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NONLINEAR OSCILLATIONS IN SYSTEMS WITH LARGE STATIC DEFLECTION OF ELASTIC ELEMENTS

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ABSTRACT. In mechanical systems the static deflection of the elastic elements is usual not appeared in the equations of motion. The reason is that either a linear model of the elastic elements or their too small static deflection assumption was accepted.

In the present paper both nonlinear model of elastic elements and their large static deflection are considered, so that the nonlinear terms in the equation of motion appear with different degrees of smallness. In this case the nonlinearity of the system depends not only on the nonlinear characteristic of the elastic element but on its static deflection. The distinguishing feature of the system under consideration is that if the elastic element has soft characteristic, the nonlinear system also belongs to the soft one. If the elastic element has hard characteristic, the system may be either soft or hard or neutral type, depending on the relation between the parameters of the elastic element and its static deflection.

The autonomous and non-autonomous systems have been studied. Analytical methods in combination with computer have been used.

The problem of nonlinear oscillations of elastic structures with large static deflection in general, and beams, plates in particular, may be studied in a similar maner.

1. INTRODUCTION

Let us consider the simplest oscillatory system which consists of a mass M and the spring as shown in the Fig. 1. The spring supporting the mass is assumed to be nonlinear with the characteristic

$$f(u) = c_0 u + \beta_0 u^3, \tag{1.1}$$

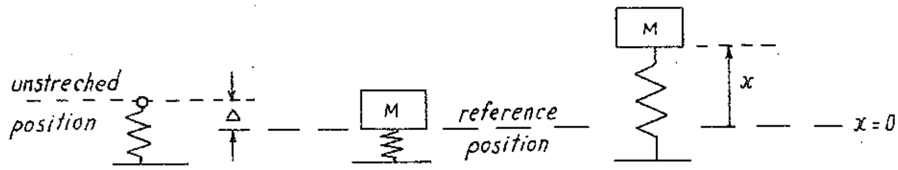

Fig. 1

so that the spring force acting on the mass M is

$$c_0(\Delta - x) + \beta_0(\Delta - x)^3,$$

where c_0 is a positive constant and β_0 is either positive (hard characteristic) or negative (soft characteristic), Δ is the deformation of the spring at the static equilibrium position. This position is chosen as the reference position. When $x = 0$, the spring force $c_0\Delta + \beta_0\Delta^3$ is equal to the gravitational force Mg, that is

$$c_0\Delta + \beta_0\Delta^3 = Mg.$$

Measuring the displacement x from the static equilibrium position with x chosen to be positive in the upward direction, and applying Newton's second law of motion to the mass M we obtain

$$M\ddot{x} + c_0 x + 3\beta_0\Delta^2 x - 3\beta_0\Delta x^2 + \beta_0 x^3 = 0.$$

It is supposed that Δ is large and x is enough small, so that in comparison with linear terms, $\beta_0 x^3$ is a small quantity of second degree and $\beta_0\Delta x^2$ is of the first degree of smallness:

$$\frac{x}{\Delta} = 0(\varepsilon), \qquad \beta_0 x^3 = 0(\varepsilon^2), \qquad \beta_0\Delta x^2 = 0(\varepsilon),$$

where ε is a small positive parameter. In this case $\beta_0\Delta^2 x$ is finite.

Taking into account the viscous damping force $h_0\dot{x}$ and exciting force $P(t,x)$ which are both assumed to be small quantities of second degree and introducing the notation

$$\omega^2 = \frac{c_0 + 3\beta_0\Delta^2}{M}, \quad \varepsilon\gamma = \frac{3\beta_0\Delta}{M}, \quad \varepsilon^2\beta = \frac{\beta_0}{M}, \quad \varepsilon^2 h = \frac{h_0}{M}, \quad \varepsilon^2 f(t,x) = \frac{1}{M}P(t,x), \tag{1.2}$$

we can write the equation of motion of the mass M in the form:

$$\ddot{x} + \omega^2 x = \varepsilon\gamma x^2 - \varepsilon^2(h\dot{x} + \beta x^3 - f(t,x)). \tag{1.3}$$

