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THE INFLUENCE OF FOUNDATION MASS ON DYNAMIC RESPONSE OF TRACK-VEHICLE INTERACTION

Phuoc T. Nguyen[1], **Trung D. Pham**[2,*], **Hoa P. Hoang**[3]
[1]*Ho Chi Minh City Open University, Vietnam*
[2]*Mientrung University of Civil Engineering, Phu Yen, Vietnam*
[3]*University of Science and Technology - The University of Danang, Vietnam*
[*]E-mail: phamdinhtrung@muce.edu.vn

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Abstract. The influence of foundation mass on the dynamic response of track-vehicle interaction is studied in this paper. The moving vehicle is modeled as a two-axle mass-spring-damper four-degrees-of-freedom system. A new dynamic foundation model, called "Dynamic foundation model" including linear elastic spring, shear layer, viscous damping and foundation mass parameter, is used to analyze the dynamic response of the track-vehicle interaction. The railway track on the new dynamic foundation model subjected to a moving vehicle is regarded as an integrated system. By means of the finite element method and dynamic balance principle, the governing equation of motion for railway track-vehicle-foundation interaction is derived and solved by the step-by-step integration method. The accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature. The influence of foundation mass parameter on the dynamic response of railway track-vehicle interaction is investigated. The numerical results show that with the new dynamic foundation model the foundation mass effects more significantly on the dynamic response of track-vehicle interaction. The study shows that the new dynamic foundation model describes the true behavior of soil in the analysis of dynamic response of structures on the foundation.

Keywords: Winkler foundation; two-parameter foundation; dynamic foundation model; foundation mass; track-vehicle interaction.

1. INTRODUCTION

The dynamic analysis of structure-vehicle interaction has always been an interesting topic in the field of transportation engineering. Nowadays, with the large increase of moving heavy vehicles and high-speed vehicles in the field highway and railway traffic, the interaction problem between moving vehicles and structures resting on foundation has been attracted much attention in during the many last decades.

Besides, the Winkler model [1], one of the most fundamental foundations suggested quite early, had been commonly used in engineering application and attracted the attention of many researchers in during many last decades [2–10]. But one of the most important deficiencies of the Winkler model is that it appears a displacement discontinuity between the loaded and the unloaded part of the foundation surface [11]. Hence, to overcome the deficiency of this model, several other foundation models had proposed to describe more real response of soil by introducing some kind of interaction between the independent springs by visualising various types of interconnections such as: an additional a thin elastic membrane stretched by a constant tension [12]; plate with flexural rigidity [13, 14]; an incompressible layer that resists only transverse shear deformation [15–19], or accounts for the effect of the neglected shear strain energy in the soil and shear forces that come from surrounding soil [20–22].

It can be seen that one of the most important deficiencies of above the foundation models is that it did not overlook the influence of the foundation mass density. In reality, the foundation always has mass density, so that the foundation mass density has to effect on the dynamic response of structure-foundation interaction during the vibration process. Recently, a new foundation model, called "Dynamic foundation model" including linear elastic spring, shear layer, viscous damping and special consideration of foundation mass parameter, was proposed [23–27]. Hence, the influence of foundation mass on the dynamic response of the track-vehicle interaction is presented in this paper. The railway track considered as a Timoshenko beam on dynamic foundation subjected to a moving vehicle is regarded as an integrated system with the moving vehicle modelled as a two-axle mass-spring-damper system having four degrees of freedom. By means of finite element method and dynamic balance principle, the governing equation of motion for railway track-vehicle interaction is derived and solved by step-by-step integration Newmark's method. The accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature. Then, the effects of foundation mass parameter on the dynamic response of railway track-vehicle interaction are discussed.

2. FORMULATION

2.1. Definition

The dynamic foundation model includes the elastic stiffness (the Winkler foundation modulus k), shear layer parameter (the shear modulus k_s of Pasternak foundation), viscous damping c, and lumped mass m at the top of the elastic spring connected between elastic layer and shear layer [23–27], as shown in Fig. 1.

The pressure-deflection relationship at the time t due to the pressure $q(x, y, t)$ and deflection $W(x, y, t)$ is determined based on dynamic balance principle, expressed mathematically as follows

$$q(x, y, t) = kw(x, y, t) + c\frac{\partial w(x, y, t)}{\partial t} + m\frac{\partial^2 w(x, y, t)}{\partial t^2} - k_s \nabla^2 w(x, y, t), \tag{1}$$

