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THE LOCAL THEORY OF ELASTOPLASTIC DEFORMATION PROCESSES AND THE STABILITY BEYOND ELASTIC LIMITS OF THIN-WALLED STRUCTURES SUBJECTED TO COMPLEX LOADING

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SUMMARY. The paper is concerned with the complete constitutive relations of elastoplastic deformation process theory. Using this theory the stability beyond elastic limits of thin-walled structures subjected to complex loading is analysed. The proposed method of loading parameter is a combination of numerical and analytical solutions. Calculations have been carried out for rectangular plates and cylindrical shells in order to compare this method and its results with other theoretical and experimental works.

1. CONSTITUTIVE RELATIONS OF THE LOCAL THEORY OF ELASTOPLASTIC DEFORMATION PROCESSES

The analysis of stress-strain states or the stability of components or structures subjected to various complex loading beyond limits of elasticity requires a plasticity theory which can describe complex elastoplastic processes of deformation. The theory of elastoplastic deformation processes, based on Ilyushin's postulate of isotropy satisfies this requirement. But up to now the stress-strain relationship has contained undetermined functionals.

Assuming some hypothesis, we have constructed a complete stress-strain relationship of the local theory of elastoplastic deformation processes using hypothesis of local determinancy and complanarity [1, 2]:

$$\begin{aligned} \dot{S}_{ij} &= -\frac{2}{3}\frac{\sigma_u f}{\sin\theta}\dot{e}_{ij} + \Big(\frac{\psi}{\cos\theta} + \frac{\sigma_u f}{\sin\theta}\Big)\frac{S_{mn}\dot{e}_{mn}}{\sigma_u^2}S_{ij}, \\ \dot{\sigma} &= 3K\dot{e}. \end{aligned} \tag{1.1}$$

This theory containes two material functions f and ψ depending upon the materials used. They are determined from experimental data [2]

$$\begin{aligned} f &\equiv f(\sigma_u,\theta,s) = -\frac{1}{s}\sin\theta\Big[1 + \Big(\frac{3Gs}{\sigma_u} - 1\Big)\Big(\frac{1-\cos\theta}{2}\Big)^{\alpha}\Big], \\ \psi &\equiv \psi(\theta,s) = \phi'(s)\cos\theta - (3G - \phi')\Big(\frac{1-\cos\theta}{2}\Big)^{\beta}, \quad \alpha \geq 1, \quad \beta > 1, \end{aligned} \tag{1.2}$$

Where $\phi'(s) = \dfrac{d\phi}{ds}$ - intantenous slope of stress versus strain characteristic.

It is significant that f and ψ can be applied for all active and passive deformation processes, i.e. the stress-strain relationship (1.1), (1.2) can describe all deformation processes with complex loading (not only loading, but unloading as well). The relationship for simple loading process, process with small and average curvature, unloading process and Prandtl-Reuss relations are considered as particular cases of this theory.

a. Simple loading process

For this process $\theta = 0$,

$$\lim_{\theta\to 0} \frac{f}{\sin\theta} = -\frac{1}{s}, \qquad \lim_{\theta\to 0} \psi = \phi'(s),$$

the relations (1.1)-(1.2) become

$$\dot{S}_{ij} = \frac{2}{3}\frac{\sigma_u}{s}\dot{e}_{ij} + \left(\phi' - \frac{\sigma_u}{s}\right)\frac{S_{ij}}{\sigma_u}v_u. \tag{1.3}$$

Otherwise, according to the small elastoplastic deformation theory for simple loading

$$S_{ij} = \frac{2\sigma_u}{3\varepsilon_u}e_{ij}$$

we obtain

$$\dot{S}_{ij} = \frac{2\sigma_u}{3\varepsilon_u}\dot{e}_{ij} + \frac{2}{3}\left(\frac{d\sigma_u}{d\varepsilon_u} - \frac{\sigma_u}{\varepsilon_u}\right)\frac{S_{ij}}{\sigma_u}\dot{\varepsilon}_u. \tag{1.4}$$

