cr}$ is called critical time and the load corresponding to this critical time is called dynamic critical buckling load $P_{cr} = c_1 t_{cr}$ (case 1) or $q_{cr} = c_2 t_{cr}$ (case 2).

4. VALIDATION

To the best of the author's knowledge, there is no any publication on the nonlinear dynamic buckling of the sandwich-FGM cylindrical shell containing full filled fluid in thermal environment. Thus, the results in this paper are compared with the fluid-free shell ($h_c = h_m = 0$). Authors compare the dynamic critical stress of fluid-free FGM cylindrical shell with the one in publication of Huaiwei Huang, Qiang Han [23] (Tab. 1), for FGM shell made of ZrO_2/Ti-6Al-4V and material properties: $E_m = 122.56$ e9Pa, $\rho_m = 4429$ kg/m^3, $\nu_m = 0.288$, $E_c = 244.27$ e9Pa, $\rho_c = 5700$ kg/m^3, $\nu_c = 0.288$.

Tab. 1 shows that, the results of this article are slightly different from the above publication. The cause of this difference is that the authors use different methods, so the results of this article can be reliable.

Table 1. Comparison of critical stress of the compressed cylindrical shell (MPa)

k	0.2	1.0	5.0
Huang & Han [23]	194.94 (2, 11)	169.94 (2, 11)	150.25 (2, 11)
Present	193.914 (1, 9)	168.685 (1, 9)	149.167 (1, 9)

5. NUMERICAL RESULTS

Consider a circular cylindrical shell made of FGM-core with geometric dimensions: $h = 0.014$ m, $h_c = h/5, h_m = h/5, L/R = 2$ and $R/h = 200$. FGM made of Aluminium and Alumina with the material properties are $E_m = 7 \times 10^9$ N/m^2; $\rho_m = 2702 \times 10^3$ kg/m^3, $\alpha_m = 2.3 \times 10^{-5}$ C^{-1}, $E_c = 3.8 \times 10^{11}$ N/m^2; $\rho_c = 3.8 \times 10^3$ kg/m^3, $\alpha_c = 5.4 \times 10^{-6}$ C^{-1}, $\varepsilon = 0.1$, the Poisson's ratio $\nu_c = 0.3$ the fluid density $\rho_L = 10^3$ kg/m^3.

- Case 1. Consider a full-filled fluid sandwich FGM circular cylinder shell under linear axial compression load varying on time $N_{01} = -ph(p = c_1 t), q = 0$.

In this case, the critical time t_{cr} can be obtained according to Budiansky–Roth criterion. The dynamic critical force $p_{cr} = c_1 t_{cr}$. The nonlinear dynamic responses of shell are shown in Figs. 2–7.

Nonlinear responses of fluid-filled and fluid-free circular cylinder shell in thermal environment are shown in Figs 2–3. From Fig. 2 we obtain $t_{cr} = 0.065$ s and P_{cr} = 68.1 GPa respectively and from Fig. 3, we can see that with fluid-filled cylinder shell, the dynamic critical force P_{cr} = 68.1 GPa increased by 4.12 times (318%) compared to the dynamic critical force of fluid-free ones P_{cr} = 16.3 GPa, $t_{cr} = 0.015$ s, respectively.

Doing the same with the next case taking into account the influence of other factors must be derived from dynamical responses to determine the critical forces.