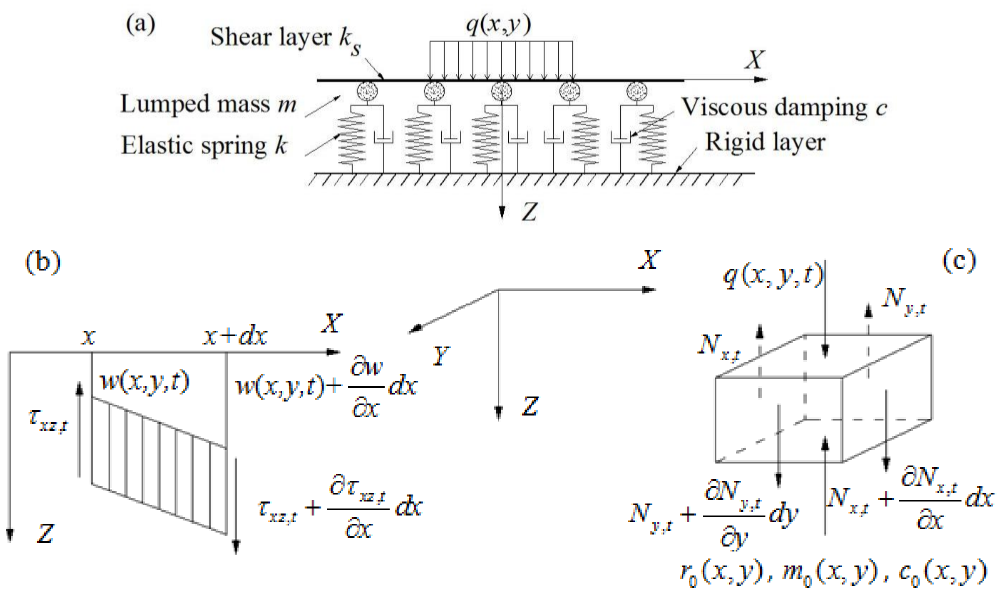


Fig. 1. The dynamic foundation model: (a) The basic model; (b) stress in the shear layer; (c) forces acting on the shear layer

where the lumped mass m is given by [23–27]

$$m = \beta\rho_f, \tag{2}$$

where β is an experimental parameter characterized by the influence of density of foundation ρ_f. It can be said that the dynamic foundation model is a quite general foundation model and accurately represents the characteristics of the soil for analyzing dynamic responses of structures-foundation interaction.

Consider a railway track modeled as a Timoshenko beam resting on a dynamic foundation subjected to a moving vehicle with constant velocity v, as shown in Fig. 2.

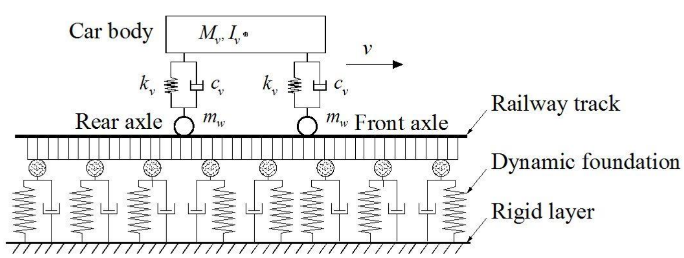


Fig. 2. The railway track subjected to a moving vehicle on the dynamic foundation

The moving vehicle is modeled as a two-axle mass-spring-damper system and consists of a car body and two axles having four degrees of freedom including three vertical displacements and one rotation displacement [28,29]. The car body is modeled as a rigid body with a mass M_v and a mass moment of inertia I_v having vertical displacement z_v and rotation θ_v about its centroid, respectively. The rear and front axles is interconnected with car body by a spring of stiffness k_v and a dashpot of damping coefficient c_v, respectively. The motions of its are described by the vertical displacement z_{w1} and z_{w2} constrained by the displacement of rails, respectively.

2.2. Formulation of element matrices

The railway track is modeled as a set of uniform elastic Timoshenko beam elements, each element has two nodes, each node having two degrees of freedom including vertical displacement and rotation displacement [30]. In the present formulation, it is assumed that: (i) the beam material is isotropic; (ii) the vibration amplitudes of the beam are sufficiently small; (iii) bonding between the beam and the dynamic foundation is perfect.

The strain energy of beam element on the dynamic foundation including the effects of both transverse shear deformation and the elastic foundation is given by

$$U_e = \frac{1}{2}\int_0^l EI\left(\frac{\partial\theta_e}{\partial x}\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^l \kappa GA\left(\frac{\partial v_e}{\partial x} - \theta_e\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^l k\left(v_e\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^l k_s\left(\frac{\partial v_e}{\partial x}\right)^2 \mathrm{d}x. \tag{3}$$

The kinetic energy of the beam element on the dynamic foundation including the rotatory inertia effect and foundation mass is written as

$$T_e = \frac{1}{2}\int_0^l \rho A\left(\frac{\partial v_e}{\partial t}\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^l \rho I\left(\frac{\partial\theta_e}{\partial t}\right)^2 \mathrm{d}x + \frac{1}{2}\int_0^l m\left(\frac{\partial v_e}{\partial t}\right)^2 \mathrm{d}x, \tag{4}$$

in which E is Young's modulus; I is the second moment of the area; G is the shear modulus; A is the cross-sectional area; κ' is the shear coefficient depending on the shape of the cross-section; x is the local location along the axis of the beam element; ρ is the mass density of beam material and t is the time.