In comparison with the classical Duffing equation, in the equation (1.3) the small terms appear with different degrees, most of them are of second degree of smallness. From the structure of the equation (1.3) one can predict that the influence of the forces on the motion of the mass M can be found in the second approximation of the solution. Some studies of stationary forced and parametric oscillations in the systems described by the equation of type (1.3) are given in [1, 2]. In the present paper a more general equation will be investigated.

$$\ddot{x} + \omega^2 x = \varepsilon\gamma x^2 + \varepsilon^2 F(\tau, \varphi(\tau), x, \dot{x}), \tag{1.4}$$

where τ is a slow time $\tau = \varepsilon t$, $F(\tau,\varphi(\tau),x,\dot{x})$ is the periodic function relatively to φ with period 2π which can be represented in the form

$$F(\tau,\varphi,x,\dot{x}) = \sum_{n=-N}^{N} e^{in\varphi} F_n(\tau,x,\dot{x}).$$

The coefficients of this expansion $F_n(\tau,x,\dot{x})$ are polynoms of x, $\dot{x}$. It is assumed that the momentary frequency $\nu(\tau) = \dfrac{d\varphi}{dt}$ is slowly changed over the time and that $F_n(\tau,x,\dot{x})$, $\nu(\tau)$ have an enough number of derivatives relatively to τ for all finite values of τ. We will be specially interested in the study of the resonance zone when ω is near to $\dfrac{p}{q}\nu$, where p and q are integers.

2. AUTONOMOUS SYSTEM

First, we study a special case of the equation (1.4) when $F(\tau,\varphi(\tau),x,\dot{x})$ does not depend on time:

$$F(\tau, \varphi(\tau), x, \dot{x}) = Q(x, \dot{x}). \tag{2.1}$$

Following to the asymptotic method of nonlinear oscillation [3, 4] the solution of the equation (1.4) in this case will be found in the form

$$x = a\cos\theta + \varepsilon u_1(a,\theta) + \varepsilon^2 u_2(a,\theta) + \dots \tag{2.2}$$

where $u_i(a,\theta)$ are periodic functions of θ with period 2π which do not contain the first harmonics $\sin\theta$, $\cos\theta$, and a, θ satisfy the equations:

$$\begin{aligned} \frac{da}{dt} &= \varepsilon A_1(a) + \varepsilon^2 A_2(a) + \dots, \\ \frac{d\theta}{dt} &= \omega + \varepsilon B_1(a) + \varepsilon^2 B_2(a) + \dots \end{aligned} \tag{2.3}$$

Substituting these expressions into the equation (1.4) and comparing the coefficients of ε and ε^2 we have:

$$\begin{aligned} \omega^2\Big(\frac{\partial^2 u_1}{\partial\theta^2} + u_1\Big) &= \gamma a^2\cos^2\theta + 2a\omega B_1\cos\theta + 2\omega A_1\sin\theta, \\ \omega^2\Big(\frac{\partial^2 u_2}{\partial\theta^2} + u_2\Big) &= 2a\gamma u_1\cos\theta + Q(a\cos\theta, -a\omega\sin\theta) + \\ &\quad + 2a\omega B_2\cos\theta + 2\omega A_2\sin\theta + R(A_1, B_1), \end{aligned} \tag{2.4}$$

where $R(0, B_1) = R(A_1, 0) \equiv 0$. Comparing the coefficients of the harmonics in the first equation of (2.4) gives:

$$A_1 = 0, \quad B_1 = 0, \quad u_1 = \frac{\gamma a^2}{2\omega^2}\big(1 - \frac{1}{3}\cos 2\theta\big). \tag{2.5}$$

Comparing the first harmonics $\sin\theta$, and $\cos\theta$ in the second equation of (2.4) yields:

$$\begin{aligned} A_2 &= -\frac{1}{\omega}\langle\sin\theta . Q(a\cos\theta, -a\omega\sin\theta)\rangle, \\ B_2 &= -\frac{1}{\omega a}\Big[\langle\cos\theta Q(a\cos\theta, -a\omega\sin\theta\rangle + \frac{5}{12}\frac{\gamma^2 a^3}{\omega^2}\Big], \end{aligned} \tag{2.6}$$

where $\langle f\rangle$ is averaged valued on time of the function f. We consider now two important examples:

Example 1. Duffing equation

Supposing that $Q(x, \dot{x}) = -h\dot{x} - \beta x^3$, we obtain

$$A_2 = -\frac{h}{2}a, \quad B_2 = \frac{\alpha}{2\omega}a^2, \quad \alpha = \frac{3}{4}\beta - \frac{5\gamma^2}{6\omega^2}. \tag{2.7}$$

Thus, in the second approximation we have

$$x = a\cos\theta + \varepsilon\frac{\gamma a^2}{2\omega^2}\big(1 - \frac{1}{3}\cos 2\theta\big), \tag{2.8}$$

where a and θ are determined from the equations

$$\begin{aligned} \frac{da}{dt} &= -\frac{\varepsilon^2}{2}ha, \\ \frac{d\theta}{dt} &= \omega + \frac{\varepsilon^2\alpha}{2\omega}a^2. \end{aligned} \tag{2.9}$$

The oscillations are damped with the frequency depending on the amplitude. With the growth of time the momentary frequency $\dfrac{d\theta}{dt}$ either increases if $\alpha < 0$ or decreases if $\alpha > 0$ or is a constant if $\alpha = 0$. This is a distingguishing feature of the system with large static deflection. The parameter α depends on the parameters c_0, β_0 (spring) and Δ (static deflection).

Example 2. Van - der - Pol equation

It is assumed that $Q(x, \dot{x}) = -\beta x^3 + D(1 - x^2)\dot{x}$, where D is a positive constant. We have

$$A_2 = \frac{aD}{2}\Big(1 - \frac{a^2}{4}\Big), \quad B_2 = \frac{\alpha}{2\omega}a^2$$

and the equations of the second approximation are

$$\begin{aligned} \frac{da}{dt} &= \varepsilon^2 \frac{aD}{2}\Big(1 - \frac{a^2}{4}\Big), \\ \frac{d\theta}{dt} &= \omega + \varepsilon^2 \frac{\alpha}{2\omega}a^2. \end{aligned} \tag{2.10}$$

The oscillation is self-excited with a constant amplitude $a_0 = 2$. The essential difference in comparison with the classical Van - der - Pol oscillator is that the momentary frequency depends on the parameter α which can be either positive or negative or zero.

3. NON-STATIONARY NON-AUTONOMOUS SYSTEM

The approximate solution of the equation (1.4) in general case will be found in the form

$$x = a\cos\Big(\frac{p}{q}\varphi + \psi\Big) + \varepsilon u_1(\tau, a, \varphi, \theta) + \varepsilon^2 u_2(\tau, a, \varphi; \theta) + \dots, \tag{3.1}$$

where $\theta = \dfrac{p}{q}\varphi + \psi$ and $u_i(\tau, a, \varphi, \theta)$ are periodic functions of φ, θ with period 2π and do not contain the first harmonics $\cos\theta$, $\sin\theta$. The unknown functions a and ψ satisfy the equations:

$$\begin{aligned} \frac{da}{dt} &= \varepsilon A_1(\tau, a, \psi) + \varepsilon^2 A_2(\tau, a, \psi) + \dots, \\ \frac{d\psi}{dt} &= \omega - \frac{p}{q}\nu(\tau) + \varepsilon B_1(\tau, a, \psi) + \varepsilon^2 B_2(\tau, a, \psi) + \dots \end{aligned} \tag{3.2}$$

