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OPTIMAL CONTROL OF TRANSVERSE VIBRATION OF EULER–BERNOULLI BEAM WITH MULTIPLE DYNAMIC VIBRATION ABSORBERS USING TAGUCHI'S METHOD

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Abstract. Vibration absorbers are frequently used to suppress the excessive vibrations in structural systems. In this paper, an imposing nodes technique is applied for vibration suppression of Euler–Bernoulli beams subjected to forced harmonic excitations by means of multiple dynamic vibration absorbers. A procedure based on Taguchi's method is proposed to determine the optimum absorber parameters to suppress the vibration amplitude of the beams. Numerical tests are performed to show the effectiveness of the proposed procedure.

Keywords: beam structures, dynamic vibration absorber, Taguchi's method, harmonic excitations, passive vibration control.

1. INTRODUCTION

Beams are conventional constructions such as house beams, suspended cable in suspension structures, air traffic control towers, wind turbine columns, etc. In the course of the work, these structures are exposed to the effects of wind exploitation and operation. Under the influence of changing external frequencies, beam structures appear to be subjected to forced vibration. Since the frequency of the external force changes over a wide band, there is the possibility of a resonance which can cause structural damage. Therefore, the reduction of the amplitude range of the structure at resonant frequency is a necessary task. In order to determine the resonance frequencies and to investigate the behavior of the system under the action of external forces, the natural frequencies of the structure must first be evaluated [1, 2]. Vibration amplitudes at different points in the structure excited by the external force in a wide frequency band are then calculated. Dynamic dampers are usually used to reduce the vibrations of the beams.

Besides, undesirable vibrations in mechanical structures have been effectively reduced by the application of dynamic vibration absorbers (DVA). The reasons for those applications of the DVA are its efficient, reliable and low-cost characteristics [3, 4]. Recently, considerable effort has been devoted to devise effective control strategies for this task. Jacquot [5] developed a technique to give the optimal parameters of DVA for the elimination of excessive vibration in sinusoidally forced Bernoulli–Euler beams. Ozguven and Candir [6] presented a procedure for determining the optimum parameters of two DVAs attached to a beam to suppress any two resonances. Lin and Cho [7] investigated dynamic characteristics of a simply supported beam traversed by multiple moving loads and a practical scheme for suppressing the resulting resonant or excessive vibration by using a damped absorber. In [8] Chtiba and his colleagues have proposed a new strategy for the optimal design of supplementary absorbers that warrant confinement with and without suppression of vibration in flexible structures. In [9] Vestroni et al. studied the pedestrian-induced vibrations in suspension footbridges via multiple tuned mas dampers. The solutions to H_∞ and H_2 optimization problems of a variant dynamic absorber applied to suppress vibration in beam structures are derived analytically by Noori and Farshidianfar [10]. The calculating results of Samani and his colleagues [11] show that, for the test cases considered, the DVAs with essentially nonlinear stiffnesses having higher power are more effective than the linear one in reducing the maximum beam deflection. In [12] Patil and Awasare have used variable stiffness vibration neutralizers to impose zero displacement or nodes to reduce vibration at desired locations on a Euler–Bernoulli beam subjected to forced harmonic excitation. Latas [13, 14] discussed the problem of optimal choice of position and parameters of the system of translational and rotational dynamic absorbers in beams.

The study of optimal design of parameters of dynamic vibration absorber installed in beam structures becomes an interesting problem in recent years. It is well known that Taguchi's method for the product design process may be divided into three stages: system design, parameter design, and tolerance design [15–22]. Taguchi's method of parameter design is successfully applied to many mechanical systems: acoustic muffler, gear/pinion system, spring, electro-hydraulic servo system, dynamic vibration absorber. In each system, the design parameters to be optimized are identified, along with the desired response. The present study deals with the determination of the optimal parameters of DVAs for the vibration reduction of Euler–Bernoulli beams subjected to forced harmonic excitation using Taguchi's method. The target function is determined by suppressing the resonance vibrations of beams.

2. DERIVATION OF TRANSVERSE VIBRATION EQUATIONS OF BEAM WITH DYNAMIC VIBRATION ABSORBERS

Let us consider the model of a Euler–Bernoulli beam of the length L and the flexural rigidity EI, which is attached with a number of DVAs at positions $x = \eta_j$ $(j = 1, \ldots, n_a)$ as shown in Fig. 1. For simplicity, it is assumed that the considered beam is homogeneous with a uniform cross section, where w is the dynamic deflection, u_j is vertical coordinate of j-absorber, k are stiffness, damping coefficients and mass of j-absorber, respectively, and $p(x, t)$ is the distributed force.

Using the method of substructures, the system is now divided into $n_a + 1$ substructures, namely, the beam structure and n_a absorbers (Fig. 2).

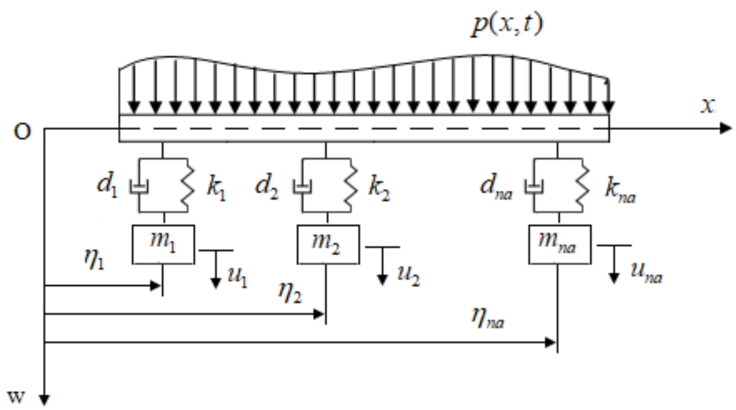
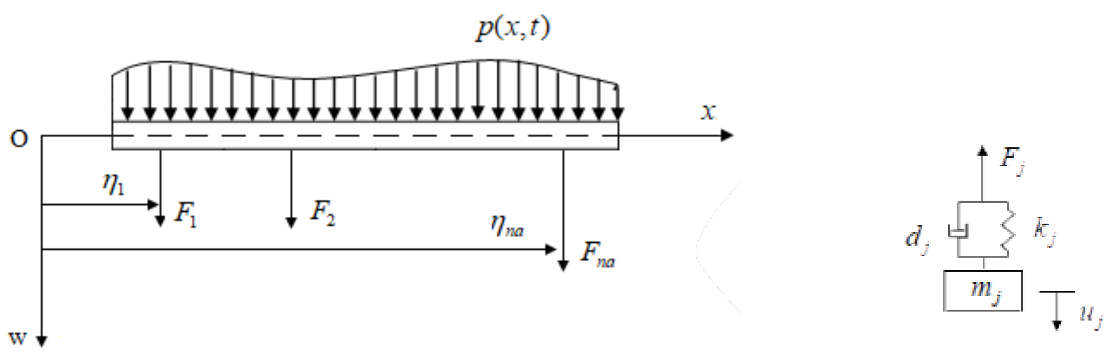