Since in this case

$$s = \varepsilon_u, \quad \dot{s} = \dot{\varepsilon}_u = v_u, \qquad \frac{d\sigma_u}{d\varepsilon_u} = \frac{d\sigma_u}{ds} = \phi'(s),$$

hence the relation (1.4) reduces to (1.3)

In the elastic stage $\frac{\sigma_u}{s} = 3G$, $\phi' = 3G$, from (1.3) we obtain Hookean relationship

$$\dot{S}_{ij} = 2G\dot{e}_{ij}. \tag{1.5}$$

b. Unloading process

The unloading process occurs when $\theta = \pi$, i.e. the direction of the tangent to the continuing deformation trajectory is opposite to the stress vector at considered point. Since

$$\lim_{\theta\to\pi} \frac{f}{\sin\theta} = -\frac{3G}{\sigma_u}, \qquad \lim_{\theta\to\pi} \psi = -3G$$

hence the relatios (1.1)-(1.2) become

$$\dot{S}_{ij} = 2G\dot{e}_{ij}. \tag{1.6}$$

c. Deformation process with average curvature

In this process the value of angle θ is small. Restricting to the second-order small values we obtain from (1.2)

$$f = -\frac{1}{s}\sin\theta \approx -\frac{\theta}{s}, \quad \psi = \phi'\cos\theta \approx \phi'\left(1 - \frac{\theta^2}{2}\right).$$

Substituting into the relations (1.1)-(1.2) gives

$$\dot{S}_{ij} = \frac{2}{3}\frac{\sigma_u}{s}\dot{e}_{ij} + \left(\phi' - \frac{\sigma_u}{s}\right)\frac{S_{mn}\dot{e}_{mn}}{\sigma_u^2}S_{ij}. \tag{1.7}$$

The relation (1.7) is a generalization of Prandtl-Reuss relation for perfectly plastic material and Prager relation for plastic strain-hardening material.

2. THE STABILITY OF THIN-WALLED STRUCTURES SUBJECTED TO COMPLEX LOADING

In recent years large development in the elastoplastic analysis of thin-walled structures has been observed. But there is no an estimation about the influence of complex loading on the stability of structures.

Suppose that components of structures are subjected to external forces which are considered as loads depending on some parameter t. When t varies, the deformation process occured by these loads in structures may be simple or complex.

The instability of the structures is expressed that with $t = t_k$ external load reaches some special value, by this load a stress state σ_{ij}^k and a strain state ε_{ij}^k occur respectively in the structure such that before and up to this state the deformation process is still determined one-by-one, but after that there exist neighbouring states, i.e. there exists a bifurcation of equilibrium states.

One of the main aims of the stability problem is to determine this value t_k. The value t_k is called a critical value of loading parameter, and respectively external load is called a critical load. As shown later, the critical load depents on the complexity of loading process.

a. Pre-buckling process

Suppose that a thin-walled structure is subjected to complex loading. At any moment t there exists a membrane plane stress state in the structure

$$\sigma_{11}, \quad \sigma_{22}, \quad \sigma_{12} \neq 0, \qquad \sigma_{33} = \sigma_{31} = \sigma_{32} = 0,$$

so that

$$\sigma = \frac{1}{3}(\sigma_{11} + \sigma_{22}), \quad \sigma_u = \left(\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3\sigma_{12}^2\right)^{1/2}.$$

The strain tensor is determined from (1.1)-(1.2) (with $\nu = 1/2$) in combination with boundary conditions:

$$\dot{\sigma}_{ij} = -\frac{2}{3}\frac{\sigma_u f}{\sin\theta}\left(\dot{\varepsilon}_{ij} + \delta_{ij}\dot{\varepsilon}_{kk}\right) + \left(\frac{\psi}{\cos\theta} + \frac{\sigma_u f}{\sin\theta}\right)\frac{\sigma_{k\ell}\dot{\varepsilon}_{k\ell}}{\sigma_u^2}\sigma_{ij} \tag{2.1}$$

where

$$\cos\theta = \frac{\sigma_{ij}\dot{\varepsilon}_{ij}}{\sigma_u v_u}, \quad v_u \equiv \frac{ds}{dt} = \frac{2}{\sqrt{3}}\left(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{22} + \dot{\varepsilon}_{12}^2\right)^{1/2}. \tag{2.2}$$

