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STABILITY OF TWO-DIMENSIONAL SEDIMENT FLOW DOWN AN INCLINED PLANE

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SUMMARY. In this paper the authors have studied the instability of 2-D flow with sediment transport, which is based on the generalized diffusion theory. The authors have constructed the boundary conditions of the bed concentration and diffusion flux at the free surface for closing equations system. After that the authors have considered the instability and obtained the neutral-curve of stability in the plane (R, k). The obtained results show that if there is no suspended sediment, the result will be identical to Chia-Shun Yih's.

The stability of two-dimensional flow has been studied before. However it was considered whether the two dimensional clear flow or sediment flow as non-Newtonian fluid, e. g., sediment concentration isn't taken into account [1-4].

In this paper the authors have studied the instability of suspended sediment flow, which is based on the generalized diffusion theory for multiphase flow [5, 6]. At last the neutral-stability curve is obtained. If one ignores the sediment concentration, it's easy to obtain again the result of C. S. Yih [1]

1. SYSTEM OF EQUATIONS

The system of 3-D dimensionaless equations describing the sediment flow for inclined axes includes [6]:

$$\nabla_i v_i = 0, \qquad \frac{dc}{dt} = -\mathrm{div} J,$$
$$\frac{dv}{dt} = f - \frac{\rho_a}{\rho}\nabla p + \frac{\rho_a}{\rho R}\Delta v - \frac{\gamma}{\rho}\frac{dJ}{dt},$$
$$\frac{dJ}{dt} + \frac{c}{k^*}J = cm - \frac{\gamma c}{\rho_s}\frac{dv}{dt} - D^* c\nabla c,$$

$$F = \frac{v_a}{\sqrt{gH\sin\alpha}}, \quad R = \frac{v_a H}{\nu_a}, \quad \nu_a = \frac{\mu}{\rho_a}, \quad f = (f_1, f_2, 0), \quad m = (m_1, m_2, 0), \quad f_1 = \frac{1}{F^2};$$
$$f_2 = \frac{\mathrm{cotg}\alpha}{F^2}, \quad m_1 = \frac{\gamma}{\rho_s F^2}, \quad m_2 = \frac{\gamma \mathrm{cotg}\alpha}{\rho_s F^2}, \quad k^* = \frac{k v_a}{H}, \quad D^* = \frac{D}{\rho_s v_a^2}, \quad \gamma = \rho_s - \rho_w, \tag{1.1}$$
$$\rho = \rho_w + \gamma c, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + (v\cdot\nabla), \quad \nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right), \quad \Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

where ρ_a, v_a - quantities averaged over the steady flow depth H of mixture density, steady flow velocity, respectively, v_i, J_i - components of mixture velocity and diffusion flux, respectively, c - suspended sediment volume concentraton, k, D - constant coefficients, R - Reynolds number, F - Froude number, ρ_s, ρ_w - densities of sediment and water, respectively, μ - viscosity.

Now we consider the steady flow with the following boundary conditions:

+ At the hard and plane bed $x_2 = 1$:

$$v_i = 0,\ i = 1,2,3, \quad c_b^0 = \frac{c^*}{2} + \delta\Big(\frac{u_*^0}{W_s}\Big)^4, \quad c^* = \gamma g \frac{\cos\alpha H}{D} \tag{1.2}$$

where δ - experiment constant, after Frank Engelund $\delta = 0.00056$, u_*^0 - shear velocity of flow, W_s - settling velocity of sediment particles.