Fig. 1. Beam with dynamic vibration absorbers



(a) Beam (b) Absorber

Fig. 2. Substructures

Reaction forces have the following form

$$F_j(t) = k_j(u_j - w_{\eta j}) + d_j(\dot{u}_j - \dot{w}_{\eta j}), \quad j = (1, 2, \ldots, n_a), \tag{1}$$

where

$$w_{\eta j} = w(\eta_j, t), \quad \dot{w}_{\eta j} = \frac{\partial w(\eta_j, t)}{\partial t}.$$

Using Newton's second law, the equation describing the vibration of j-absorber can be expressed in the form

$$m_j \ddot{u}_j = -F_j \Rightarrow F_j(t) = -m_j \ddot{u}_j. \tag{2}$$

Substitution of Eq. (2) into Eq. (1) yields the vibration equation of j-absorber

$$m_j \ddot{u}_j + d_j \dot{u}_j + k_j u_j = k_j w_{\eta j} + d_j \dot{w}_{\eta j}, \quad (j = 1, 2, \ldots, n_a). \tag{3}$$

Applying the basic principles of dynamics, the equation that describes transverse vibration of beam including internal friction is [1,2]

$$\mu\left(\frac{\partial^2 w}{\partial t^2}+c^{(e)}\frac{\partial w}{\partial t}\right)+EI\left[\frac{\partial^4 w}{\partial x^4}+c^{(i)}\frac{\partial^5 w}{\partial x^4\partial t}\right]=p(x,t)+\sum_{j=1}^{n_a}F_j\delta(x-\eta_j). \tag{4}$$

In Eq. (4), μ denotes mass per length unit, $c^{(e)}, c^{(i)}$ are damping coefficient and internal friction coefficient per length unit of beam, respectively, and the Delta-Dirac function $\delta(x-\eta_j)$ is defined by

$$\delta(x-\eta_j)=\begin{cases}1 & \text{when } x=\eta_j\\ 0 & \text{when } x\neq\eta_j\end{cases} \tag{5}$$

The vibration equations according to Eqs. (3) and (4) are a mixed set of ordinary and partial differential equations. Four boundary conditions, two at $x=0$ and two at $x=L$, and the initial conditions must be specified to find the solution of this set.

Using Ritz–Galerkin method, the solution of Eqs. (3) and (4) can be found in the form

$$w(x,t)=\sum_{r=1}^{n_b}X_r(x)q_r(t), \tag{6}$$

where $X_r(x)$ denotes the mode shape of the beam and $q_r(t)$ is the generalized displacement to be determined. Substituting Eq. (6) into Eqs. (4) and (3), we find

$$\begin{aligned}&\ddot{q}_k(t)+(c^{(e)}+c^{(i)}\omega_k^2)\dot{q}_k(t)+\omega_k^2 q_k(t)\\&=\frac{\int_0^l p(x,t)X_k(x)\mathrm{d}x}{\mu\int_0^l X_k^2(x)\mathrm{d}x}+\frac{\sum_{j=1}^{n_a}\int_0^l F_j(t)X_k(x)\delta(x-\eta_j)\mathrm{d}x}{\mu\int_0^l X_k^2(x)\mathrm{d}x},\quad (k=1,\ldots,n_b)\end{aligned} \tag{7}$$

and

$$m_j\ddot{u}_j(t)+d_j\dot{u}_j(t)+k_ju_j(t)-d_j\sum_{r=1}^{n_b}X_r(\eta_j)\dot{q}_r(t)-k_j\sum_{r=1}^{n_b}X_r(\eta_j)q_r(t)=0,\ (j=1,2,\ldots,n_a) \tag{8}$$

where ω_k is eigenfrequency of the beam [1,2]. Using the notations

$$2\delta_k=(c^{(e)}+c^{(i)}\omega_k^2),\quad D_k=\mu\int_0^l X_k^2(x)\mathrm{d}x=\text{const}. \tag{9}$$

it follows from Eq. (7) that

$$\ddot{q}_k(t)+2\delta_k\dot{q}_k(t)+\omega_k^2q_k(t)=h_k(t)+\frac{1}{D_k}\sum_{j=1}^{n_a}\int_0^l F_j(t)X_k(x)\delta(x-\eta_j)\mathrm{d}x. \tag{10}$$

$$\alpha_k(t) = \int_0^l p(x,t)X_k(x)\mathrm{d}x, \quad h_k(t) = \frac{\alpha_k(t)}{D_k} \tag{11}$$

Substitution of Eq. (6) into Eq. (1) yields

$$\begin{aligned} F_j(t) &= d_j\left[\dot{u}_j(t) - \dot{w}_{\eta j}\right] + k_j\left[u_j(t) - w_{\eta j}\right] \\ &= d_j\dot{u}_j(t) + k_j u_j(t) - d_j\sum_{r=1}^{n_b} X_r(\eta_j)\dot{q}_r(t) - k_j\sum_{r=1}^{n_b} X_r(\eta_j)q_r(t), \end{aligned} \tag{12}$$

in which $d_j, k_j, X_r(\eta_j)\ (j = 1, 2, \ldots, n_a)$ are the known constants. According to the property of the Delta–Dirac function we have

$$\sum_{j=1}^{n_a}\int_0^l F_j(t)X_k(x)\delta(x-\eta_j)\mathrm{d}x = \sum_{j=1}^{n_a} F_j(t)X_k(\eta_j). \tag{13}$$

Substitution of Eq. (12) into Eq. (13) one obtains

$$\begin{aligned} \sum_{j=1}^{n_a} F_j(t)X_k(\eta_j) =& \sum_{j=1}^{n_a} d_j\dot{u}_j(t)X_k(\eta_j) + \sum_{j=1}^{n_a} k_j u_j(t)X_k(\eta_j) \\ &- \sum_{j=1}^{n_a}\sum_{r=1}^{n_b} d_j X_r(\eta_j)X_k(\eta_j)\dot{q}_r(t) - \sum_{j=1}^{n_a}\sum_{r=1}^{n_b} k_j X_r(\eta_j)X_k(\eta_j)q_r(t) \\ =& \sum_{j=1}^{n_a} X_k(\eta_j)\left[d_j\dot{u}_j(t) + k_j u_j(t)\right] - \sum_{j=1}^{n_a} X_k(\eta_j)\left\{\sum_{r=1}^{n_b}\left[d_j\dot{q}_r(t) + k_j q_r(t)\right]X_r(\eta_j)\right\}. \end{aligned} \tag{14}$$