b. Equations of stability

Suppose, that at the moment t_k a bifurcation of equilibrium states appears such that with an infinitesimal small increment of external load there are possible increments of deformation (including the bending deformation) in the structure

$$\delta\varepsilon_{ij} = \delta\varepsilon^*_{ij} - Z\delta\chi_{ij}$$

where

$$\delta\varepsilon^*_{ij} = \frac{1}{2}\Big(\frac{\partial\delta u_i}{\partial x_j} + \frac{\partial\delta u_j}{\partial x_i}\Big) - k_{ij}\delta w, \quad \delta\chi_{ij} = \frac{\partial^2\delta w}{\partial x_i\partial x_j},$$

δu_i, δw - increment of in-plane displacement and deflection of middle surface.
$\delta\chi_{ij}$ - increment of curvature and torsion associated with the instability.
k_{ij} - principal curvature of the shell, ($k_{12} = 0$).
Respectively, we determine stress increments according to (1.1), (1.2) (with $\nu = 1/2$):

$$\delta\sigma_{ij} = \frac{2}{3}A\big(\delta\varepsilon_{ij} + \delta_{ij}\delta\varepsilon_{kk}\big) + (P - A)\frac{\sigma_{k\ell}\delta\varepsilon_{k\ell}}{\sigma_u^2}\sigma_{ij}. \tag{2.3}$$

where

$$\begin{aligned} A &= -\frac{\sigma_u f}{\sin\theta} = \frac{\sigma_u}{s} + \Big(3G - \frac{\sigma_u}{s}\Big)\Big(\frac{1-\cos\theta}{2}\Big)^{\alpha}, \\ P &= \frac{\psi}{\cos\theta} = \phi'(s) - \frac{1}{\cos\theta}(3G - \phi')\Big(\frac{1-\cos\theta}{2}\Big)^{\beta}. \end{aligned} \tag{2.4}$$

In the case of the deformation process with average curvature A and B become

$$A \equiv N = \frac{\sigma_u}{s}, \qquad P \equiv \phi'. \tag{2.5}$$

Using (2.3), (2.4) we write increments of membrane forces and bending moments in the form

$$\begin{aligned} \delta T_{ij} &= \int_{-h/2}^{h/2} \delta\sigma_{ij}dZ = \frac{2}{3}A_1\big(\delta\varepsilon^*_{ij} + \delta_{ij}\delta\varepsilon^*_{kk}\big) - \frac{2}{3}A_2\big(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}\big) + \\ &\quad + \frac{\sigma_{ij}}{\sigma_u}\big[(P_1 - A_1)\varepsilon - (P_2 - A_2)\chi\big], \\ \delta M_{ij} &= \int_{-h/2}^{h/2} \delta\sigma_{ij}ZdZ = \frac{2}{3}A_2\big(\delta\varepsilon^*_{ij} + \delta_{ij}\delta\varepsilon^*_{kk}\big) - \frac{2}{3}A_3\big(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}\big) + \\ &\quad + \frac{\sigma_{ij}}{\sigma_u}\big[(P_2 - A_2)\varepsilon - (P_3 - A_3)\chi\big], \end{aligned} \tag{2.6}$$

where

$$P_m = \int_{-h/2}^{h/2} PZ^{m-1}dZ, \quad A_m = \int_{-h/2}^{h/2} AZ^{m-1}dZ, \quad \varepsilon = \frac{\sigma_{ij}}{\sigma_u}\delta\varepsilon^*_{ij}, \quad \chi = \frac{\sigma_{ij}}{\sigma_u}\delta\chi_{ij}.$$