+ At the free surface $x_2 = 0$

$$\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} = 0, \qquad -p + \frac{2}{R}\frac{\partial v_2}{\partial x_2} = 0, \qquad J_2 = 0. \tag{1.3}$$

We easily have found solution of (1.1) - (1.3) for steady flow:

$$c^0 = c^* x_2 + c_\varsigma^0, \quad c_\varsigma^0 = c_b^0 - c*, \quad p^0 = \mathrm{cotg}\alpha\big(\rho_\varsigma^0 x_2 + \frac{1}{2}\gamma c^* x_2^2\big)/F^2\rho_a,$$
$$v_1^0 = R\Big(\frac{\rho_\varsigma^0}{2\rho_a} + \frac{\gamma c^*}{6\rho_a} - \frac{\rho_\varsigma^0 x_2^2}{2\rho_a} - \frac{\gamma c^*}{6\rho_a x_2^3}\Big)/F^2.$$
$$v_2^0 = v_3^0 = 0, \quad J_1^0 = k^* m_1, \quad J_2^0 = J_3^0 = 0,$$

where the subscrits ς, b refer in turn to values at the surface and bed. From here we obtain the relation of Froude number and Reynolds number:

$$F^2 = R\big(\frac{\rho_\varsigma^0}{3} + \frac{\gamma c^*}{8}\big)/\rho_a \tag{1.4}$$

If one ignores sediment concentration then (1.4) is identical to Chia-Shun Yih's result [1].

2. PROBLEM OF DISTURBANCES AND SQUIRE'S THEOREM

Let's consider the 3-D disturbances as cylindrical waves propagating at the angle θ to the flow direction [2]. Here, the xyz Cartesian coordinates, in which the y axis is directed along the x_2 - axis and x - axis is orthogonal to the wave front, are used instead of $x_1\ x_2\ x_3$. Therefore such disturbances have the form:

$$q^1 = q^*(y)e^{ik(x-\sigma t)}, \qquad \sigma = \sigma_r + i\sigma_i \tag{2.1}$$

with $q^*(y)$ - complex amplitudes, and steady solution of the primary flow becomes:

$$u^0 = v_1^0\cos\theta, \quad w^0 = -v_1^0\sin\theta, \quad v^0 = v_2^0 = 0, \quad p^0(y) \equiv p^0(x_2),$$
$$J_x^0 = J_1^0\cos\theta, \quad J_y^0 = J_2^0 = 0, \quad J_z^0 = -J_1\sin\theta, \tag{2.2}$$
$$f = (f_1\cos\theta, f_2, -f_1\sin\theta), \quad m = (m_1\cos\theta, m_2, -m_1\sin\theta).$$

Let

$$q = q^0 + q^1, \qquad |q_1| \ll 1. \tag{2.3}$$

Where q is u, v, w, c, p, J_x, J_y and J_z in turn. Substituting (2.3) into (1.1) in xyz coordinate system we obtain equations for disturbances q^1, here q^1 is u^1, v^1, w^1, c^1, p^1, J_x^1, J_y^1, J_z^1 in turn.

The boundary conditions for disturbances are the followings:

+ At the free surface $y = \varsigma(x,t)$ reduced to $y = 0$:

$$
\begin{aligned}
&-\frac{\cos\theta\rho_\varsigma^0 R\varsigma}{\rho_a F^2} + \frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x} = 0, \quad \frac{\sin\theta\rho_\varsigma^0 R\varsigma}{\rho_a F^2} + \frac{\partial w^1}{\partial y} = 0,\\
&-\frac{\mathrm{cotg}\alpha\rho_\varsigma^0\varsigma}{F^2\rho_a} - p^1 + \frac{2}{R}\frac{\partial v^1}{\partial y} + S\frac{\partial^2\varsigma}{\partial x^2} = 0, \quad J_y^1 = 0, \quad S = \frac{T}{H\rho_a v_a^2}\,.
\end{aligned}
\tag{2.4}
$$

+ At the plane hard bed $y = 1$:

$$
u^1 = v^1 = w^1 = 0, \quad c_b^1 = \frac{2\delta\cos\theta}{R^2 W_s^{*4}}\frac{dv_1^0}{dy}\Big(\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}\Big), \quad W_s^* = \frac{W_s}{v_a}\,. \tag{2.5}
$$