Substitution of Eq. (14) into Eq. (10) yields

$$\begin{aligned} \ddot{q}_k(t) + 2\delta_k\dot{q}_k(t) + \omega_k^2 q_k(t) &= h_k(t) + \frac{1}{D_k}\sum_{j=1}^{n_a} X_k(\eta_j)\left[d_j\dot{u}_j(t) + k_j u_j(t)\right] \\ -\frac{1}{D_k}\sum_{j=1}^{n_a} X_k(\eta_j)&\left\{\sum_{r=1}^{n_b}\left[d_j\dot{q}_r(t) + k_j q_r(t)\right]X_r(\eta_j)\right\}, \quad (k = 1, 2, \ldots, n_b). \end{aligned} \tag{15}$$

It follows from Eq. (8) that

$$\sum_{r=1}^{n_b}\left[d_j\dot{q}_r(t) + k_j q_r(t)\right]X_r(\eta_j) = m_j\ddot{u}_j(t) + d_j\dot{u}_j(t) + k_j u_j(t) \tag{16}$$

By substituting Eq. (16) into Eq. (15) leads to

$$\begin{aligned} \ddot{q}_k(t) + 2\delta_k\dot{q}_k(t) + \omega_k^2 q_k(t) &= h_k(t) + \frac{1}{D_k}\sum_{j=1}^{n_a} X_k(\eta_j)\left[d_j\dot{u}_j(t) + k_j u_j(t)\right] \\ -\frac{1}{D_k}\sum_{j=1}^{n_a} X_k(\eta_j)&\left\{m_j\ddot{u}_j(t) + d_j\dot{u}_j(t) + k_j u_j(t)\right\}, \ k = 1, 2, \ldots, n_b. \end{aligned} \tag{17}$$

Eqs. (17) and (8) consist of a system of $n = n_a + n_b$ ordinary differential equations that describes the vibration of the beam with dynamic vibration absorbers.

We consider now the vibration of the beams under harmonic excitation $p(t) = p_0 \sin \Omega t$. According to Eqs. (8) and (11) we have

$$\alpha_k(t) = \left[p_0 \int_0^l X_k(x) \mathrm{d}x \right] \sin \Omega t, \quad h_k(t) = \frac{p_0 \int_0^l X_k(x) \mathrm{d}x}{D_k} \sin \Omega t.$$

It follows that

$$h_k(t) = \widehat{h}_k \sin \Omega t, \quad \widehat{h}_k = \frac{p_0 \int_0^l X_k(x) \mathrm{d}x}{D_k}. \tag{18}$$

In this case, Eq. (17) has the following form

$$\ddot{q}_k(t) + 2\delta_k \dot{q}_k(t) + \omega_k^2 q_k(t) = \widehat{h}_k \sin \Omega t - \frac{1}{D_k} \sum_{j=1}^{n_a} m_j X_k(\eta_j) \ddot{u}_j \,, \quad k = (1, 2 \ldots, n_b). \tag{19}$$

Using the notations

$$\frac{d_j}{m_j} = 2\delta_{jc} \,, \quad \frac{k_j}{m_j} = \omega_{jc}^2 \,,$$

it follows from Eq. (8) that

$$\ddot{u}_j(t) + 2\delta_{jc} \dot{u}_j(t) + \omega_{jc}^2 u_j(t) = 2\delta_{jc} \sum_{r=1}^{n_b} X_r(\eta_j) \dot{q}_r(t) + \omega_{jc}^2 \sum_{r=1}^{n_b} X_r(\eta_j) q_r(t) \,, \quad (j = 1, 2, \ldots, n_a). \tag{20}$$

Eqs. (19) and (20) are a system of $n = n_a + n_b$ differential equations describing the transverse vibration of beam with a lot of dynamic vibration absorbers under the harmonic distributed force, in which $X_k(x)$ is the eigenfunction of beam. The concrete form of $X_k(x)$ depends on the boundary conditions of beam.

3. THE COMPLEX FREQUENCY RESPONSE FUNCTION

In this section, we consider cases that often occur in structural, when the excitation frequency Ω is approximately equal to the fundamental frequency ω_1 of the beam ($\Omega \approx \omega_1$). It follows from Eq. (19) and Eq. (20) that

$$\ddot{q}_1(t) + 2\delta_1 \dot{q}_1(t) + \omega_1^2 q_1(t) = \widehat{h}_1 \sin \Omega t - \frac{1}{D_1} \sum_{j=1}^{n_a} m_j X_k(\eta_j) \ddot{u}_j \,, \tag{21}$$

$$\ddot{u}_j(t) + 2\delta_{jc} \dot{u}_j(t) + \omega_{jc}^2 u_j(t) = 2\delta_{jc} X_1(\eta_j) \dot{q}_1(t) + \omega_{jc}^2 X_1(\eta_j) q_1(t), \ (j = 1, 2, \ldots, n_a). \tag{22}$$

We use the following notations

$$x_s = q_1 \,, \quad \omega_s = \omega_1 \,, \quad \delta_s = \delta_1 \,, \quad \widehat{h}_s = \widehat{h}_1 \,, \tag{23}$$

Eq. (21) can be written in the following form

$$\ddot{x}_s(t) + 2\delta_s \dot{x}_s(t) + \omega_s^2 x_s(t) = \widehat{h}_1 \sin \Omega t - \frac{1}{D_1} \sum_{j=1}^{n_a} m_j X_1(\eta_j) \ddot{u}_j. \tag{24}$$

The solution of Eqs. (22) and (24) can now be found using the method of frequency response function. We note that $\cos \Omega t = \operatorname{Re} e^{i\Omega t}$, $\sin \Omega t = \operatorname{Im} e^{i\Omega t}$, Eq. (24) can thus be written as follows

$$\ddot{x}_s(t) + 2\delta_s \dot{x}_s(t) + \omega_s^2 x_s(t) = \widehat{h}_s e^{i\Omega t} - \frac{1}{D_1} \sum_{j=1}^{n_a} m_j X_1(\eta_j) \ddot{u}_j. \tag{25}$$

We find the solutions of Eqs. (22) and (25) in the form

$$x_s(t) = H_s e^{i\Omega t}, \;\; u_j(t) = H_{jc} e^{i\Omega t}. \tag{26}$$

Substitution of Eq. (26) into Eqs. (25) and (22) lead to the system of linear algebraic equations

$$\left[\omega_s^2 - \Omega^2 + i2\delta_s \Omega\right] H_s = \widehat{h}_s + \frac{\Omega^2}{D_1} \sum_{j=1}^{n_a} m_j X_1(\eta_j) H_{jc}\,, \tag{27}$$

$$\left[\omega_{jc}^2 - \Omega^2 + 2i\delta_{jc}\Omega\right] H_{jc} = \left[\omega_{jc}^2 + 2i\delta_{jc}\Omega\,\right] X_1(\eta_j) H_s\,, \;\; (j = 1, 2, \ldots, n_a). \tag{28}$$