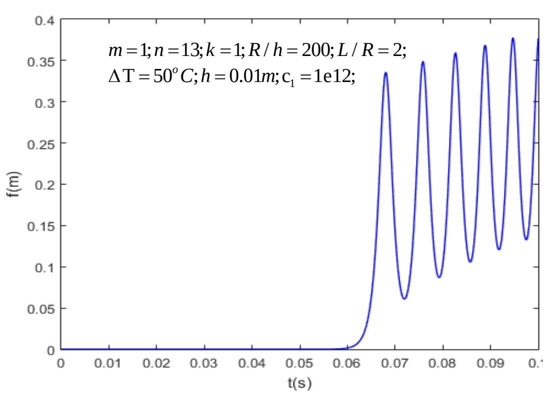

Fig. 2. Nonlinear dynamic response of fluid-filled circular cylinder shell

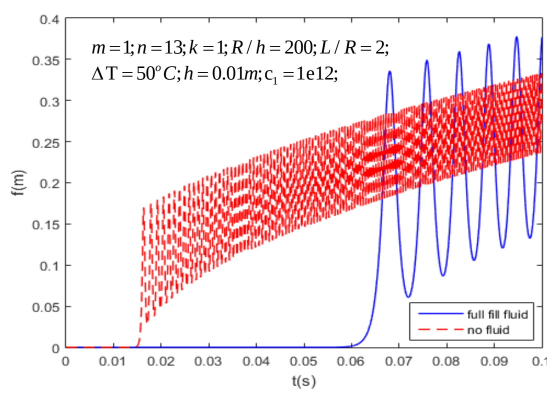

Fig. 3. Effects of fluid on dynamic response of circular cylinder shell

Fig. 4 shows nonlinear dynamic responses of cylinder shell when volume-fraction index k changes. It can be seen that, if k increases the dynamic critical force of shell will decrease. That means the load-bearing capability of cylinder shell decreases.

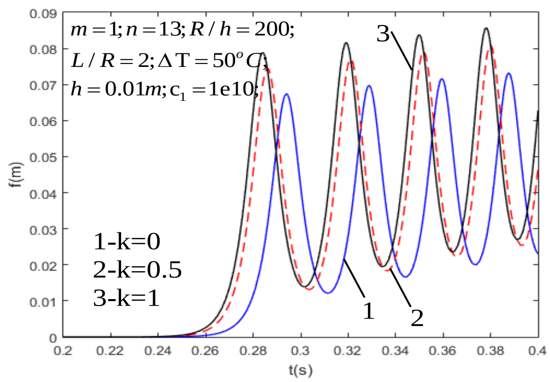

Fig. 4. Dynamic response of fluid-filled cylinder shell when *k* changes

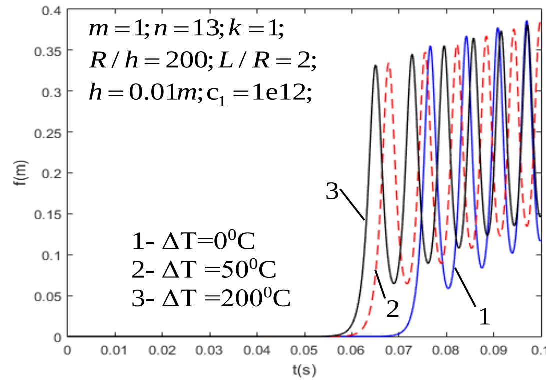

Fig. 5. Temperature effect on nonlinear dynamic responses of fluid-filled cylinder shell

The effect of thermal environment on nonlinear dynamic responses of circular cylinder shell is shown in Fig. 5. From the graph it is observed that when the temperature increases, the dynamic critical force of shell will decrease. From P_{cr} = 76.6 GPa at 0°C to P_{cr} = 65 GPa at 200°C. That means, the load-bearing capability of the shell will decrease when temperature increases.

The effect of geometric parameters (L/R ratio) on nonlinear dynamic responses of cylinder shells made of sandwich-FGM filled with fluid is shown in Fig. 6. The dynamic critical force of cylinder shell decreases when increasing R/L ratio. That means increasing length of the shell, the stability of the shell structure will decrease.

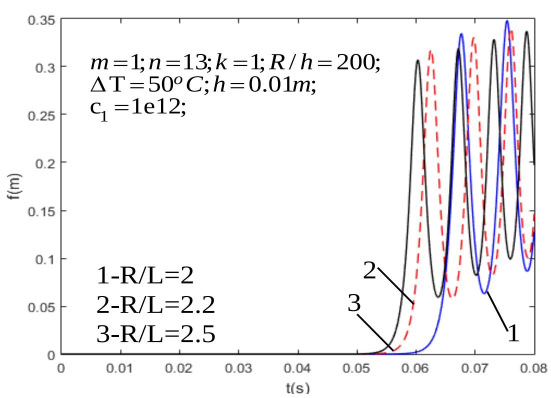

Fig. 6. Effect of geometric parameters on dynamic responses of fluid-filled cylinder shell