From the first of disturbance equations it is shown that there exists a function $\psi(x,y)$ such that $u^1 = \dfrac{\partial\psi}{\partial y}$ $v^1 = -\dfrac{\partial\psi}{\partial x}$. We look for the solution of (2.4) in the form:

$$
q_j = \phi_j(y)e^{ik(x-\sigma t)} \tag{2.6}
$$

with $q_1 = \psi$, $q_2 = c^1$, $q_3 = p^1$, $q_4 = J_x^1$, $q_5 = J_y^1$, $q_6 = w^1$, $q_7 = J_z^1$

Substituting (2.6) into the disturbance equations we obtain the differential equations system:

$$
\begin{aligned}
&a_3\phi_2 - ikc^*\phi_1 + ik\phi_4 + \phi_5' = 0,\\
&a_1\phi_1' - ikv_1^{0\prime}\cos\theta\phi_1 + \frac{ik\rho_a\phi_3}{\rho^0} - \frac{\rho_a}{R\rho^0}\phi_1''' - \frac{\gamma}{\rho^0 F^2}\phi_2 + \frac{\gamma a_3\phi_4}{\rho^0} = 0,\\
&-ika_1\phi_1 + \frac{\rho_a ik}{\rho^0 R}\phi_1'' - \frac{\gamma\mathrm{cotg}\alpha}{\rho^0 F^2}\phi_2 + \frac{\rho_a\phi_3'}{\rho^0} + \frac{\gamma a_3\phi_5}{\rho_0} = 0,\\
&a_2\phi_4 + \frac{\gamma c^0 a_3\phi_1'}{\rho_s} - \frac{\gamma c^0 ikv_1^{0\prime}\cos\theta\phi_1}{\rho_s} + D^*c^0 ik\phi_2 = 0,\\
&a_2\phi_5 - \frac{ik\gamma c^0 a_3\phi_1}{\rho_s} + D^*c^0\phi_2' = 0,\\
&a_1\phi_6 + ik\sin\theta v_1^0\phi_1 - \frac{\rho_a}{\rho^0 R}\phi_6'' - \frac{\rho_a\gamma\sin\theta}{R\rho^{02}}v_1^{0\prime\prime}\phi_2 + \frac{\gamma a_3\phi_7}{\rho^0} = 0,\\
&a_2\phi_7 + \frac{\gamma c^0 a_3\phi_6}{\rho_s} + \frac{\gamma ikc^0\sin\theta v_1^{0\prime}\phi_1}{\rho_s} = 0,\\
&a_1 = a_3 + \frac{\rho_a k^2}{\rho^0 R}, \quad a_2 = a_3 + \frac{c^0}{k^*}, \quad a_3 = ik(v_1^0\cos\theta - \sigma).
\end{aligned}
\tag{2.7}
$$

Here the accents denote differentiation with respect to y. And now the boundary conditons for ϕ_j are

$$
\begin{aligned}
&\phi_1'(1) = 0, \quad \phi_1(1) = 0, \quad \phi_6(1) = 0, \quad \phi_2(1) = -N\phi_1''(1),\\
&\Big(k^2 - \frac{R\rho_\varsigma^0\cos\theta}{F^2\rho_a\overline{\sigma}}\Big)\phi_1(0) + \phi_1''(0) = 0, \quad \frac{R\sin\theta\rho_\varsigma^0}{F^2\rho_a\overline{\sigma}}\phi_1(0) + \phi_6'(0) = 0,\\
&\Big(\frac{\mathrm{cotg}\alpha\rho_\varsigma^0}{F^2\rho_a} + Sk^2\Big)\frac{\phi_1(0)}{\overline{\sigma}} + \frac{2ik}{R}\phi_1'(0) + \phi_3(0) = 0, \quad \phi_5(0) = 0,\\
&\overline{\sigma} = \sigma - v_1^0(0)\cos\theta, \quad N = \frac{2\delta\cos\theta}{RW_s^{*4}F^2}\,.
\end{aligned}
\tag{2.8}
$$

It's easily seen that the system of eqs. (2.7) with the boundary (2.8) is separated into two groups, one of which only contains ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5. This is related to problem in the xy

plane with velocity profile $v_1^0 cos\theta$. From here it is easily proved that it satisfies Squire's theorem. Therefore it is enough to consider the two-dimensional one with $\theta = 0$. Here we have to remark that Squire's theorem can be proved only in the case of cylindrical waves. For the general case of clear flow we can see [7].