From Eq. (28) one has

$$H_{jc} = \frac{\left[\omega_{jc}^2 + i2\delta_{jc}\Omega\,\right]}{\left[\omega_{jc}^2 - \Omega^2 + i2\delta_{jc}\Omega\right]} X_1(\eta_j) H_s\,, \;\; (j = 1, 2, \ldots, n_a). \tag{29}$$

Substitution of Eq. (29) into Eq. (27) yields

$$\left[\omega_s^2 - \Omega^2 + i2\delta_s \Omega\right] H_s = \widehat{h}_s + H_s \frac{\Omega^2}{D_1} \sum_{j=1}^{n_a} \left(m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2 + i2\delta_{jc}\Omega\,\right]}{\left[\omega_{jc}^2 - \Omega^2 + i2\delta_{jc}\Omega\right]} \right). \tag{30}$$

By introducing

$$a_{11} = \sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2 + i2\delta_{jc}\Omega\,\right]}{\left[\omega_{jc}^2 - \Omega^2 + i2\delta_{jc}\Omega\,\right]}. \tag{31}$$

Eq. (30) can be written in the following form

$$\left(\left[\omega_s^2 - \Omega^2 + i2\delta_s \Omega\right] + \frac{\Omega^2}{D_k} a_{11} \right) H_s = \widehat{h}_s. \tag{32}$$

From Eq. (32) one has

$$H_s = \frac{\widehat{h}_s}{\left[\omega_s^2 - \Omega^2 + i2\delta_s \Omega\right] + \dfrac{\Omega^2}{D_1} a_{11}}, \tag{33}$$

in which

$$
\begin{aligned}
a_{11} &= \sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2 + i2\delta_{jc}\Omega\right]}{\left[\omega_{jc}^2 - \Omega^2 + i2\delta_{jc}\Omega\right]} \\
&= \sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2 + i2\delta_{jc}\Omega\right]\left[\omega_{jc}^2 - \Omega^2 - i2\delta_{jc}\Omega\right]}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]} \\
&= \sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2\left(\omega_{jc}^2 - \Omega^2\right) + 4\delta_{jc}^2\Omega^2 - 2i\delta_{jc}\Omega^3\right]}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]}.
\end{aligned} \tag{34}
$$

The denominator in Eq. (33) takes the form

$$
\begin{aligned}
&\left[\omega_s^2 - \Omega^2 + i2\delta_1\Omega\right] + \frac{\Omega^2}{D_1}a_{11} = \left[\omega_s^2 - \Omega^2 + i2\delta_s\Omega\right] \\
&+ \frac{\Omega^2}{D_1}\sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2\left(\omega_{jc}^2 - \Omega^2\right) + 4\delta_{jc}^2\Omega^2\right]}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]} - i\sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{2\Omega^3\delta_{jc}}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]}.
\end{aligned}
$$

By introducing the following notations

$$
a = \left[\omega_s^2 - \Omega^2\right] + \frac{\Omega^2}{D_1}\sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{\left[\omega_{jc}^2\left(\omega_{jc}^2 - \Omega^2\right) + 4\delta_{jc}^2\Omega^2\right]}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]}, \tag{35}
$$

$$
b = 2\delta_s\Omega - \frac{\Omega^2}{D_1}\sum_{j=1}^{n_a} m_j X_1^2(\eta_j) \frac{2\Omega^3\delta_{jc}}{\left[\left(\omega_{jc}^2 - \Omega^2\right)^2 + 4\delta_{jc}^2\Omega^2\right]}, \tag{36}
$$

function H_s can be written in the form

$$
H_s = \frac{\widehat{h}_s}{a + ib} = \frac{\widehat{h}_s(a - ib)}{a^2 + b^2}. \tag{37}
$$

The modulus of the complex frequency response function H_s can now be calculated by the following formula

$$
H = \left|\widetilde{H}_s\right| = \frac{\widehat{h}_s}{\sqrt{a^2 + b^2}}. \tag{38}
$$

The optimum problem is stated as follows: Find the parameters of dynamic vibration absorbers m_j, c_j, k_j $(j = 1, 2, \ldots, n_a)$ which minimize the objective function according to Eq. (38).

4. USING THE TAGUCHI METHOD FOR DETERMINATION OF THE OPTIMUM ABSORBER PARAMETERS

4.1. The idea of Taguchi method

Taguchi developed his methods in the 1950s and 1960s. Taguchi's approach to the product design process may be divided into three stages: system design, parameter design, and tolerance design [14–16]. System design is the conceptual design stage where the system configuration is developed. Parameter design, sometime called robust design, identifies factors that reduce the system sensitivity to noise, thereby enhancing the system's robustness. Tolerance design specifics the allowable deviations in the parameter values, loosening tolerances if possible and tightening tolerances if necessary. Taguchi's objective functions for robust design arise from quality measures using quadratic loss functions. In the extension of this definition to design optimisation. Taguchi suggested the signal-to-noise (SNR), $-10\log_{10}$ (MSD), as a measure of the mean squared deviation (MSD) in the performance. The use of SNR in system analysis provides a quantitative value for response variation comparison.

The mathematical basis of the Taguchi method is mathematical methods of statistics. The Taguchi method allows determining the optimal condition of many parameters of the research object. This method is applied to solve the multi-objective optimization problem in mechanical engineering, civil engineering, and transportation engineering. In this paper, Taguchi's method is applied to optimize the parameters of DVA to reduce the vibration amplitude of primary system. By using the Taguchi method, we must note the following two important points. The first is that we need to determine the quality characteristics of the problem. The second option is that we need to select the orthogonal arrays. The Taguchi's methods begin with the definition of the word quality. Taguchi employs a revolutionary definition: "Quality is the loss imparted to society from the time a product is shipped" [20]. In this paper the quality characteristics are also called the signal-to-noise ratio (SNR). It is defined for a nominal-the-best procedure as [15,16]

$$\eta = \mathrm{SNR} = -10\log_{10}(H_{actual} - H_{\min})^2,$$

where H_{actual} is the target function in experiment j, and $H_{\min}$ is desired value of target function. Taguchi developed the orthogonal array method to study the systems in a convenient and rapid way, whose performance is affected by different factors when the considered system becomes more complicated with increasing number of influence factors.

4.2. A procedure for optimal design

This subsection aims to present numerical results that verify the procedure discussed above by using Taguchi's method [22]. Fig. 3 shows a beam with three dynamic translational vibration absorbers with damping placed at the points $\eta_1 = {}^{3L}/_{8}$, $\eta_2 = {}^{4L}/_{8}$, $\eta_3 = {}^{5L}/_{8}$. The parameters of the beam are listed in Tab. 1.