Written quantities satisfy the stability equations of plates or shells

$$\begin{aligned} &\frac{\partial\delta T_{ij}}{\partial x_j} = 0, \\ &\frac{\partial^2\delta M_{ij}}{\partial x_i\partial x_j} + T_{ij}\delta\chi_{ij} + k_{ij}\delta T_{ij} = 0, \\ &\frac{\partial^2\delta\varepsilon_{11}^*}{\partial x_2^2} + \frac{\partial^2\delta\varepsilon^*_{22}}{\partial x_1^2} - 2\frac{\partial^2\delta\varepsilon^*_{12}}{\partial x_1\partial x_2} = k_{11}\frac{\partial^2\delta w}{\partial x_2^2} + k_{22}\frac{\partial^2\delta w}{\partial x_1^2}. \end{aligned} \tag{2.7}$$

Equation (2.1)-(2.7) form a system of fundamental equations for solving a stability problem of thin plates or shells

3. CALCULATIONS FOR RECTANGULAR PLATES AND CYLINDRICAL SHELLS

The stability of rectangular plates subjected to complex biaxial compression was analysed in [3].

Now let's consider the stability problem of a cylindrical shell of radius R, thickness h and length L subjected to complex loading. In this case we choose x lying along the generatrix of the shell, $y = R\theta$ and Z - in the direction of the normal, so that $k_{11} = 0$, $k_{22} = 1/R$.

Suppose that the deformation process in the shell is determined by the constitutive relations (2.2), (2.5). Since A and P do not depend on variable Z, so that $A_2 = P_2 = 0$, δT_{ij} depend only on $\delta\varepsilon^*_{ij}$, and δM_{ij} - only on $\delta\chi_{ij}$.

Stability equations (2.7) become

$$\alpha_1\frac{\partial^4\delta w}{\partial x^4} + \alpha_2\frac{\partial^4\delta w}{\partial x^3\partial y} + \alpha_3\frac{\partial^4\delta w}{\partial x^2\partial y^2} + \alpha_4\frac{\partial^4\delta w}{\partial x\partial y^3} + \alpha_5\frac{\partial^4\delta w}{\partial y^4} - \frac{9}{Nh^2}\left(\sigma_{11}\frac{\partial^2\delta w}{\partial x^2} + \sigma_{22}\frac{\partial^2\delta w}{\partial y^2} + 2\sigma_{12}\frac{\partial^2\delta w}{\partial x\partial y} - \frac{1}{R}\frac{\partial^2\varphi}{\partial x^2}\right) = 0, \quad (3.1)$$

$$\beta_1\frac{\partial^4\varphi}{\partial x^4} + \beta_2\frac{\partial^4\varphi}{\partial x^3\partial y} + \beta_3\frac{\partial^4\varphi}{\partial x^2\partial y^2} + \beta_4\frac{\partial^4\varphi}{\partial x\partial y^3} + \beta_5\frac{\partial^4\varphi}{\partial y^4} + \frac{N}{R}\frac{\partial^2\delta w}{\partial x^2} = 0, \quad (3.2)$$

where

$$\delta T_{11} = \frac{\partial^2\varphi}{\partial y^2}, \qquad \delta T_{22} = \frac{\partial^2\varphi}{\partial x^2}, \qquad \delta T_{12} = -\frac{\partial^2\varphi}{\partial x\partial y},$$

$$\alpha_1 = 1 - \frac{3}{4}\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{11}^2}{\sigma_u^2}, \qquad \alpha_2 = -3\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{11}\sigma_{12}}{\sigma_u^2},$$

$$\alpha_3 = 2 - \frac{3}{4}\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{12}^2}{\sigma_u^2} - \frac{3}{2}\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{11}\sigma_{22}}{\sigma_u^2},$$

$$\alpha_4 = -3\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{12}\sigma_{22}}{\sigma_u^2}, \qquad \alpha_5 = 1 - \frac{3}{4}\left(1 - \frac{\phi'}{N}\right)\frac{\sigma_{22}^2}{\sigma_u^2},$$

$$\beta_1 = 1 + \frac{1}{4}\left(\frac{N}{\phi'} - 1\right)\left(\frac{2\sigma_{22} - \sigma_{11}}{\sigma_u}\right)^2, \quad \beta_2 = -3\left(\frac{N}{\phi'} - 1\right)\frac{\sigma_{12}(2\sigma_{22} - \sigma_{11})}{\sigma_u^2},$$