The generalized displacements of the beam element can be expressed in terms of the element nodal displacement vector $\{\mathbf{q}_e\}$ as

$$v_e = \begin{bmatrix} N_{v1} & N_{v2} & N_{v3} & N_{v4} \end{bmatrix} \begin{Bmatrix} v_{ei} \\ \theta_{ei} \\ v_{ej} \\ \theta_{ej} \end{Bmatrix} = [\mathbf{N}_v]\{\mathbf{q}_e\}, \tag{5}$$

$$\theta_e = \begin{bmatrix} N_{\theta 1} & N_{\theta 2} & N_{\theta 3} & N_{\theta 4} \end{bmatrix} \begin{Bmatrix} v_{ei} \\ \theta_{ei} \\ v_{ej} \\ \theta_{ej} \end{Bmatrix} = [\mathbf{N}_\theta]\{\mathbf{q}_e\}, \tag{6}$$

where $[\mathbf{N}_v]$ and $[\mathbf{N}_\theta]$ are the interpolation functions, given by

$$
\begin{aligned}
N_{w1} &= \left[1 - 3\xi^2 + 2\xi^3 + (1-\xi)\,\Phi\right] / \left(1+\Phi\right), \\
N_{w2} &= \left[\xi - 2\xi^2 + \xi^3 + \left(\xi - \xi^2\right)\Phi/2\right] l / \left(1+\Phi\right), \\
N_{w3} &= \left[3\xi^2 - 2\xi^3 + \xi\Phi\right] / \left(1+\Phi\right), \\
N_{w4} &= \left[-\xi^2 + \xi^3 - \left(\xi - \xi^2\right)\Phi/2\right] l / \left(1+\Phi\right), \\
N_{\theta 1} &= 6\left(-\xi + \xi^2\right) / \left[l\left(1+\Phi\right)\right], \\
N_{\theta 2} &= \left[1 - 4\xi + 3\xi^2 + (1-\xi)\,\Phi\right] / \left(1+\Phi\right), \\
N_{\theta 3} &= 6\left(\xi - \xi^2\right) / \left[l\left(1+\Phi\right)\right], \\
N_{\theta 4} &= \left[-2\xi + 3\xi^2 + \xi\Phi\right] / \left(1+\Phi\right),
\end{aligned}
\tag{7}
$$

where $\xi = x/l$ is the nondimensional axial coordinate and $\Phi = 12EI/(\kappa' GAl^2)$ is the shear deformation parameter. The flexural strain κ_e and the shear strain ϕ_e within the element are defined as

$$\kappa_e = \frac{\partial \theta_e}{\partial x} = [\mathbf{B}_b]\{\mathbf{q}_e\}, \tag{8}$$

$$\phi_e = \frac{\partial v_e}{\partial x} - \theta_e = [\mathbf{B}_s]\{\mathbf{q}_e\}, \tag{9}$$

in which

$$[\mathbf{B}_b] = \frac{1}{l}\frac{\partial}{\partial \xi}[\mathbf{N}_\theta], \tag{10}$$

$$[\mathbf{B}_s] = \frac{1}{l}\frac{\partial}{\partial \xi}[\mathbf{N}_v] - [\mathbf{N}_\theta] = [\mathbf{B}_v] - [\mathbf{N}_\theta]. \tag{11}$$

Substituting Eqs. (5)–(11) into Eqs. (3)–(4) and rearrangement of this equations. The strain U_e and kinetic energy T_e can be expressed in terms of the element nodal displacement vector $\{\mathbf{q}_e\}$ as

$$U_e = \frac{1}{2}\{\mathbf{q}\}_e^T[\mathbf{K}_v]\{\mathbf{q}\}_e^T + \frac{1}{2}\{\mathbf{q}\}_e^T[\mathbf{K}_\theta]\{\mathbf{q}\}_e^T + \frac{1}{2}\{\mathbf{q}\}_e^T[\mathbf{K}_w]\{\mathbf{q}\}_e^T + \frac{1}{2}\{\mathbf{q}\}_e^T[\mathbf{K}_s]\{\mathbf{q}\}_e^T, \tag{12}$$

$$T_e = \frac{1}{2}\{\dot{\mathbf{q}}\}_e^T[\mathbf{M}_v]\{\dot{\mathbf{q}}\}_e^T + \frac{1}{2}\{\dot{\mathbf{q}}\}_e^T[\mathbf{M}_\theta]\{\dot{\mathbf{q}}\}_e^T + \frac{1}{2}\{\dot{\mathbf{q}}\}_e^T[\mathbf{M}_F]\{\dot{\mathbf{q}}\}_e^T, \tag{13}$$

where the superposed dots denote differentiation with respect to time t.