Form Tab. 1 we can calculate $\omega_1 = 110.3005$ rad/s, $m_b = 245$ kg. In which m_b is the mass of the beam, and $\omega_1 = 110.3005$ rad/s is the fundamental frequency of the beam.

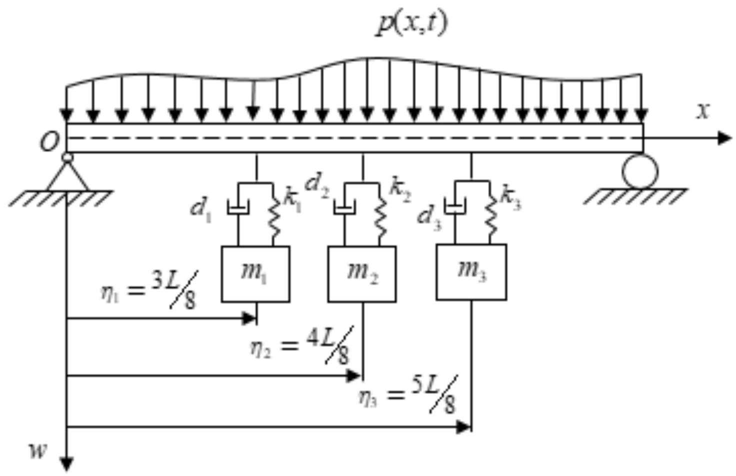


Fig. 3. Beam simply supported at both ends with three absorbers

Table 1. Parameters of beam

Parameters	Variable	Value	Unit
Fexural rigidity	EI	3.06×10^7	Nm^2
Length of beam	L	10	m
Mass per length unit	μ	24.5	kg/m
Damping coefficient	$c^{(e)}$	0.4	1/s
Internal friction coefficient	$c^{(i)}$	0.0005	s/m
Distributed force	$p = p_0 \sin \Omega t$	$p_0 = 100$	N/m
Coordinate of j-absorber	η_1, η_2, η_3	$3L/8, 4L/8, 5L/8$	m

Step 1: Selection of control factors and target function

In the design experience, the engineer normally selects that the mass of DVAs is equal to 1% of the mass of beam. Accordingly, the masses of the DVAs can be chosen as follows

$$m_1 = \frac{0.1}{100} m_b = 0.245 \text{ kg}, \quad m_2 = \frac{0.8}{100} m_b = 1.960 \text{ kg}, \quad m_3 = \frac{0.1}{100} m_b = 0.245 \text{ kg}.$$

The control factors are chosen as follows

$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [d_1 \ k_1 \ d_2 \ k_2 \ d_3 \ k_3]^T .$$

The target function H is chosen according to the formula (64)

$$H = |H_s| = \frac{\left|\widehat{h}_s\right|}{\sqrt{a^2 + b^2}}.$$

Three levels of each control factor are given in Tab. 2.

Table 2. Control factors and levels of each control factor

Levels	Control factors					
	d_1 [Ns/m]	k_1 [N/m]	d_2 [Ns/m]	k_2 [N/m]	d_3 [Ns/m]	k_3 [N/m]
1	0.005	1.0×10^4	0.1	2.0×10^4	0.004	0.5×10^4
2	0.010	1.5×10^4	0.2	2.5×10^4	0.006	1.0×10^4
3	0.015	2.0×10^4	0.3	3.0×10^4	0.008	1.5×10^4

Step 2: Selection of orthogonal array and calculation of signal-to-noise ratio (SNR)

Three levels of each control factor are applied, necessitating the use of an L18 orthogonal array (Tab. 3) [15,16]. Coding stage 1, stage 2, stage 3 of the control parameters by the symbols 1, 2, 3. By performing the experiments and then calculating the corresponding response results, we have the values of the target function H as shown in the Tab. 3, in which a target value of $H_{\min} = 0$ is selected.

Table 3. Experimental design using L18 orthogonal array

Trial	Control factors						Results	
	d_1	k_1	d_2	k_2	d_3	k_3	H	SNR
1	1	1	1	1	1	1	0.0044340966	47.0638969423
2	1	2	2	2	2	2	0.0012036545	58.3899629081
3	1	3	3	3	3	3	0.0042376395	47.4575198849
4	2	1	1	2	2	3	0.0012017250	58.4038977293
5	2	2	2	3	3	1	0.0041602303	47.6176524925
6	2	3	3	1	1	2	0.0043690768	47.1922063525
7	3	1	2	1	3	2	0.0043767090	47.1770466279
8	3	2	3	2	1	3	0.0012067156	58.3679013846
9	3	3	1	3	2	1	0.0041617143	47.6145548175
10	1	1	3	3	2	2	0.0042128399	47.5085009203
11	1	2	1	1	3	3	0.0043474735	47.2352611858
12	1	3	2	2	1	1	0.0011980787	58.4302931033
13	2	1	2	3	1	3	0.0042203721	47.4929851721
14	2	2	3	1	2	1	0.0044391926	47.0539202335
15	2	3	1	2	3	2	0.0012021408	58.4008931815
16	3	1	3	2	3	1	0.0011986789	58.4259426044
17	3	2	1	3	1	2	0.0042176774	47.4985327364
18	3	3	2	1	2	3	0.0043506134	47.2289902086

The experimental results are then analyzed by means of the mean square deviation of the target function for each control parameter, namely the calculation of the SNR of the control factors according to the formula

$$\eta_j = (\mathrm{SNR})_j = -10\log(H_j - H_{opt})^2, \;\; j = 1, \dots, 18, \tag{39}$$

where H_j is the target function in experiment j, and H_{opt} is desired value of target function.