$$\beta_3 = 2 + \frac{1}{2}\left(\frac{N}{\phi'} - 1\right)\frac{(2\sigma_{11} - \sigma_{22})(2\sigma_{22} - \sigma_{11})}{\sigma_u^2} + 9\left(\frac{N}{\phi'} - 1\right)\frac{\sigma_{12}^2}{\sigma_u^2},$$

$$\beta_4 = -3\left(\frac{N}{\phi'} - 1\right)\frac{\sigma_{12}(2\sigma_{11} - \sigma_{22})}{\sigma_u^2}, \quad \beta_5 = 1 + \frac{1}{4}\left(\frac{N}{\phi'} - 1\right)\left(\frac{2\sigma_{11} - \sigma_{22}}{\sigma_u^2}\right)^2.$$

Satisfying kinematic boundary conditions with butt-ends simply supported, we can find solutions of the form

$$\delta w = A\sin\left(\frac{m\pi x}{L} - \frac{ny}{R}\right), \qquad \varphi = B\sin\left(\frac{m\pi x}{L} - \frac{ny}{R}\right). \quad (3.3)$$

Consider some particular cases:

a. Cylindrical shell subjected to compression along generatrix

In this case

$$\sigma_{11} = -p, \qquad \sigma_{22} = \sigma_{12} = 0, \qquad \sigma_u = |\sigma_{11}|,$$
$$\alpha_1 = \frac{1}{4} + \frac{3}{4}\frac{\phi'}{N}, \qquad \alpha_3 = 2, \quad \alpha_5 = 1, \quad \alpha_2 = \alpha_4 = 0,$$
$$\beta_1 = \frac{3}{4} + \frac{1}{4}\frac{N}{\phi'}, \qquad \beta_3 = 3 - \frac{N}{\phi'}, \quad \beta_5 = \frac{N}{\phi'}, \quad \beta_2 = \beta_4 = 0.$$

Substituting these values and δw, φ by (3.3) into stability equations (3.1), (3.2) and taking in account of the existence of nontrivial solution, we receive a relation for finding critical force

$$i^2 = \frac{N}{p\lambda_m^2}\left(\alpha_1\lambda_m^4 + \alpha_3\lambda_m^2 n^2 + \alpha_5 n^4 + \frac{\lambda_m^4 i^2}{\beta_1\lambda_m^4 + \beta_3\lambda_m^2 n^2 + \beta_5 n^4}\right)$$

or

$$i^2 = \frac{N}{p}\frac{(\alpha_1\lambda_m^4 + \alpha_3\lambda_m^2 n^2 + \alpha_5 n^4)(\beta_1\lambda_m^4 + \beta_3\lambda_m 2n^2 + \beta_5 n^4)}{\lambda_m^2(\beta_1\lambda_m^4 + \beta_3\lambda_m^2 n^2 + \beta_5 n^4) - \lambda_m^4\dfrac{N}{p}} \tag{3.4}$$

where

$\lambda_m = m\pi R/L$,

$i = 3R/h$ - stiffness of the shell.

Denoting by $X = n^2$, $Y = \lambda_m^2/n^2$ rewrite (3.4) as follows

$$i^2 = \frac{N}{p}\frac{X^2\left(\alpha_1 Y + \alpha_3 + \dfrac{\alpha_5}{Y}\right)\left(\beta_1 Y + \beta_3 + \dfrac{\beta_5}{Y}\right)}{X\left(\beta_1 Y + \beta_3 + \dfrac{\beta_5}{Y}\right) - \dfrac{N}{p}} \tag{3.5}$$

Minimizing (3.5)

$$\frac{\partial i^2}{\partial X} = 0, \qquad \frac{\partial i^2}{\partial Y} = 0$$

gives us

$$i^2 = 4\frac{N^2}{p^2}\frac{\phi'}{N}\frac{2 + \sqrt{1 + 3\dfrac{\phi'}{N}}}{3\dfrac{\phi'}{N} - 1 + \sqrt{1 + 3\dfrac{\phi'}{N}}}$$

Hence

$$i = 2\frac{N}{p}\sqrt{\frac{\phi'}{N}}\sqrt{\frac{2 + \sqrt{1 + 3\dfrac{\phi'}{N}}}{3\dfrac{\phi'}{N} - 1 + \sqrt{1 + 3\dfrac{\phi'}{N}}}} \tag{3.6}$$

where $N = \sigma_u/s$

Since s is determined from simple expression

$$\sigma_u = p = \phi(s), \quad \text{so that} \quad s = \phi^{-1}(p),$$

then critical force p_k can be found from (3.6).