It can be seen that the respective overall stiffness and mass element matrices include effect of $[\mathbf{K}_v]$, bending stiffness matrix; $[\mathbf{K}_\theta]$, shear stiffness matrix; $[\mathbf{K}_v]$, elastic foundation stiffness matrix; $[\mathbf{K}_s]$, shear foundation stiffness matrix; $[\mathbf{M}_v]$, consistent mass matrix for translational inertia; $[\mathbf{M}_\theta]$, consistent mass matrix for rotatory inertia; and $[\mathbf{M}_F]$, consistent mass matrix for foundation mass inertia, respectively, given by

$$[\mathbf{K}_e] = [\mathbf{K}_v] + [\mathbf{K}_\theta] + [\mathbf{K}_w] + [\mathbf{K}_s], \tag{14}$$

$$[\mathbf{M}_e] = [\mathbf{M}_v] + [\mathbf{M}_\theta] + [\mathbf{M}_F], \tag{15}$$

in which

$$[K_v] = \int_0^l [B_b]^T EI [B_b] \, l\mathrm{d}\xi, [K_\theta] = \int_0^l [B_s]^T \kappa GA [B_s] \, l\mathrm{d}\xi, \tag{16}$$

$$[K_w] = \int_0^l [N_v]^T k [N_v] \, l\mathrm{d}\xi, [K_s] = \int_0^l [N_s]^T k_s [N_s] \, l\mathrm{d}\xi, \tag{17}$$

and

$$[M_v] = \int_0^l [N_v]^T \rho A [N_v] \, l\mathrm{d}\xi, \quad [M_\theta] = \int_0^l [N_\theta]^T \rho I [N_\theta] \, l\mathrm{d}\xi, \tag{18}$$

$$[M_F] = \int_0^l [N_v]^T m [N_v] \, l\mathrm{d}\xi. \tag{19}$$

The viscous damping property of the foundation is considered to be the dashpots system, the damping matrix of foundation can be expressed as

$$[\mathbf{C}_e] = \int_0^l [\mathbf{N}_v]^T c [\mathbf{N}_v] \, l\mathrm{d}\xi. \tag{20}$$

It can be computed by using the dissipation energy of these dashpots.

2.3. The governing equation of motion

It is assumed that each axle vehicle always keeps in contact with the upper railway; the horizontal distance of two contact points between vehicle and beam is ξ_1 and ξ_2 from the left endpoint of the i_1-th and i_2-th beam elements at time t, respectively; and that for the convenience of describing the equations of motion of railway vehicle-track-foundation interaction element the i_1-th element and the i_2-th element are not adjacent, shown in Fig. 3. The governing differential equation for the displacement of moving railway vehicle can be obtained by considering the free bodies diagram shown in Fig. 4.

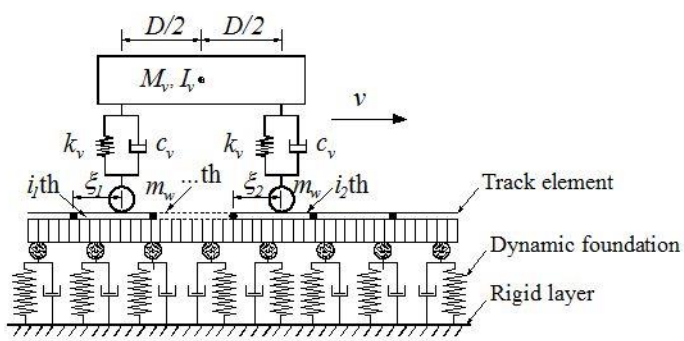


Fig. 3. Model of a railway vehicle-track-foundation interaction element

The interaction forces between the car body and each wheel are given by

$$\begin{aligned} &f_{vg} = M_v g, \quad f_{vI} = M_v \ddot{z}_v, \quad m_I = I_v \ddot{\varphi}_v, \quad f_{wI1} = m_w \ddot{z}_{w1}, \quad f_{wI2} = m_w \ddot{z}_{w2}, \\ &f_{s1} = k_v \left(z_v - 0.5\varphi_v D - z_{w1} - \delta_s \right), \quad f_{s2} = k_v \left(z_v + 0.5\varphi_v D - z_{w2} - \delta_s \right), \\ &f_{d1} = c_v \left(\dot{z}_v - 0.5\varphi_v D - z_{w1} \right), \quad f_{d2} = c_v \left(\dot{z}_v + 0.5\varphi_v D - z_{w2} \right), \quad f_{wg} = m_w g, \end{aligned} \tag{21}$$

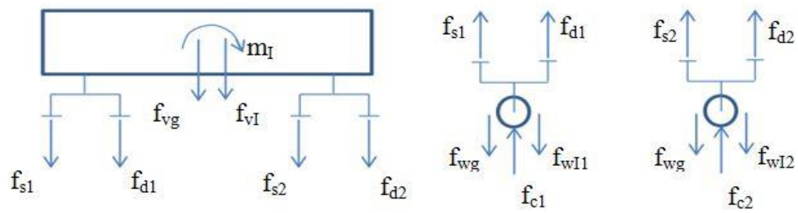


Fig. 4. Model of the free bodies diagram of moving railway vehicle

where δ_s is the static displacement of spring k_v, defined as

$$k_v \delta_s = \frac{1}{2} M_v g. \tag{22}$$

Based on the dynamic balance principle of each free body diagram, the governing equation of motion of the moving vehicle can be written as follows