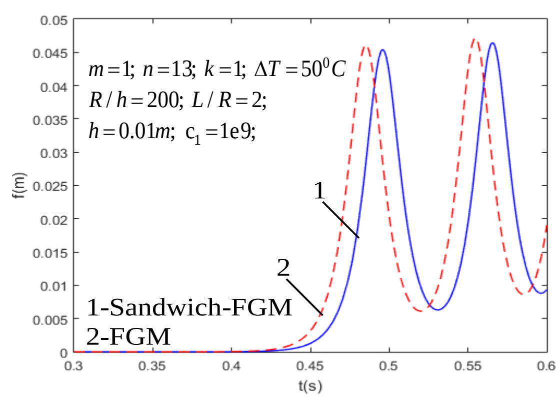

Fig. 7. Dynamic responses of FGM and sandwich-FGM circular cylinder shell

Fig. 7 indicates nonlinear dynamic responses of circular cylinder shell made of FGM and sandwich-FGM filled with fluid. For the structure made of sandwich-FGM, the critical force is P_{cr} = 0.496 GPa, and for FGM ones, the critical force is P_{cr} = 0.485 GPa. That means, with the same geometry dimensions, the workability of sandwich-FGM cylinder shell is better than FGM ones.

- Case 2. Consider a full-filled fluid sandwich FGM circular cylinder shell under a uniform pre-axial compression load and an external uniformly distributed pressure varying on time: N_{01} = const, $q = c_2 t$ (c_2-loading speed).

The nonlinear dynamic responses of circular cylinder shell are shown in Figs. 8–13. Nonlinear dynamic responses of fluid-filled and fluid-free sandwich FGM circular cylinder shell are depicted in Figs. 8–9. From Fig. 8 we obtain $t_{cr} = 0.01$ s and $q_{cr} =$ 147 MPa respectively, from the Fig. 9, it is observed that fluid remarkably increases the dynamic critical force of the shell (from q_{cr} = 25 MPa at $t_{cr} = 0.002$ s in case fluid-fee shell to $q_{cr} = 147$ MPa at $t_{cr} = 0.01$ s in case shell containing fluid, i.e. the critical force increased by 5.88 times by 488%).

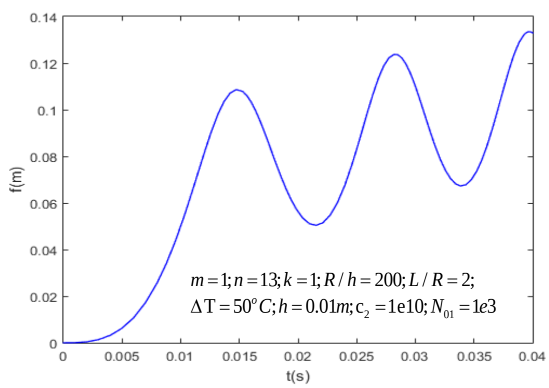

Fig. 8. Nonlinear dynamic responses of full-filled fluid circular cylinder shell

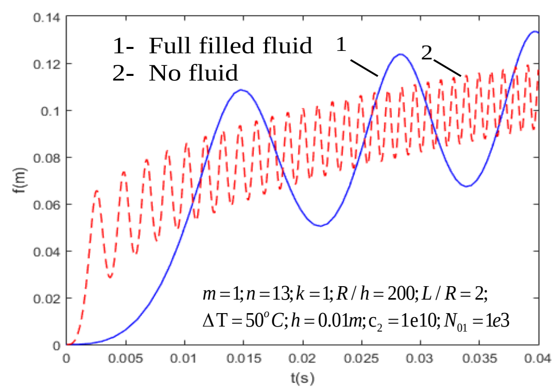

Fig. 9. Effect of fluid on dynamic responses of circular cylinder shell

Similarly, we make other cases when taking into account the influence of other factors derive from dynamic response curves to determine dynamic critical forces.

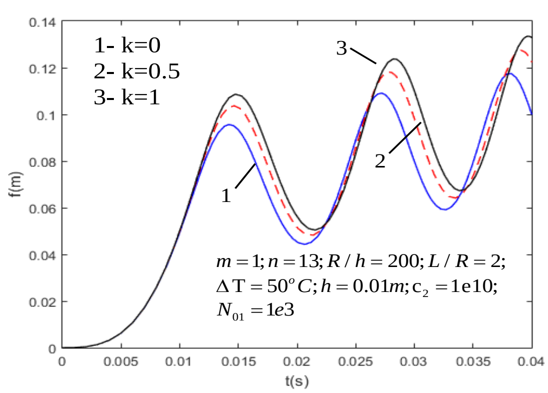

Fig. 10. Dynamic responses of fluid-filled circular cylinder shell with k changes