Eliminating unknown functions ϕ_3, ϕ_4, ϕ_5 we obtain two Orr-Sommerfeld equations for ϕ_1, ϕ_2 as follows:

$$\begin{aligned} &c_1\phi_1 + c_2\phi_2 + c_3\phi_3' + c_4\phi_1'' + c_5\phi_2' - \frac{\rho_a}{\rho^0 R}\phi_1'''' = 0, \\ &d_1\phi_1 + d_2\phi_2 - \left(\frac{D^* c^0 \phi_2'}{a_2}\right)' = 0, \end{aligned} \tag{2.10}$$

with

$$\begin{aligned} b_1 &= a_1 - \frac{\gamma^2 c^0 a_3^2}{\rho^0 \rho_s a_2}, \quad b_2 = -iku^{0\prime} + \frac{\gamma^2 ikc^0 u^{0\prime} a_3}{\rho^0 \rho_s a_2}, \quad b_3 = \frac{\gamma}{\rho^0 F^2} + \frac{\gamma D^* c^0 ika_3}{\rho^0 a_2}, \\ c_1 &= \frac{(\rho^0 b_2)'}{\rho^0} - k^2 a_1 + \frac{\gamma^2 a_3^2 k^2 c^0}{\rho^0 \rho_s a_2}, \quad c_3 = \frac{(\rho^0 b_1)'}{\rho^0} + b_2, \\ c_2 &= \frac{ik\gamma \text{cotg}\alpha}{\rho^0 F^2} - \frac{(\rho^0 b_3)'}{\rho^0}, \quad c_4 = b_1 + \frac{\rho_a k^2}{\rho^0 R}, \quad c_5 = -b_3 + \frac{ikD^* c^0 \gamma a_3}{\rho^0 a_2}, \\ d_1 &= -ikc^* + \left(\frac{ikc^0 a_3 \gamma}{\rho_s a_2}\right)' - \frac{\gamma c^0 k^2 u^{0\prime}}{\rho_s a_2}, \quad d_2 = a_3 + \frac{D^* c^0 k^2}{a_2}. \end{aligned}$$

3. SOLUTION FOR LONG WAVES

In this case $k \ll 1$, hence we can look for approximate solution in k as follows:

$$\phi_1 = \phi_{10} + k\phi_{11}, \quad \phi_2 = \phi_{20} + k\phi_{21}, \quad \sigma = \sigma_0 + k\sigma_1. \tag{3.1}$$

For the first approximation we have:

$$\begin{aligned} &\phi_{10} = -\frac{R\gamma A_1}{6F^2\rho_a}y^3 + \frac{A_2}{2}y^2 + A_3 y + A_4, \quad \phi_{20} = A_1, \quad \overline{\sigma}_0 = \frac{R\rho_\varsigma^0(RN\gamma - 3F^2\rho_a)}{6F^2\rho_a(RN\gamma - F^2\rho_a)}, \\ &A_1 = -\frac{NR\rho_\varsigma^0 A_4}{(F^2\rho_a - NR\gamma)\overline{\sigma}_0}, \quad A_2 = \frac{R\rho_\varsigma^0 A_4}{F^2\rho_a\overline{\sigma}_0}, \quad A_3 = -\frac{R\rho_\varsigma^0}{F^2\rho_a\overline{\sigma}_0}\left[\frac{RN\gamma}{2(F^2\rho_a - RN\gamma)} + 1\right]A_4 \end{aligned}$$

A_4 - arbitrary constant, we can choose $A_4 = 1$. If one ignores sediment concentration then $\overline{\sigma}_0 = 3/2$, identical to the result obtained by Chia-Shun Yih [1].