$$M_v \ddot{z}_v + c_v \left(2\dot{z}_v - \dot{z}_{w1} - \dot{z}_{w2}\right) + k_v \left(2z_v - z_{w1} - z_{w2}\right) = 0, \tag{23}$$

$$I_v \ddot{\varphi}_v + 0.5D \left[c_v \left(D\dot{\varphi}_v + \dot{z}_{w1} - \dot{z}_{w2}\right) + k_v \left(D\varphi_v + z_{w1} - z_{w2}\right)\right] = 0, \tag{24}$$

$$m_w \ddot{z}_{w1} + c_v \left(-\dot{z}_v + 0.5\dot{\varphi}_v d + \dot{z}_{w1}\right) + k_v \left(-z_v + 0.5\varphi_v d + z_{w1}\right) = -\left(0.5M_v + m_w\right) g + f_{c1}, \tag{25}$$

$$m_w \ddot{z}_{w2} + c_v \left(-\dot{z}_v - 0.5\dot{\varphi}_v d + \dot{z}_{w2}\right) + k_v \left(-z_v - 0.5\varphi_v d + z_{w2}\right) = -\left(0.5M_v + m_w\right) g + f_{c2}, \tag{26}$$

or it can be expressed in the matrix form as

$$\begin{bmatrix} M_v & 0 & 0 & 0 \\ 0 & I_v & 0 & 0 \\ 0 & 0 & m_w & 0 \\ 0 & 0 & 0 & m_w \end{bmatrix} \begin{Bmatrix} \ddot{z}_v \\ \ddot{\varphi} \\ \ddot{z}_{w1} \\ \ddot{z}_{w2} \end{Bmatrix} + \begin{bmatrix} 2c_v & 0 & -c_v & -c_v \\ 0 & 0.5D^2 c_v & 0.5Dc_v & -0.5Dc_v \\ -c_v & 0.5Dc_v & c_v & 0 \\ -c_v & -0.5Dc_v & 0 & c_v \end{bmatrix} \begin{Bmatrix} \dot{z}_v \\ \dot{\varphi} \\ \dot{z}_{w1} \\ \dot{z}_{w2} \end{Bmatrix} +$$
$$\begin{bmatrix} 2k_v & 0 & -k_v & -k_v \\ 0 & 0.5D^2 k_v & 0.5Dk_v & -0.5Dk_v \\ -z_v & 0.5Dk_v & k_v & 0 \\ -z_v & -0.5Dk_v & 0 & k_v \end{bmatrix} \begin{Bmatrix} z_v \\ \varphi \\ z_{w1} \\ z_{w2} \end{Bmatrix} \tag{27}$$
$$= \begin{Bmatrix} 0 \\ 0 \\ -\left(0.5M_v + m_w\right) g \\ -\left(0.5M_v + m\right)_w g \end{Bmatrix} + \begin{Bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{Bmatrix} \begin{Bmatrix} f_{c1} \\ f_{c2} \end{Bmatrix},$$

where f_{c1} and f_{c2} are the contact forces between the rear and front axle and railway, respectively,

$$\begin{aligned} f_{c1} &= m_w \ddot{z}_{w1} + 0.5M_v \ddot{z}_v - I_v {}^{\ddot{\varphi}_v}\!/_{d} + \left(0.5M_v + m_w\right) g, \\ f_{c2} &= m_w \ddot{z}_{w2} + 0.5M_v \ddot{z}_v + I_v {}^{\ddot{\varphi}_v}\!/_{d} + \left(0.5M_v + m_w\right) g. \end{aligned} \tag{28}$$

It is assumed that each axle vehicle is assumed to be always in contact with the railway, the displacement, velocity and acceleration field of each axle at the contact position of each axle (ξ_1 and ξ_2), can be expressed in terms of the nodal displacement, velocity and acceleration vector as

$$\begin{aligned} z_{w1} &= [\mathbf{N}_{v1}]\{\mathbf{q}_e\}_{i_1th}, \quad \dot{z}_{w1} = [\mathbf{N}_{v1}]\{\dot{\mathbf{q}}_e\}_{i_1th}, \quad \ddot{z}_{w1} = [\mathbf{N}_{v1}]\{\ddot{\mathbf{q}}_e\}_{i_1th}, \\ z_{w2} &= [\mathbf{N}_{v2}]\{\mathbf{q}_e\}_{i_2th}, \quad \dot{z}_{w2} = [\mathbf{N}_{v2}]\{\dot{\mathbf{q}}_e\}_{i_2th}, \quad \ddot{z}_{w2} = [\mathbf{N}_{v2}]\{\ddot{\mathbf{q}}_e\}_{i_2th}, \end{aligned} \tag{29}$$

where $[\mathbf{N}_{v1}]$ and $[\mathbf{N}_{v2}]$ are the values of interpolation function depending on the coordinate ξ_1 and ξ_2 corresponding with the position of each axle on the railway element i_1-th and i_2-th at the time t, respectively. At each time step, the governing differential equation of the railway element resting on the dynamic foundation subjected to moving railway vehicle at time t can be expressed as