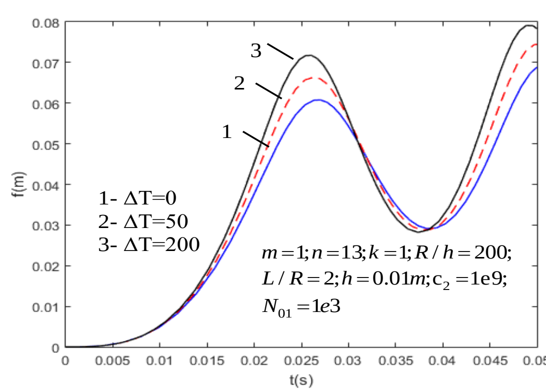

Fig. 11. Effect of thermal on the dynamic response of circular cylinder shells

Figs. 10–11 show dynamic responses of circular cylinder shell filled with fluid with various volume-fraction index k and the effect of thermal environment on dynamic responses of circular cylinder shells. From the graph as can see that if temperature increases the dynamic critical force decreases. That means if the temperature increases then the stability of the shell structure will decrease.

Effects of geometric parameters on nonlinear dynamic response of full-filled fluid circular cylinder shells are surveyed and presented in Fig. 12. Dynamic critical force of the shell decreases with increasing the ratio of length to radius L/R. That means if the length of shell increases, the stability of the shell will decrease.

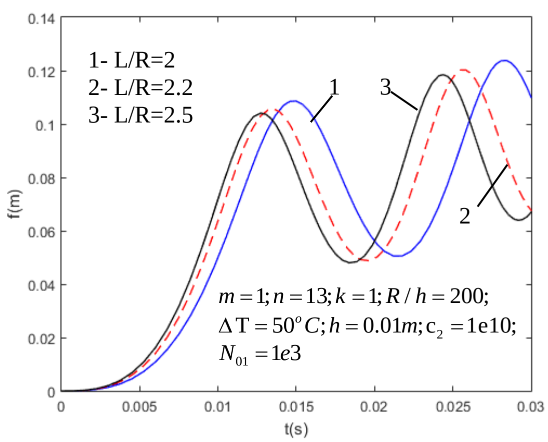

Fig. 12. Nonlinear dynamic responses of circular cylinder shell with L/R changes

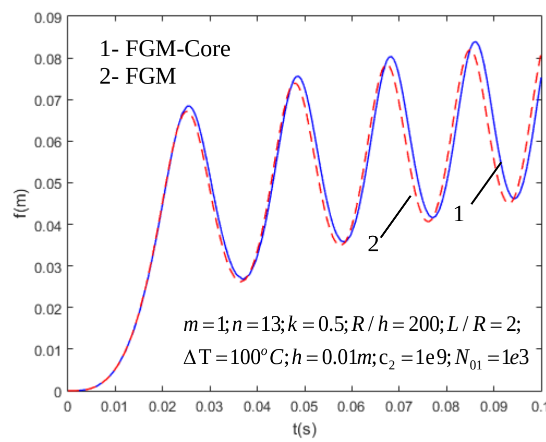

Fig. 13. Effect of material structure on dynamic response of shell

Nonlinear responses of FGM and sandwich-FGM circular cylinder shell filled with fluid are shown in Fig. 13. The critical force of full-filled fluid sandwich-FGM circular cylinder shell is higher than those of FGM ones. That means, with the same geometry dimensions, sandwich-FGM cylinder shell structures will work better than FGM ones.

6. CONCLUSIONS

This paper established nonlinear dynamic equations of fluid-filled circular cylinder shells made of sandwich-FGM under mechanical load including the effect of temperature. Dynamic responses of the simply supported shell are obtained by using Galerkin method and Runge–Kutta method. Based on dynamic responses, critical dynamic loads are obtained by using the Budiansky–Roth criterion. Some conclusions can be obtained from the present analysis:

- Dynamic critical force of full-filled fluid sandwich-FGM circular cylinder shell is remarkably higher than those of fluid-free ones. That means, the fluid enhances the stability of sandwich-FGM cylinder shell.

- Temperature reduces dynamic critical force of sandwich-FGM cylinder shell. That means, temperature reduces stability of shell.