Step 3: Analysis of signal-to-noise ratio (SNR)

From Tab. 3 we can calculate the mean value of the SNR of the control parameter of $d_1 = x_1$ corresponds to the levels 1, 2, 3

$$\mathrm{SNR}(x_1^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(2) + \mathrm{SNR}(3) + \mathrm{SNR}(10) + \mathrm{SNR}(11) + \mathrm{SNR}(12))\,/6 = 51.0142391574,$$

$$\mathrm{SNR}(x_1^2) = (\mathrm{SNR}(4) + \mathrm{SNR}(5) + \mathrm{SNR}(6) + \mathrm{SNR}(13) + \mathrm{SNR}(14) + \mathrm{SNR}(15))\,/6 = 51.0269258602,$$

$$\mathrm{SNR}(x_1^3) = (\mathrm{SNR}(7) + \mathrm{SNR}(8) + \mathrm{SNR}(9) + \mathrm{SNR}(16) + \mathrm{SNR}(17) + \mathrm{SNR}(18))\,/6 = 51.0521613966.$$

In which $\mathrm{SNR}(x_1^1), \mathrm{SNR}(x_1^2), \mathrm{SNR}(x_1^3)$ are the mean square deviation of the control parameter d_1 at the levels 1, 2, 3, respectively. Similarly we calculate the mean square deviation of the SNR for the levels 1, 2, 3 of the control parameter $k_1 = x_2, d_2 = x_3, k_2 = x_4, d_3 = x_5, k_3 = x_6$

$$\mathrm{SNR}(x_2^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(4) + \mathrm{SNR}(7) + \mathrm{SNR}(10) + \mathrm{SNR}(13) + \mathrm{SNR}(16))\,/6 = 51.0120449994,$$

$$\mathrm{SNR}(x_2^2) = (\mathrm{SNR}(2) + \mathrm{SNR}(5) + \mathrm{SNR}(8) + \mathrm{SNR}(11) + \mathrm{SNR}(14) + \mathrm{SNR}(17))\,/6 = 51.0272051568,$$

$$\mathrm{SNR}(x_2^3) = (\mathrm{SNR}(3) + \mathrm{SNR}(6) + \mathrm{SNR}(9) + \mathrm{SNR}(12) + \mathrm{SNR}(15) + \mathrm{SNR}(18))\,/6 = 51.0540762581,$$

$$\mathrm{SNR}(x_3^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(4) + \mathrm{SNR}(9) + \mathrm{SNR}(11) + \mathrm{SNR}(15) + \mathrm{SNR}(17))\,/6 = 51.0361727655,$$

$$\mathrm{SNR}(x_3^2) = (\mathrm{SNR}(2) + \mathrm{SNR}(5) + \mathrm{SNR}(7) + \mathrm{SNR}(12) + \mathrm{SNR}(13) + \mathrm{SNR}(18))\,/6 = 51.0561550854,$$

$$\mathrm{SNR}(x_3^3) = (\mathrm{SNR}(3) + \mathrm{SNR}(6) + \mathrm{SNR}(8) + \mathrm{SNR}(10) + \mathrm{SNR}(14) + \mathrm{SNR}(16))\,/6 = 51.0009985634,$$

$$\mathrm{SNR}(x_4^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(6) + \mathrm{SNR}(7) + \mathrm{SNR}(11) + \mathrm{SNR}(14) + \mathrm{SNR}(18))\,/6 = 47.1585535918,$$

$$\mathrm{SNR}(x_4^2) = (\mathrm{SNR}(2) + \mathrm{SNR}(4) + \mathrm{SNR}(8) + \mathrm{SNR}(12) + \mathrm{SNR}(15) + \mathrm{SNR}(16))\,/6 = 58.4031484852,$$

$$\mathrm{SNR}(x_4^3) = (\mathrm{SNR}(3) + \mathrm{SNR}(5) + \mathrm{SNR}(9) + \mathrm{SNR}(10) + \mathrm{SNR}(13) + \mathrm{SNR}(17))\,/6 = 47.5316243373,$$

$$\mathrm{SNR}(x_5^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(6) + \mathrm{SNR}(8) + \mathrm{SNR}(12) + \mathrm{SNR}(13) + \mathrm{SNR}(17))\,/6 = 51.0076359485,$$

$$\mathrm{SNR}(x_5^2) = (\mathrm{SNR}(2) + \mathrm{SNR}(4) + \mathrm{SNR}(9) + \mathrm{SNR}(10) + \mathrm{SNR}(14) + \mathrm{SNR}(18))\,/6 = 51.0333044696,$$

$$\mathrm{SNR}(x_5^3) = (\mathrm{SNR}(3) + \mathrm{SNR}(5) + \mathrm{SNR}(7) + \mathrm{SNR}(11) + \mathrm{SNR}(15) + \mathrm{SNR}(16))\,/6 = 51.0523859962,$$

$$\mathrm{SNR}(x_6^1) = (\mathrm{SNR}(1) + \mathrm{SNR}(5) + \mathrm{SNR}(9) + \mathrm{SNR}(12) + \mathrm{SNR}(14) + \mathrm{SNR}(16))\,/6 = 51.0343766989,$$

$$\mathrm{SNR}(x_6^2) = (\mathrm{SNR}(2) + \mathrm{SNR}(6) + \mathrm{SNR}(7) + \mathrm{SNR}(10) + \mathrm{SNR}(15) + \mathrm{SNR}(17))\,/6 = 51.0278571211,$$

$$\mathrm{SNR}(x_6^3) = (\mathrm{SNR}(3) + \mathrm{SNR}(4) + \mathrm{SNR}(8) + \mathrm{SNR}(11) + \mathrm{SNR}(13) + \mathrm{SNR}(18))\,/6 = 51.0310925942.$$

In which $\mathrm{SNR}(x_1^1), \mathrm{SNR}(x_1^2), \mathrm{SNR}(x_1^3)$ are the mean square deviation of the control parameter d_1 at the levels 1, 2, 3, respectively. Similarly we calculate the mean square deviation of the SNR for the levels 1, 2, 3 of the control parameter $k_1 = x_2, d_2 = x_3, k_2 = x_4, d_3 = x_5, k_3 = x_6$. Then SNR Ratio can be plotted to use for optimization of seat displacement as shown in Fig. 4.

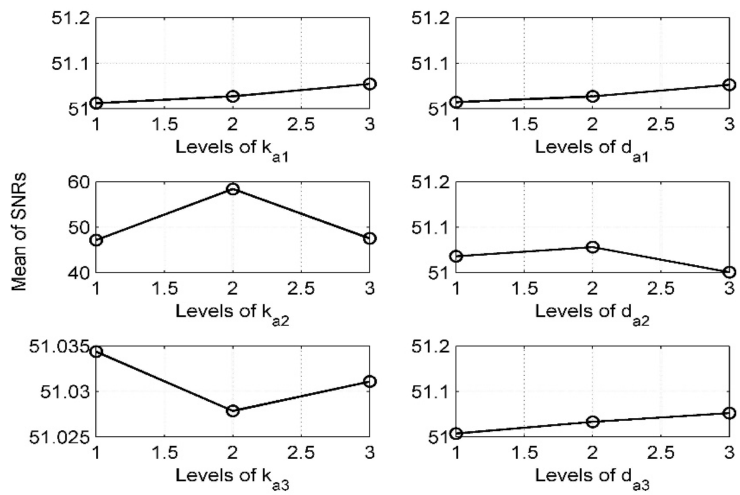


Fig. 4. SNR Ratio plot for optimization of seat displacement of control parameters $d_1, k_1, d_2, k_2, d_3, k_3$