By substituting the expressions (3.1) and (3.2) into the equation (1.4) and comparing the coefficients of ε and ε^2 we obtain

$$\begin{aligned} &\nu^2(\tau)\frac{\partial^2 u_1}{\partial\varphi^2} + 2\omega\nu(\tau)\frac{\partial^2 u_1}{\partial\varphi\partial\theta} + \omega^2\frac{\partial^2 u_1}{\partial\theta^2} + \omega^2 u_1 = \gamma a^2\cos^2\theta - \\ &- \Big[\Big(\omega - \frac{p}{q}\nu(\tau)\Big)\frac{\partial A_1}{\partial\psi} - 2a\omega B_1\Big]\cos\theta + \Big[\Big(\omega - \frac{p}{q}\nu(\tau)\Big)a\frac{\partial B_1}{\partial\psi} + 2\omega A_1\Big]\sin\theta, \end{aligned} \tag{3.3}$$

$$\nu^2(\tau)\frac{\partial^2 u_2}{\partial\varphi^2}+2\omega\nu(\tau)\frac{\partial^2 u_2}{\partial\varphi\partial\theta}+\omega^2\frac{\partial^2 u_2}{\partial\theta^2}+\omega^2 u_2=2a\gamma u_1\cos\theta+F(\tau,\varphi,a\cos\theta,-a\omega\sin\theta)$$
$$-\Big[\big(\omega-\frac{p}{q}\nu(\tau)\big)\frac{\partial A_2}{\partial\psi}-2a\omega B_2+A_1\frac{\partial A_1}{\partial a}+B_1\frac{\partial A_1}{\partial\psi}+\frac{\partial A_1}{\partial\tau}-aB_1^2\Big]\cos\theta$$
$$+\Big[\big(\omega-\frac{p}{q}\nu(\tau)\big)a\frac{\partial B_2}{\partial\psi}+2\omega A_2+2A_1B_1+aA_1\frac{\partial B_1}{\partial a}+aB_1\frac{\partial B_1}{\partial\psi}+a\frac{\partial B_1}{\partial\tau}\Big]\sin\theta$$
$$-\Big\{2\omega\frac{\partial^2 u_1}{\partial\tau\partial\theta}+2\nu(\tau)\frac{\partial^2 u_1}{\partial\tau\partial\varphi}+2\nu(\tau)A_1\frac{\partial^2 u_1}{\partial a\partial\varphi}+2\omega A_1\frac{\partial^2 u_1}{\partial a\partial\theta}+2\nu(\tau)B_1\frac{\partial^2 u_1}{\partial\varphi\partial\theta}$$
$$+2\omega B_1\frac{\partial^2 u_1}{\partial\theta^2}+\big(\omega-\frac{p}{q}\nu(\tau)\big)\Big(\frac{\partial B_1}{\partial\psi}\frac{\partial u_1}{\partial\theta}+\frac{\partial A_1}{\partial\psi}\frac{\partial u_1}{\partial a}\Big)+\frac{\partial u_1}{\partial\varphi}\frac{d\nu(\tau)}{d\tau}\Big\}. \tag{3.4}$$

The unknown functions A_1, B_1 and u_1 will be determined from the equation (3.3). By comparing the coefficients of harmonics in (3.3) we obtain:

$$A_1=0,\quad B_1=0,\quad u_1=\frac{\gamma a^2}{2\omega^2}\big(1-\frac{1}{3}\cos 2\theta\big),\quad \theta=\frac{p}{q}\varphi+\psi. \tag{3.5}$$

Analogeously, we can find A_2, B_2 and u_2 from the equation (3.4) for the general form of the function $F(\tau,\varphi,x,\dot{x})$. However, we will concentrate attention on two important cases:

Case 1. The passage of the system through the principal resonance zone

It is supposed that the function $F(\tau,\varphi,x,\dot{x})$ is of the form

$$F(\tau,\varphi,x,\dot{x})=-h\dot{x}-\beta x^3+E\sin\varphi(t),\quad p=q=1, \tag{3.6}$$

where E is a constant. In this case the equation (3.4) becomes:

$$\nu^2(\tau)\frac{\partial^2 u_2}{\partial\varphi^2}+2\omega\nu(\tau)\frac{\partial^2 u_2}{\partial\varphi\partial\theta}+\omega^2\frac{\partial^2 u_2}{\partial\theta^2}+\omega^2 u_2=2a\gamma u_1\cos\theta+ha\omega\sin\theta-\beta a^3\cos^3\theta$$
$$+E\sin\varphi(t)-\Big[\big(\omega-\nu(\tau)\big)\frac{\partial A_2}{\partial\psi}-2a\omega B_2\Big]\cos\theta+\Big[\big(\omega-\nu(\tau)\big)a\frac{\partial B_2}{\partial\psi}+2\omega A_2\Big]\sin\theta. \tag{3.7}$$