For a long cylindrical shell $\lambda_m^2 \ll n^2$ we obtain

$$i^2 = \frac{N}{p\lambda_m^2}\left(\alpha_5 n^4 + \frac{\lambda_m^4 i^2}{\beta_5 n^4}\right)$$

A minimization gives

$$i = \frac{2}{p}\sqrt{N\phi'} \tag{3.7}$$

b. Cylindrical shell subjected to compression and external pressure

Pre-buckling process is of the form

$$\sigma_{11} = -p(t), \quad \sigma_{22} = -\tilde{q}\frac{R}{h} = -q(t), \quad \sigma_{12} = 0, \quad \sigma_u^2 = p^2 - pq + q^2,$$

where t is a loading parameter. Respectively, deformation increments are determined by following equations

$$\dot{\varepsilon}_{11} = \frac{1}{N}\left(-\dot{p} + \frac{1}{2}\dot{q}\right) - \left(\frac{1}{\phi'} - \frac{1}{N}\right)\frac{\left(p\dot{p} + q\dot{q} - \frac{1}{2}p\dot{q} - \frac{1}{2}q\dot{p}\right)\left(p - \frac{1}{2}q\right)}{p^2 - pq + q^2},$$

$$\dot{\varepsilon}_{22} = -\frac{1}{N}\left(\dot{q} - \frac{1}{2}\dot{p}\right) - \left(\frac{1}{\phi'} - \frac{1}{N}\right)\frac{\left(p\dot{p} + q\dot{q} - \frac{1}{2}p\dot{q} - \frac{1}{2}q\dot{p}\right)\left(q - \frac{1}{2}p\right)}{p^2 - pq + q^2},$$

where

$$N = \frac{\sigma_u}{s} = \frac{1}{s}\left(p^2 - pq + q^2\right)^{1/2}.$$

The arc-length of the deformation trajectory is found by

$$\frac{ds}{dt} = \frac{2}{\sqrt{3}}\left(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{22}\right)^{1/2} \equiv F(s, p, q). \tag{3.8}$$

Since p, q depend on loading parameter $p = p(t)$, $q = q(t)$, so from (3.8) s is determined as a function of t.

In this cases the coefficients of the stability equations (3.1), (3.2) are the following

$$\alpha_1 = 1 - \frac{3}{4}\left(1 - \frac{\phi'}{N}\right)\frac{p^2}{p^2 - pq + q^2},$$

$$\alpha_3 = 2 - \frac{3}{2}\left(1 - \frac{\phi'}{N}\right)\frac{pq}{p^2 - pq + q^2},$$

$$\alpha_5 = 1 - \frac{3}{4}\left(1 - \frac{\phi'}{N}\right)\frac{p^2}{p^2 - pq + q^2},$$

$$\alpha_2 = \alpha_4 = 0.$$

$$\beta_1 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2q-p)^2}{p^2 - pq + q^2},$$
$$\beta_3 = 2 + \frac{1}{2}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p-q)(2q-p)}{p^2 - pq + q^2},$$
$$\beta_5 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p-q)^2}{p^2 - pq + q^2},$$
$$\beta_2 = \beta_4 = 0.$$

By the same method we get an expression for finding critical load

$$i^2 = \frac{N}{p\lambda_m^2 + qn^2}\Big(\alpha_1\lambda_m^4 + \alpha_3\lambda_m^2 n^2 + \alpha_5 n^4 + \frac{\lambda_m^4 i^2}{\beta_1\lambda_m^4 + \beta_3\lambda_m^2 n^2 + \beta_5 n^4}\Big).$$