- When the volume-fraction index k increases (it means the volume fraction of metal increases),

the critical force decreases (the stability of the shell structure will decrease) .

- Dynamic critical force of the shell decreases when increasing ratio of length to radius (L/R). On the other hand, length of shell decreases stability of shell.

- With the same geometry dimensions, sandwich-FGM circular cylinder shell structures will work better than FGM one.

ACKNOWLEDGEMENTS

This research is funded by National Foundation for Science and Technology Development of Vietnam (NAFOSTED) under grant number 107.02-2018.324.

REFERENCES

[1] D. H. Bich and N. X. Nguyen. Nonlinear vibration of functionally graded circular cylindrical shells based on improved Donnell equations. *Journal of Sound and Vibration*, **331**, (25), (2012), pp. 5488–5501. https://doi.org/10.1016/j.jsv.2012.07.024.

[2] Y. W. Kim. Free vibration analysis of FGM cylindrical shell partially resting on Pasternak elastic foundation with an oblique edge. *Composites Part B: Engineering*, **70**, (2015), pp. 263–276. https://doi.org/10.1016/j.compositesb.2014.11.024.

[3] N. D. Duc and P. T. Thang. Nonlinear dynamic response and vibration of shear deformable imperfect eccentrically stiffened S-FGM circular cylindrical shells surrounded on elastic foundations. *Aerospace Science and Technology*, **40**, (2015), pp. 115–127. https://doi.org/10.1016/j.ast.2014.11.005.

[4] N. D. Duc, N. D. Tuan, P. Tran, N. T. Dao, and N. T. Dat. Nonlinear dynamic analysis of Sigmoid functionally graded circular cylindrical shells on elastic foundations using the third order shear deformation theory in thermal environments. *International Journal of Mechanical Sciences*, **101**, (2015), pp. 338–348. https://doi.org/10.1016/j.ijmecsci.2015.08.018.

[5] R. Bahadori and M. M. Najafizadeh. Free vibration analysis of two-dimensional functionally graded axisymmetric cylindrical shell on Winkler–Pasternak elastic foundation by First-order Shear Deformation Theory and using Navier-differential quadrature solution methods. *Applied Mathematical Modelling*, **39**, (16), (2015), pp. 4877–4894. https://doi.org/10.1016/j.apm.2015.04.012.

[6] D. H. Bich, D. V. Dung, and V. H. Nam. Nonlinear dynamical analysis of eccentrically stiffened functionally graded cylindrical panels. *Composite Structures*, **94**, (8), (2012), pp. 2465–2473. https://doi.org/10.1016/j.compstruct.2012.03.012.

[7] D. H. Bich, D. V. Dung, V. H. Nam, and N. T. Phuong. Nonlinear static and dynamic buckling analysis of imperfect eccentrically stiffened functionally graded circular cylindrical thin shells under axial compression. *International Journal of Mechanical Sciences*, **74**, (2013), pp. 190–200. https://doi.org/10.1016/j.ijmecsci.2013.06.002.

[8] B. Mirzavand, M. R. Eslami, and J. N. Reddy. Dynamic thermal postbuckling analysis of shear deformable piezoelectric-FGM cylindrical shells. *Journal of Thermal Stresses*, **36**, (3), (2013), pp. 189–206. https://doi.org/10.1080/01495739.2013.768443.

[9] N. D. Duc and P. T. Thang. Nonlinear response of imperfect eccentrically stiffened ceramic–metal–ceramic FGM thin circular cylindrical shells surrounded on elastic foundations and subjected to axial compression. *Composite Structures*, **110**, (2014), pp. 200–206. https://doi.org/10.1016/j.compstruct.2013.11.015.

[10] N. D. Duc, P. T. Thang, N. T. Dao, and H. V. Tac. Nonlinear buckling of higher deformable S-FGM thick circular cylindrical shells with metal–ceramic–metal layers surrounded on elastic foundations in thermal environment. *Composite Structures*, **121**, (2015), pp. 134–141. https://doi.org/10.1016/j.compstruct.2014.11.009.

[11] D. H. Bich, N. X. Nguyen, and H. V. Tung. Postbuckling of functionally graded cylindrical shells based on improved Donnell equations. *Vietnam Journal of Mechanics*, **35**, (1), (2013), pp. 1–15. https://doi.org/10.15625/0866-7136/35/1/2894.