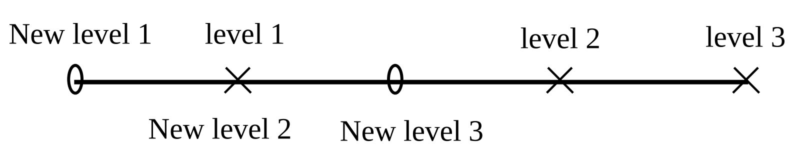
From Fig. 4 the optimal signal-to-noise ratio of the control parameters can be derived as follows

$$\begin{aligned} &(\mathrm{SNR})x_1 = 51.0521613966, \ (\mathrm{SNR})x_2 = 51.0540762581, \\ &(\mathrm{SNR})x_3 = 51.0561550854, \ (\mathrm{SNR})x_4 = 58.4031484852, \\ &(\mathrm{SNR})x_5 = 51.0523859962, \ (\mathrm{SNR})x_6 = 51.0343766989. \end{aligned} \tag{40}$$

Step 4: Selection of new levels for control factors

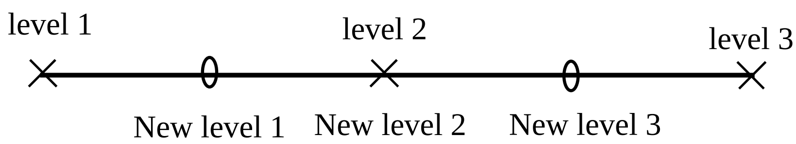
From Eq. (40) it can be seen that the optimal SNR of the control parameters is different. This makes it easy to perform iterative calculation. Firstly new levels for control parameters are selected. Based on the level distribution diagram of the parameter as shown in Fig. 4, we choose the new levels of control parameters as follows. The optimal parameters are levels with the largest value of the parameters, namely, d_1 level 3, k_1 level 3, d_2 level 2, k_2 level 2, d_3 level 3, k_3 level 1. Therefore, we have the values of the new levels as follows:

If level 1 is optimal then the next levels are



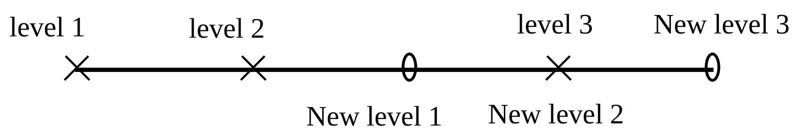
$$\begin{cases} level2\_new = level1\_old \\ level1\_new = level1\_old - \dfrac{level2\_old - level1\_old}{2} \\ level3\_new = level1\_old + \dfrac{level2\_old - level1\_old}{2} \end{cases}$$

If level 2 is optimal then the next levels are



$$\begin{cases} level2\_new = level2\_old \\ level1\_new = level2\_old - \dfrac{level2\_old - level1\_old}{2} \\ level3\_new = level2\_old + \dfrac{level3\_old - level2\_old}{2} \end{cases}$$

If level 3 is optimal then the next levels are



$$\begin{cases} level2\_new = level3\_old \\ level1\_new = level3\_old - \dfrac{level3\_old - level2\_old}{2} \\ level3\_new = level3\_old + \dfrac{level3\_old - level2\_old}{2} \end{cases}$$

According to the above rule, we have the new levels of control parameters as shown in Tab. 4.

Table 4. Control factors and new levels of control factors

Levels	Control factors					
	d_1 [Ns/m]	k_1 [N/m]	d_2 [Ns/m]	k_2 [N/m]	d_3 [Ns/m]	k_3 [N/m]
1	0.0125	1.75×10^4	0.1500	2.25×10^4	0.0700	0.25×10^4
2	0.0150	2.00×10^4	0.2000	2.50×10^4	0.0800	0.50×10^4
3	0.0175	2.25×10^4	0.2500	2.75×10^4	0.0900	0.75×10^4

Then the analysis of signal-to-noise ratio (SNR) is performed as the step 2.

Step 5: Check the convergence condition of the signal-to-noise ratio and determine the optimal parameters of the DVA

After 24 iterations, we obtain the optimal noise values of the control parameters. The calculation results are recorded in Tab. 5.

If the optimal signal-to-noise ratio of the control parameters is equal (or approximately equal) we move on to step 5. If otherwise we return to step 2. According to

Table 5. Noise values of the control parameter $(\text{SNR})_i$ of the control parameters

Trial	Optimal noise values $(\text{SNR})_i$					
	$(\text{SNR})x_1$	$(\text{SNR})x_2$	$(\text{SNR})x_3$	$(\text{SNR})x_4$	$(\text{SNR})x_5$	$(\text{SNR})x_6$
1	51.052161	51.054076	51.056155	58.403148	51.052386	51.034377
2	54.879695	54.866990	54.859337	58.311646	54.865234	54.870096
3	63.488241	63.473320	63.495179	78.961093	63.496090	63.533600
...	...	...	...	...	...	...
23	91.588713	91.588713	91.588714	91.588713	91.588713	91.588713
24	91.588714	91.588714	91.588714	91.588714	91.588714	91.588714

the above analysis, we obtain the optimal values of the parameter of absorber after 24 iterations as

$$\begin{aligned} &m_1 = 0.2450 \text{ (kg)},\ d_1 = 0.0145 \text{ (Ns/m)},\ k_1 = 1.9652 \times 10^4 \text{ (N/m)}, \\ &m_2 = 1.960 \text{ (kg)},\ d_2 = 0.2125 \text{ (Ns/m)},\ k_2 = 2.3846 \times 10^4 \text{ (N/m)}, \\ &m_3 = 0.2450 \text{ (kg)},\ d_3 = 0.0779 \text{ (Ns/m)},\ k_3 = 0.4470 \times 10^4 \text{ (N/m)}. \end{aligned} \tag{41}$$

Step 6: Determine the vibration of the primary system and of the DVA

Knowing the parameters of the DVAs, using Eq. (41) we can easily calculate the vibration of the beam with DVAs and without DVAs. Fig. 5 shows the response of the beam at the $x = L/2$ with DVAs and without DVAs. It can be clearly seen that the vibration amplitude of the beam at $x = L/2$ without DVAs is 7.267 (mm), and reduces to 0.03068 (mm) with 3 DVAs at $3L/8, 4L/8, 5L/4$ and the reducing rate is 99.57% at the excitation frequency equal to the first natural frequency of the beam.