$$[\mathbf{M}_e]\{\ddot{\mathbf{q}}_e\} + [\mathbf{C}_e]\{\dot{\mathbf{q}}_e\} + [\mathbf{K}_e]\{\mathbf{q}_e\} = -\delta(x_i - vt)[\mathbf{N}_{vi}] f_{ci}, \tag{30}$$

where $\delta(x_i - vt)$ is the Dirac delta function and i (i_1-th or i_2-th) denotes contact element between railway and contact force, respectively. By means the finite element theory, the corresponding degrees of freedom of the railway element on dynamic foundation and moving railway vehicle (Eqs. (23) and (24)) are connected in the global coordinate, the equation of motion of the system railway track-vehicle interaction on dynamic foundation in each time step is defined as follows

$$\mathbf{M}\{\ddot{\mathbf{U}}\} + \mathbf{C}\{\dot{\mathbf{U}}\} + \mathbf{K}\{\mathbf{U}\} = \{\mathbf{F}\}, \tag{31}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the overall mass, damping and stiffness matrices of the system, respectively; $\{\mathbf{U}\}$ and $\{\mathbf{F}\}$ are the nodal displacement vector and the external force vector of the system, respectively. Eq. (31) is used for studying the dynamic response of the railway track-vehicle-foundation interaction and solved by means of the direct step-by-step integration method based on Newmark's algorithm.

3. NUMERICAL RESULTS

3.1. Verified examples

Before studying numerical results, in order to verify the accuracy of the above formulation and the computer program using MATLAB developed by the authors, the results obtained from the present study are compared with available results in the literature. In the first example, free vibration of a simple support Timoshenko beam resting on a dynamic foundation without the effect of foundation mass is analyzed. The dimensionless parameters K_1 and K_2 representing the stiffness of the springs and shear layer of the dynamic foundation and the dimensionless natural frequency λ are defined as follows

$$K_1 = \frac{kL^4}{EI}, \quad K_2 = \frac{k_S L^2}{\pi^2 EI}, \quad \lambda = \omega L^2 \sqrt{\frac{\rho A}{EI}}, \tag{32}$$

where ω is the natural circular frequency of the beam. The convergences of the lowest natural frequencies are compared with the results in the literature, shown in Tab. 1.

In the next example, the dynamic response of a simple support beam without foundation subjected to a moving two-axle vehicle is investigated. The data adopted for the beam and moving vehicle are similar to those in [28]. The time history of vertical displacement of the midpoint is compared with those of this literature, shown in Fig. 5.

Table 1. The dimensionless natural frequencies λ of the beam

L/h	K_1	K_2	Present	Reference [31]
10	0	1	13.8163	13.8162
	10		14.1708	14.1709
	10^2		17.0327	17.0326
	10^3		34.3964	34.3963
	10^4		100.5570	100.5564
	10^5		313.8910	314.9778

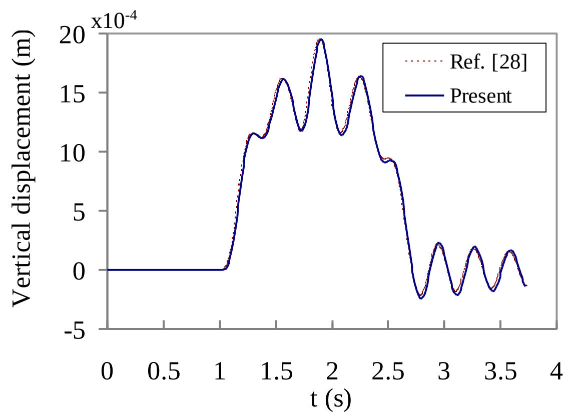


Fig. 5. Time history of vertical displacement of the midpoint of the beam

Through above examples, the numerical results from the program based on the suggested formulation show quite good agreement with numerical results in the literature. Therefore, the program which will analyze the influence of many parameters on the dynamic response of the railway track-vehicle-foundation interaction is reliable.

3.2. Numerical investigation

In this section, the effects of foundation mass on the dynamic response of railway track-vehicle-foundation interaction are analyzed. The physical and geometric properties of the railway track, vehicle and dynamic foundation are listed in Tab. 2.

In this subsection the influence of foundation mass parameter on the dynamic behavior of the railway track-vehicle interaction is investigated. The time history vertical displacement of the midpoint of the track is plotted in Fig. 6.