[12] V. H. Nam, N. T. Phuong, D. H. Bich, and D. V. Dung. Nonlinear static and dynamic buckling of eccentrically stiffened functionally graded cylindrical shells under axial compression surrounded by an elastic foundation. *Vietnam Journal of Mechanics*, **36**, (1), (2014), pp. 27–47. https://doi.org/10.15625/0866-7136/36/1/3470.

[13] G. G. Sheng and X. Wang. Thermomechanical vibration analysis of a functionally graded shell with flowing fluid. *European Journal of Mechanics-A/Solids*, **27**, (6), (2008), pp. 1075–1087. https://doi.org/10.1016/j.euromechsol.2008.02.003.

[14] G. G. Sheng and X. Wang. Dynamic characteristics of fluid-conveying functionally graded cylindrical shells under mechanical and thermal loads. *Composite Structures*, **93**, (1), (2010), pp. 162–170. https://doi.org/10.1016/j.compstruct.2010.06.004.

[15] Z. Iqbal, M. N. Naeem, N. Sultana, S. H. Arshad, and A. G. Shah. Vibration characteristics of FGM circular cylindrical shells filled with fluid using wave propagation approach. *Applied Mathematics and Mechanics*, **30**, (11), (2009), pp. 1393–1404. https://doi.org/10.1007/s10483-009-1105-x.

[16] A. G. Shah, T. Mahmood, M. N. Naeem, and S. H. Arshad. Vibrational study of fluid-filled functionally graded cylindrical shells resting on elastic foundations. *ISRN Mechanical Engineering*, **2011**, (2011), pp. 1–13. https://doi.org/10.5402/2011/892460.

[17] F. M. A. da Silva, R. O. P. Montes, P. B. Goncalves, and Z. J. G. N. Del Prado. Nonlinear vibrations of fluid-filled functionally graded cylindrical shell considering a time-dependent lateral load and static preload. *Journal of Mechanical Engineering Science*, **230**, (1), (2016), pp. 102–119. https://doi.org/10.1177/0954406215587729.

[18] H. L. Dai, W. F. Luo, T. Dai, and W. F. Luo. Exact solution of thermoelectroelastic behavior of a fluid-filled FGPM cylindrical thin-shell. *Composite Structures*, **162**, (2017), pp. 411–423. https://doi.org/10.1016/j.compstruct.2016.12.002.

[19] P. V. Khuc, B. H. Dao, and D. X. Le. Analysis of nonlinear thermal dynamic responses of sandwich functionally graded cylindrical shells containing fluid. *Journal of Sandwich Structures & Materials*, (2017), pp. 1–22. https://doi.org/10.1177/1099636217737235.

[20] D. O. Brush and B. O. Almroth. *Buckling of bars, plates, and shells*. McGraw-Hill, New York, (1975).

[21] A. S. Volmir. *The nonlinear dynamics of plates and shells*. Science edition, Moscow, (1975).

[22] B. Budiansky and R. S. Roth. Axisymmetric dynamic buckling of clamped shallow spherical shells. *NASA Technical Note*, **510**, (1962), pp. 597–606.

[23] H. Huang and Q. Han. Nonlinear dynamic buckling of functionally graded cylindrical shells subjected to time-dependent axial load. *Composite Structures*, **92**, (2), (2010), pp. 593–598. https://doi.org/10.1016/j.compstruct.2009.09.011.

APPENDIX A

Stiffness coefficients and quantities related to thermal load in Eq. (8)

$$A_{11} = A_{22} = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} dz = \frac{E_1}{1-\nu^2}; A_{12} = \int_{-h/2}^{h/2} \frac{\nu E}{1-\nu^2} dz = \frac{\nu E_1}{1-\nu^2}; A_{66} = \int_{-h/2}^{h/2} \frac{E}{2(1+\nu)} dz = \frac{E_1}{2(1+\nu)};$$
$$B_{11} = B_{22} = \int_{-h/2}^{h/2} \frac{E.z}{1-\nu^2} dz = \frac{E_2}{1-\nu^2}; B_{12} = \int_{-h/2}^{h/2} \frac{\nu Ez}{1-\nu^2} dz = \frac{\nu E_2}{1-\nu^2}; B_{66} = \int_{-h/2}^{h/2} \frac{Ez}{2(1+\nu)} dz = \frac{E_2}{2(1+\nu)};$$
$$D_{11} = D_{22} = \int_{-h/2}^{h/2} \frac{E.z^2}{1-\nu^2} dz = \frac{E_3}{1-\nu^2}; D_{12} = \int_{-h/2}^{h/2} \frac{\nu Ez^2}{1-\nu^2} dz = \frac{\nu E_3}{1-\nu^2}; B_{66} = \int_{-h/2}^{h/2} \frac{Ez^2}{2(1+\nu)} dz = \frac{E_3}{2(1+\nu)};$$