4.3. Problem formulation for determining optimal parameters of DVAs in frequency domain

When a primary system is damped, the "fixed-points" feature no longer exists. However, as shown in the work of Pannestri [23], when a DVA with a small mass ratio is attached to lightly or moderately damped primary systems, the normalized amplitude curves roughly join at two points. When the damping ratio of the primary system approaches zero, these two points converge to the "fixed-points". Therefore, it is justified to assume that the "fixed-point" theory also approximately holds even for the case when a DVA is attached to a lightly or moderately damped primary system. Based on this assumption, it is reasonable to assume that $H(\Omega)$ has two distinct resonance points, Liu and Coppola [24]. These are denoted by A and B with frequencies Ω_A and Ω_B $(\Omega_A < \Omega_B)$, respectively. This leads to the equations

$$H(\Omega_A) = \max |H(\Omega)| \quad \text{and} \quad H(\Omega_B) = \max |H(\Omega)|. \tag{42}$$

It is well recognized that each fixed point very close to the corresponding resonance point, and that the trade off relation between $H(\Omega_A) = \max |H(\Omega)|$ and $H(\Omega_B) = \max |H(\Omega)|$ can be postulated. On this assumption, it is guaranteed that the optimum

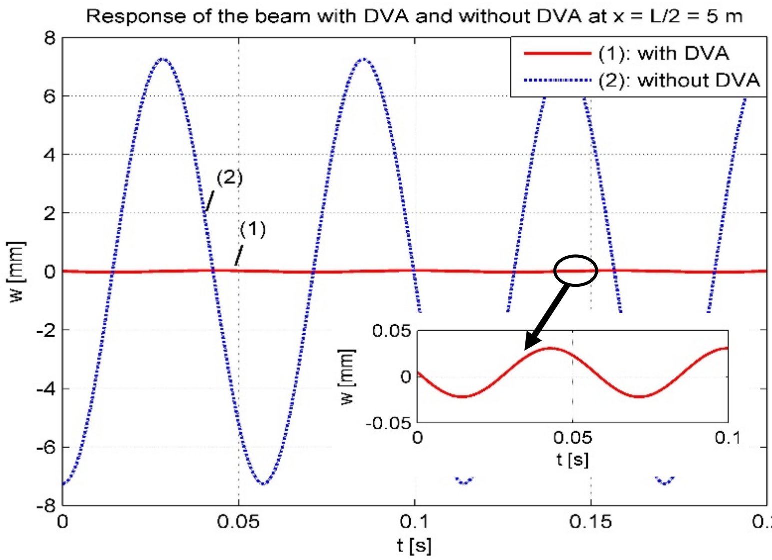


Fig. 5. The response of the beam at $x = L/2 = 5$ m without and with 3 DVAs at $3L/8, 4L/8, 5L/8$

design is derived using equivalent resonance magnification factors

$$\max|H(\Omega)| = |H(\Omega_A)| = |H(\Omega_B)|. \tag{43}$$

The problem can also be formulated as the one that minimizes the following two functions

$$\begin{aligned} f_1 &= \frac{1}{2}\left|H\left(\Omega_A\right) - H\left(\Omega_B\right)\right|, \\ f_2 &= \frac{1}{2}\left|H\left(\Omega_s\right) + H\left(\Omega_B\right)\right|. \end{aligned} \tag{44}$$

A target function can be defined as

$$f = \varepsilon_1 f_1 + \varepsilon_2 f_2 \quad \to \min, \tag{45}$$

where ε_1 and ε_2 are weighting factors used to impose different emphasis on each of the target functions.

Using the optimal parameters of the DVAs in Eq. (41), we can plot the amplitude-frequency curve as shown in Fig. 6. Figs. 5 and 6 show the response of the beam at $x = L/2 = 5$ m without DVAs is 7.267 mm and reduces to 1.721 mm with three DVAs in in a narrow band of the resonance frequency. The reducing rate is 76.32% .It can be seen form Fig. 6 that three resonance nodes at the frequencies $\Omega_A, \Omega_s, \Omega_B$. From there we have $\Omega_A = 0.94\Omega_s, \Omega_B = 1.1053\Omega_s$.

According to Eq. (41), we define the new optimal parameters of DVAs by Taguchi's method. Thus the amplitude-frequency curves with the weighting factors $\varepsilon_1 = 0.8, \varepsilon_2 = 0.3$ with will be drawn as in Fig. 7. The amplitude-frequency curve is also called the compliance curve. From Fig. 7, it is found that the maximum vale of the vibration amplitude corresponds to the case of the beam having installed 3 DVAs is 1.67 mm. The vibration amplitude of the beam without DVAs at $x = L/2 = 5$ m is 7.267 mm. The vibration amplitude of the beam decreased by 77.02%.

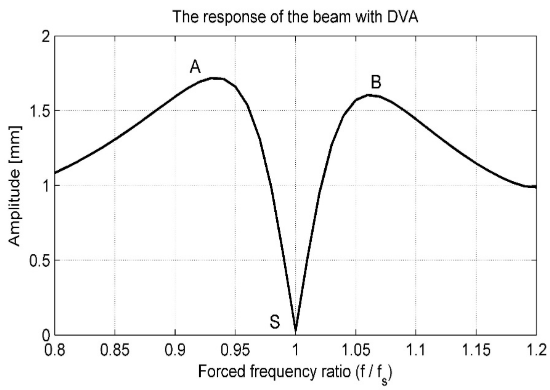


Fig. 6. Amplitude-Frequency curve in a narrow band of the resonance frequency

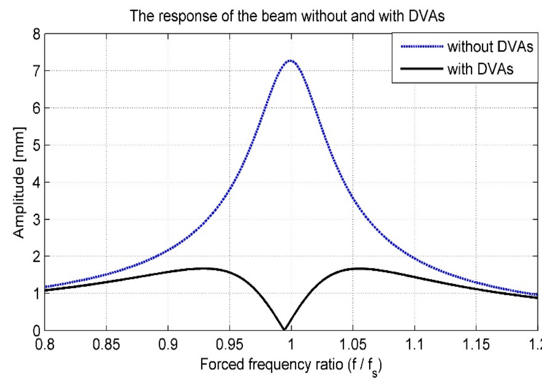


Fig. 7. The compliance curve in in a narrow band of the resonance frequency

5. CONCLUSIONS

When a damped primary system is excited by a harmonic force, its vibration can be suppressed by attaching a DVA. The DVA has the effect of reducing vibrations in the resonance region, and has almost ineffective far out of the resonance region. In this paper, the performance of the optimal design of parameters of a number of DVAs installed in an Euler–Bernoulli beam was investigated from the viewpoint of suppressing vibration amplitude of the beam. Based on the obtained results, the following concluding remarks can be reached.

- A general method for derivation of transverse vibration equations of beam with dynamic vibration absorbers is presented.

- A procedure based on the Taguchi's method for designing the optimal parameters for the DVAs attached to the beam is proposed. The use of the Taguchi's method to design the optimal parameters of the DVAs installed in damped primary system is relatively simple and convenient.

- The Taguchi's method has the good effect of reducing vibration in a narrow band of the resonance frequency (the ratio is approximately 80%).

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