Using previous notation X, Y we rewrite

$$i^2 = \frac{N(\alpha_1 Y + \alpha_3 + \frac{\alpha_5}{Y})(\beta_1 Y + \beta_3 + \frac{\beta_5}{Y})X^2}{X(p + \frac{q}{Y})(\beta_1 Y + \beta_3 + \frac{\beta_5}{Y}) - N}. \tag{3.9}$$

Minimization of i^2 gives

$$G(s,p,q,Y) \equiv (\beta_1 Y + \beta_3 + \frac{\beta_5}{Y})(\alpha_1 - \frac{\alpha_5}{Y^2}) - (\alpha_1 Y + \alpha_3 + \frac{\alpha_5}{Y})(\beta_1 - \frac{\beta_5}{Y^2}) + \\ + 2(\alpha_1 Y + \alpha_3 + \frac{\alpha_5}{Y})(\beta_1 Y + \beta_3 + \frac{\beta_5}{Y})\frac{q}{Y^2(p + \frac{q}{Y})} = 0; \tag{3.10}$$

$$i = \frac{2N}{p + \frac{q}{Y}}\left(\frac{\alpha_1 Y + \alpha_3 + \frac{\alpha_5}{Y}}{\beta_1 Y + \beta_3 + \frac{\beta_5}{Y}}\right)^{1/2} \equiv H(s,p,q,Y). \tag{3.11}$$

From equation (3.10) Y is determined as a function of s, p, q and then substituted into (3.11). Expressing p, q through loading parameter t, from (3.8) and (3.11) we can find critical value of loading parameter t_k by numerical calculation.

Critical loads are the following

$$p_k = p(t_k), \qquad q_k = q(t_k)$$

For a long cylindrical shell $\lambda_m^2 \ll n^2$ we obtain

$$i^2 = \frac{N}{p\lambda_m^2 + qn^2}\Big(\alpha_5 n^4 + \frac{\lambda_m^4 i^2}{\beta_5 n^4}\Big)$$

Minimization of i^2 gives us

$$i = \frac{2N}{p}\sqrt{\frac{1 - \frac{3}{4}(1 - \frac{\phi'}{N})\frac{q^2}{p^2 - pq + q^2}}{1 + \frac{1}{4}(\frac{N}{\phi'} - 1)\frac{(2p-q)^2}{p^2 - pq + q^2} + 4N\frac{q}{p^2}}} \equiv H(s,p,q). \tag{3.12}$$

Expressing $p = p(t)$, $q = q(t)$, we can find the critical value of loading parameter t_k from (3.12) and (3.8) by numerical calculation.

CONCLUSIONS

1. Constitutive relations are formulated completely. Contained in these relations material functions are already determined.

2. The local theory of elastoplastic deformation processes can be applied to the stability problems of thin-walled structures, when both Pre-buckling process and Post-buckling processes are complicated.

3. Proposed method of loading parameter gives a way to solve stability problem for all types of loading.

4. Complex loading process has an essential influence on the stability of structures. Critical loads are lower than that is for simple loading.

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LÝ THUYẾT QUÁ TRÌNH BIẾN DẠNG ĐÀN DẺO VÀ BÀI TOÁN ỔN ĐỊNH NGOÀI GIỚI HẠN ĐÀN HỒI CỦA CÁC KẾT CẤU THÀNH MỎNG CHỊU TẢI PHỨC TẠP

Dựa trên định đề đẳng hướng và giả thuyết xác định địa phương tác giả đã xây dựng được dạng hoàn chỉnh của lý thuyết quá trình biến dạng đàn dẻo. Lý thuyết chứa đựng hai hàm vật liệu mô tả tính chất vô hướng và tính chất vectơ của vật liệu. Các hàm này cũng đã được thiết lập từ các số liệu thực nghiệm.

Sử dụng lý thuyết trên vào bài toán ổn định ngoài giới hạn đàn hồi của kết cấu thành mỏng khi quá trình biến dạng trước và sau khi mất ổn định đều là phức tạp. Đưa ra phương pháp giải loại bài toán này thông qua tham số tải. Giải bài toán cụ thể về bản mỏng chữ nhật và vỏ trụ chịu tải phức tạp.