in which

$$E_1 = \int_{-h/2}^{h/2} E(z)\, dz = E_m h + E_{cm} h_c + \frac{E_{cm} h_x}{k+1};$$
$$E_2 = \int_{-h/2}^{h/2} E(z)\, z dz = \frac{E_{cm} h_c h}{2} - \frac{E_{cm} h_c^2}{2} + \frac{E_{cm}}{k+1}\left(\frac{h}{2} - h_c\right) h_x - \frac{E_{cm} h_x^2}{(k+1)(k+2)};$$
$$E_3 = \int_{-h/2}^{h/2} E(z)\, z^2 dz = \frac{E_{cm}}{k+1}\left(\frac{h}{2} - h_c\right)^2 h_x - \frac{2E_{cm}}{(k+1)(k+2)}\left(\frac{h}{2} - h_c\right) h_x^2 + \frac{2E_{cm}}{(k+1)(k+2)(k+3)} h_x^3$$
$$+ \frac{E_c h_c^3}{3} + \frac{E_c h h_c}{2}\left(\frac{h}{2} - h_c\right) + \frac{E_m}{3}\left[h_m^3 + \frac{3hh_m}{2}\left(\frac{h}{2} - h_m\right)\right] + \frac{E_m}{3}\left[h_x^3 - 3(h/2 - h_m)(h/2 - h_c) h_x\right];$$

$$\Phi_a = \frac{1}{1-\nu} \int_{-h/2}^{h/2} E(z)\, \alpha(z)\, \Delta T dz, \quad \Phi_b = \frac{1}{1-\nu} \int_{-h/2}^{h/2} E(z)\, \alpha(z)\, \Delta T z dz.$$

If ΔT = const then $\Phi_a = \frac{1}{1-\nu} P \Delta T$.

For FGM-core:

$$P = E_m \alpha_m h + E_c \alpha_c h_c + E_m \alpha_m (h - h_c) + \frac{E_m \alpha_{cm} h_x}{k+1} + \frac{E_{cm} \alpha_m h_x}{k+1} + \frac{E_{cm} \alpha_{cm} h_x}{2k+1},$$

where $h_x = h - h_c - h_m$; $E_{cm} = E_c - E_m$.

APPENDIX B

Extended stiffness coefficients in Eq. (9) and Eq. (10)

$$A_{11}^* = \frac{A_{11}}{A_{11}A_{22} - A_{12}^2}; A_{12}^* = \frac{A_{12}}{A_{11}A_{22} - A_{12}^2}; A_{22}^* = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2}; B_{11}^* = \frac{A_{22}B_{11} - A_{12}B_{12}}{A_{11}A_{22} - A_{12}^2};$$
$$B_{12}^* = \frac{A_{22}B_{12} - A_{12}B_{22}}{A_{11}A_{22} - A_{12}^2}; B_{21}^* = \frac{A_{11}B_{12} - A_{12}B_{11}}{A_{11}A_{22} - A_{12}^2}; B_{22}^* = \frac{A_{11}B_{22} - A_{12}B_{12}}{A_{11}A_{22} - A_{12}^2}; A_{66}^* = \frac{1}{A_{66}}; B_{66}^* = \frac{B_{66}}{A_{66}};$$
$$D_{11}^* = D_{11} - B_{11}B_{11}^* - B_{12}B_{21}^*; \quad D_{12}^* = D_{12} - B_{11}B_{12}^* - B_{12}B_{22}^*;$$
$$D_{21}^* = D_{12} - B_{12}B_{11}^* - B_{22}B_{21}^*; \quad D_{22}^* = D_{22} - B_{12}B_{12}^* - B_{22}B_{22}^*; D_{66}^* = D_{66} - B_{66}B_{66}^*.$$