For the second approximation we have:

$$\begin{aligned} \phi_{11} &= \frac{M_0}{6}y^3 + \frac{B_3}{2}y^2 + B_4 y + B_5 + i\sum_{n=1}^{5}\frac{M_n}{(n+1)(n+2)(n+3)}y^{n+1}, \\ \phi_{21} &= \frac{i}{D^* k^*}\left[-\frac{y^4}{24}\left(\frac{R\rho_\varsigma^0}{F^2\rho_a}A_1 + c^* A_2\right) - \frac{c^* A_3}{6}y^3 - \frac{c^* + \overline{\sigma}_0 A_1}{2}y^2\right] + B_2, \end{aligned}$$

$$M_0 = -\frac{iR\bar\sigma_0\rho_\varsigma^0 A_3}{\rho_a} + B_1 - \frac{R\gamma B_2}{F^2\rho_a}, \quad M_1 = E_1R + E_2, \quad M_2 = E_3R, \quad M_3 = E_4R,$$

$$M_4 = E_5R, \quad M_5 = E_6R, \quad E_1 = -\gamma c^*\bar\sigma_0 A_3/\rho_a, \quad E_2 = r\gamma \mathrm{cotg}\alpha A_1/\rho_a,$$

$$E_3 = r\rho_\varsigma^0\Big(\frac{\gamma c^*}{\rho_a} + \frac{A_3\rho_\varsigma^0}{\rho_a} + \frac{\gamma\bar\sigma_0 A_1}{\rho_a}\Big)/2\rho_a + \frac{r\gamma\rho_s\mu(c^* + \bar\sigma_0 A_1)}{2\rho_a^2 kD},$$

$$E_4 = r\gamma c^*\Big(\frac{5\rho_\varsigma^0 A_3}{3\rho_a} + \frac{\gamma c^*}{\rho_a} + \frac{\gamma\bar\sigma_0 A_1}{\rho_a}\Big)/2\rho_a + \frac{r\gamma c^* A_3\rho_s\mu}{6\rho_a^2 kD},$$

$$E_5 = r\gamma\Big(\frac{r\rho_\varsigma^{02}A_1}{4\rho_a^2} + \frac{\rho_\varsigma^0 c^* A_2}{4\rho_a} + \frac{\gamma c^{*2}A_3}{\rho_a}\Big)/3\rho_a + r\gamma\rho_s\mu\Big(\frac{r\rho_\varsigma^0 A_1}{\rho_a} + c^*A_2\Big)/24\rho_a^2 kD,$$

$$E_6 = r\gamma^2 c^*\Big(\frac{r\rho_\varsigma^0 A_1}{\rho_a} + c^*A_2\Big)/12\rho_a^2, \quad r = \rho_a/\Big(\frac{\rho_\varsigma^0}{3} + \frac{\gamma c^*}{8}\Big).$$

The constants B_i are defined by the following equations:

$$B_1 = -\frac{iR\rho_\varsigma^0 \mathrm{cotg}\alpha A_4}{F^2\rho_a\bar\sigma_0}, \quad B_3 - \frac{R\rho_\varsigma^0}{F^2\rho_a\bar\sigma_0}B_5 = D_1, \quad -\frac{R\gamma}{2F^2\rho_a}B_2 + B_3 + B_4 = iD_2,$$

$$-\frac{R\gamma}{6F^2\rho_a}B_2 + \frac{1}{2}B_3 + B_4 + B_5 = iD_3, \quad B_2\Big(1 - \frac{NR\gamma}{F^2\rho_a}\Big) + NB_3 = iD_4 \tag{3.2}$$