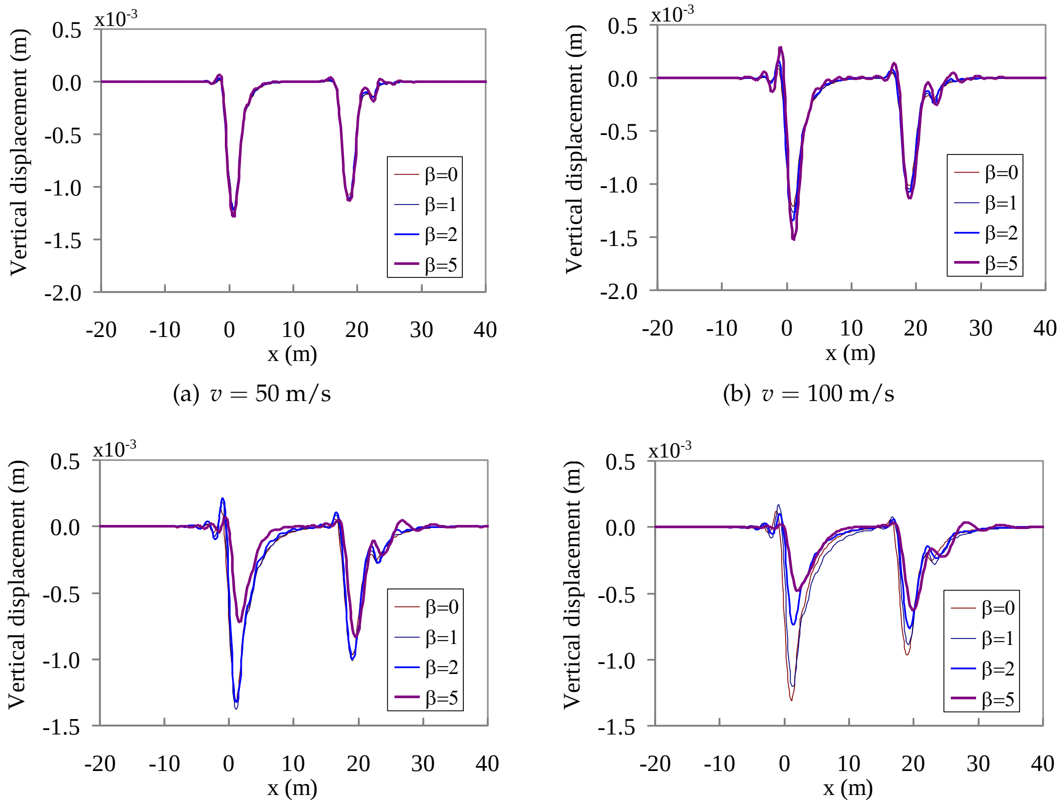
It is seen that the mass density of dynamic foundation effects significantly on the dynamic analysis of the railway track-vehicle-foundation interaction. It increases the vertical displacement of the midpoint of the track (see in Figs. 6(a) and 6(b)) but sometimes it also decreases the dynamic response of the track with an increase of value of the foundation mass parameter (see in Figs. 6(c) and 6(d)). Hence, it can be seen that the

Table 2. Properties of railway track, vehicle and dynamic foundation

Item	Notation	Unit	Value
Railway track [32]			
Young's modulus	E	GPa	210
Shear modulus	G	GPa	77
Mass density	ρ	kg/m^3	7850
Cross sectional area	A	m^2	7.690×10^{-3}
Second moment of area	I	m^4	3.055×10^{-5}
Shear coefficient	κ		0.4
Moving Railway Vehicle [28]			
Mass of car body	M_v	kg	4.8×10^4
Mass moment of inertia	I_v	kgm^2	2.5×10^6
Spring stiffness	k_v	N/m	1.5×10^6
Dashpot coefficient	c_v	Ns/m	8.5×10^4
Mass of each axle	m_w	kg	5×10^3
Longitudinal distance	D	m	18
Dynamic foundation [32]			
Elastic parameter	k	N/m^2	10^8
Shear parameter	k_s	N	66687500
Viscous damping	c	Ns/m^2	1.5×10^6
Foundation mass density	ρ_f	kg/m^3	1800

mass density of dynamic foundation has an influence on the dynamic response of the railway track-vehicle-foundation interaction.

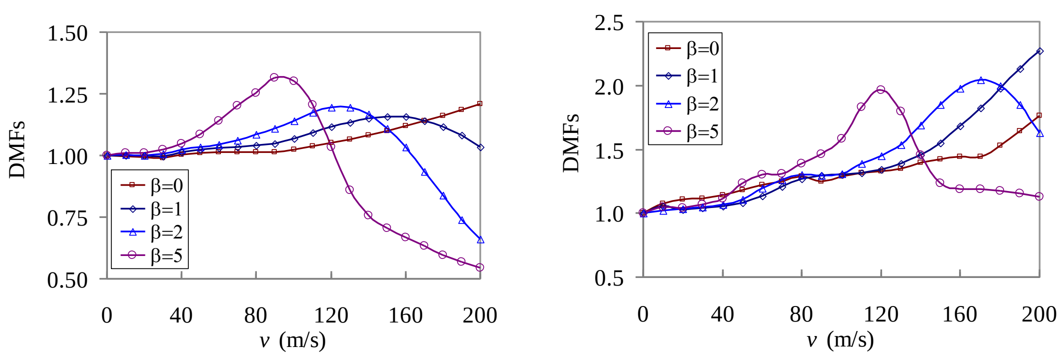
To show more clearly the influence of the foundation mass parameter on dynamic analysis of the interaction between railway track-vehicle and foundation, the effects of the mass density of foundation on dynamic magnification factor (DMF) are investigated for various values of the velocity of the moving vehicle. Figs. 7–9 show that in the range of high velocity, the effects of the mass density of foundation on the DMFs of vertical displacement of the track are so quite clear, and the comparisons show that the mass density of foundation increases and also decreases the DMFs of the track for the present model more than for the foundation model without the mass density ($\beta = 0$).