$$D_1 = -\frac{R\rho_\varsigma^0\sigma_1 A_4}{F^2\rho_a\bar\sigma_0^2}, \quad D_2 = -\sum_{n=1}^{5}\frac{M_n}{(n+1)(n+2)} + \frac{R\rho_\varsigma^0\mathrm{cotg}\alpha}{2F^2\rho_a\bar\sigma_0}A_4 + \frac{R\rho_\varsigma^0\bar\sigma_0}{2\rho_a}A_3,$$

$$D_3 = -\sum_{n=1}^{5}\frac{M_n}{(n+1)(n+2)(n+3)} + \frac{R\rho_\varsigma^0\bar\sigma_0 A_3}{6\rho_a} + \frac{R\rho_\varsigma^0\mathrm{cotg}\alpha}{6F^2\rho_a\bar\sigma_0},$$

$$D_4 = N\Big[\frac{R\rho_\varsigma^0\bar\sigma_0 A_3}{\rho_a} + \frac{R\rho_\varsigma^0\mathrm{cotg}\alpha}{F^2\rho_a\bar\sigma_0} - \sum_{n=1}^{5}\frac{M_n}{n+1}\Big] +$$

$$+ \frac{1}{D^*k^*}\Big[\frac{1}{24}\Big(\frac{R\rho_\varsigma^0 A_1}{F^2\rho_a} + c^*A_2\Big) + \frac{c^*A_3}{6} + \frac{c^* + \bar\sigma_0 A_1}{2}\Big].$$

From the condition of solution existence for (3.2) we have:

$$\sigma_1 = i\bar\sigma_0\Big[D_3 - D_2 - \frac{R\gamma D_4}{3(F^2\rho_a - NR\gamma)}\Big], \quad \text{or} \quad \mathrm{Im}\sigma = k\bar\sigma_0\Big[D_3 - D_2 - \frac{R\gamma D_4}{3(F^2\rho_a - NR\gamma)}\Big]. \tag{3.3}$$

From (3.3) it is easily seen that in the plane (R, k) the neutral curves include the curve $k = 0$ and the one:

$$D_3 - D_2 - \frac{R\gamma D_4}{3(F^2\rho_a - NR\gamma)} = 0. \tag{3.4}$$

From (3.4) it's easy to follow the critical Reynolds number for stability with assumption of small suspended sediment at free surface:

$$R = \frac{r\gamma E_{10} - (\rho_a - \gamma c^*)E_8}{(\rho_a - \gamma c^*)E_7 - r\gamma E_9} \tag{3.5}$$

$$E_7 = \frac{3E_1}{8} + \frac{E_3}{5} + \frac{E_4}{8} + \frac{3E_5}{35} + \frac{E_6}{6} - \frac{\rho_\varsigma^0 \overline{\sigma}_0 A_3}{\rho_a},$$

$$E_8 = \frac{3E_2}{8} - \frac{r\rho_\varsigma^0 \mathrm{cotg}\alpha}{\rho_a \overline{\sigma}_0}, \quad E_{10} = \frac{c^* \rho_\varsigma^0 \mathrm{cotg}\alpha}{\rho_a \overline{\sigma}_0} - \frac{c^* E_2}{2r},$$

$$E_9 = \frac{c^* \rho_\varsigma^0 \overline{\sigma}_0 A_3}{r\rho_a} - \frac{c^*}{r}\left(\frac{E_1}{2} + \frac{E_3}{3} + \frac{E_4}{4} + \frac{E_5}{5} + \frac{E_6}{6}\right) +$$

$$+ \rho_s \mu \frac{\dfrac{\frac{r\rho_\varsigma^0 A_1}{\rho_a} + c^* A_2}{24} + \dfrac{c^* A_3}{6} + \dfrac{c^* + \overline{\sigma}_0 A_1}{2}}{\rho_a k D}.$$