(a) $v = 50$ m/s (b) $v = 100$ m/s

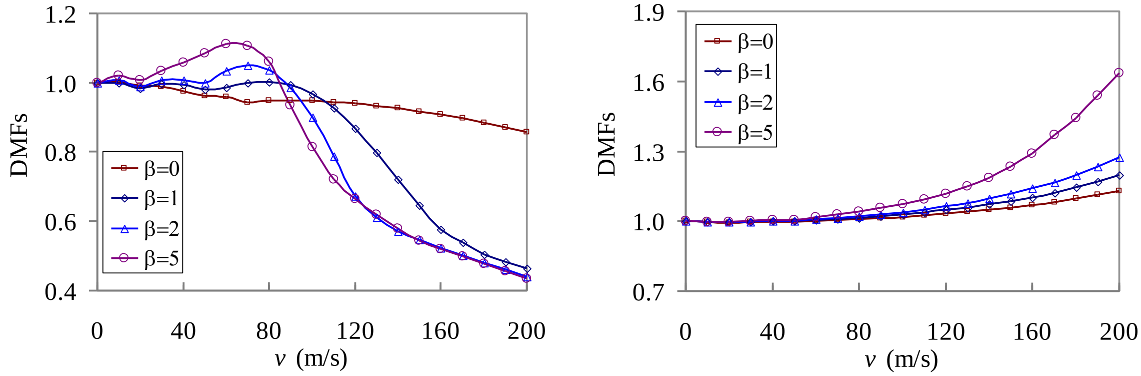
(c) $v = 150$ m/s (d) $v = 200$ m/s

Fig. 6. Time history of vertical displacement of the midpoint of the track



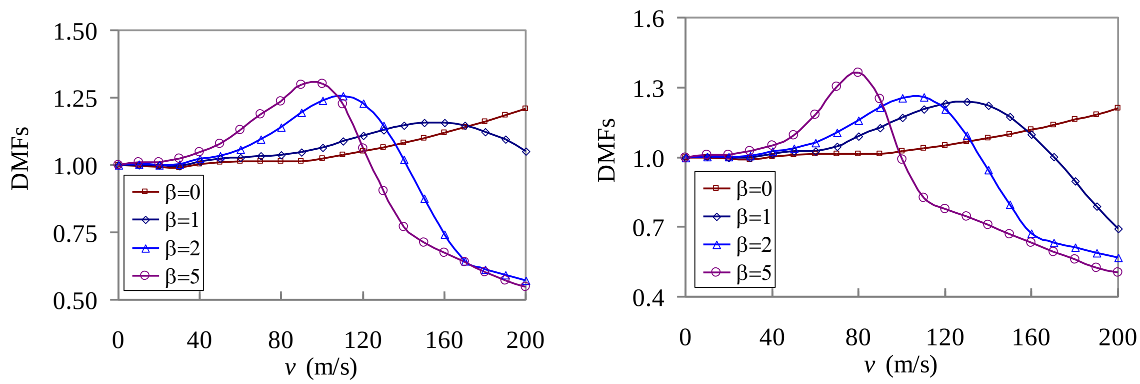
(a) $k = 10^8$ N/m^2 (b) $k = 5 \times 10^8$ N/m^2

Fig. 7. The DMFs of vertical displacement of the railway track for various spring stiffness



(a) $k_s = 10^7$ N (b) $k_s = 10^9$ N

Fig. 8. The DMFs of vertical displacement of the railway track for various shear layer stiffness



(a) $\rho_f = 1700\ \mathrm{kg/m^3}$ (b) $\rho_f = 1900\ \mathrm{kg/m^3}$

Fig. 9. The DMFs of vertical displacement of the railway track for the various mass density of foundation

4. CONCLUSIONS

The influences of foundation mass density on dynamic analysis of railway track under moving railway vehicle are investigated by means of the finite element method. The railway track modeled as a Timoshenko beam subjected to a moving vehicle modeled as a two-axle mass-spring-damper system on a dynamic foundation model including linear elastic spring, shear layer, viscous damping and mass density of foundation. The railway track, vehicle and dynamic foundation are regarded as an integrated system and the governing equation of motion of the system is solved by the step-by-step integration method. The parametric analysis has been performed to investigate the effects of mass density of dynamic foundation with various values of the stiffness of foundation and velocity of the vehicle on the dynamic analysis of the system. A comparison shows that the mass density of the dynamic foundation effects significantly on the dynamic response of the system in the range of high velocity.

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