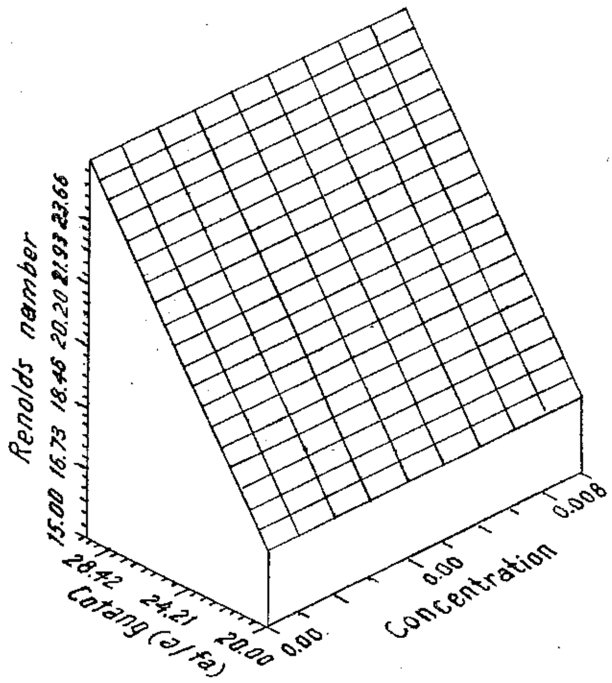

Fig. 1

Neutral-stability surface wiht $\mu = 0.01 g/cm^3$, $kD = 0.01 g/cms^2$ in 3-D space $(R, \mathrm{cotg}\alpha, c^*)$

From fig. 1 we can see stability region in three dimensional space and dependence of Reynolds number on the inclined angle α and sediment concentration.

When the sediment concentration is very small, we approximate (3.5) in c^* and obtain the condition of stability as follows:

$$R \le R_c - \frac{5\gamma c}{24\rho_w}\mathrm{cotg}\alpha\left[\frac{5}{3}\eta + \frac{1}{2}\right] \le R_c,$$

$$R_c = \frac{5}{6}\mathrm{cotg}\alpha, \quad \eta = \frac{\rho_s \mu}{\rho_w K}, \quad K = kD \text{ - diffusion coefficient}$$

If one ignores sediment, it's easy to obtain again the result of Chia-Shun Yih [1]. From numerical and analytical solutions for Raynolds number we can see that the sediment flow is more unstable than the clear one.

4. DISCUSSION

Based on generalized diffusion theory the two-dimensional model of suspended sediment transport as well as the boundary conditions of bed concentration and diffusion flux at free surface has been constructed. On this model the 2-D problem of disturbances has been studied. At last it is obtained flow stability condition for Reynolds according to sediment concentration. From obtained results it's shown that disturbances of suspended sediment flow are more unstable. If one ignores sediment then the Chia-Shun Yih's result [1] is obtained again.

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ỔN ĐỊNH CỦA DÒNG CHẢY HAI CHIỀU MANG BÙN CÁT

Trong bài báo này các tác giả đã nghiên cứu sự ổn định của dòng chảy hai chiều mang bùn cát trên mặt phẳng nằm nghiêng. Hệ phương trình toán học mô tả dòng chảy được thu nhận dựa theo lý thuyết khuyếch tán suy rộng; các điều kiện biên trên đáy và trên mặt thoáng đã được xây dựng. Đã thu nhận được hệ phương trình đóng kín đủ để nghiên cứu bài toán ổn định. Định lý Squire đã được chỉ ra, cho phép chỉ cần nghiên cứu sự phát triển của các kích động hai chiều trong bài toán ổn định. Đối với các kích động dạng sóng dài, phương trình Orr - Sommerfeld đã được khảo sát số và khảo sát giải tích (khi các nồng độ bùn cát bé). Đã chỉ ra rằng bùn cát lơ lửng làm dòng chảy sớm mất ổn định so với trường hợp không có bùn cát.