Comparing the coefficients of $\sin\theta$ and $\cos\theta$ in (3.7) we obtain

$$\big(\omega-\nu(\tau)\big)\frac{\partial A_2}{\partial\psi}-2a\omega B_2=-\alpha a^3-E\sin\psi,$$
$$\big(\omega-\nu(\tau)\big)a\frac{\partial B_2}{\partial\psi}+2\omega A_2=-ha\omega-E\cos\psi,$$
$$\alpha=\frac{3}{4}\beta-\frac{5\gamma^2}{6\omega^2}.$$

Solving these equations we have

$$A_2=-\frac{ha}{2}-\frac{E}{\omega+\nu(\tau)}\cos\psi,$$
$$B_2=\frac{\alpha}{2\omega}a^2+\frac{E}{a[\omega+\nu(\tau)]}\sin\psi. \tag{3.8}$$

Comparing the coefficients of the other harmonics in (3.7) and solving the equation obtained we get

$$u_2=\frac{1}{16\omega^2}\Big(\frac{\gamma^2}{3\omega^2}+\frac{\beta}{2}\Big)a^3\cos 3\theta. \tag{3.9}$$

Thus, in the second approximation we have:

$$x = a\cos\theta + \frac{\varepsilon\gamma a^2}{2\omega^2}\left(1 - \frac{1}{3}\cos 2\theta\right), \quad \theta = \varphi(t) + \psi(t), \tag{3.10}$$

where a and ψ satisfy the equations:

$$\begin{aligned} \frac{da}{dt} &= -\varepsilon^2\left[\frac{h}{2}a + \frac{E}{\omega + \nu(\tau)}\cos\psi\right], \\ \frac{d\psi}{dt} &= \omega - \nu(\tau) + \frac{\varepsilon^2\alpha}{2\omega}a^2 + \frac{\varepsilon^2 E}{a[\omega + \nu(\tau)]}\sin\psi. \end{aligned} \tag{3.11}$$

These equations are solved on the personal computer by using the finite difference method for the parameters $\frac{\varepsilon^2 h}{\omega} = 0.5 \cdot 10^{-3}$, $\frac{\varepsilon^2 E}{\omega^2} = 0.158 \cdot 10^{-3}$ and $\frac{\varepsilon^2\alpha}{\omega^2} = +0.1$ (Fig. 2), $\frac{\varepsilon^2\alpha}{\omega^2} = -0.1$ (Fig. 3) with the initial values: $t = 0$, $a_0 = 10^{-5}$, $\psi_0 = 0$. The parameter $\eta = \frac{\nu}{\omega}$ for Fig. 2 is $\eta = 0.97 + 10^{-6}t$ (curve 1, $\Delta t = 0.04$), $\eta = 0.97 + 10^{-5}t$ (curve 2, $\Delta t = 0.4$), $\eta = 1.03 - 10^{-6}t$ (curve 3), $\eta = 1.03 - 10^{-5}t$ (curve 4) and for Fig. 3 is $\eta = 1.02 - 10^{-6}t$ (curve 1), $\eta = 1.02 - 10^{-5}t$ (curve 2), $\eta = 0.97 + 10^{-6}t$ (curve 3, $\Delta t = 0.04$), $\eta = 0.97 + 10^{-5}t$ (curve 4, $\Delta t = 0.4$).

The stationary amplitudes corresponding to the constant values of the frequency ν are presented in the Fig. 4 for the values mentioned above of f, E and $\frac{\varepsilon^2\alpha}{\omega^2} = +0.1$ (curve 1), $\alpha = 0$ (curve 2), $\frac{\varepsilon^2\alpha}{\omega^2} = -0.1$ (curve 3). The heavy (dashed) lines in this figure correspond to the stability (instability) of oscillations.

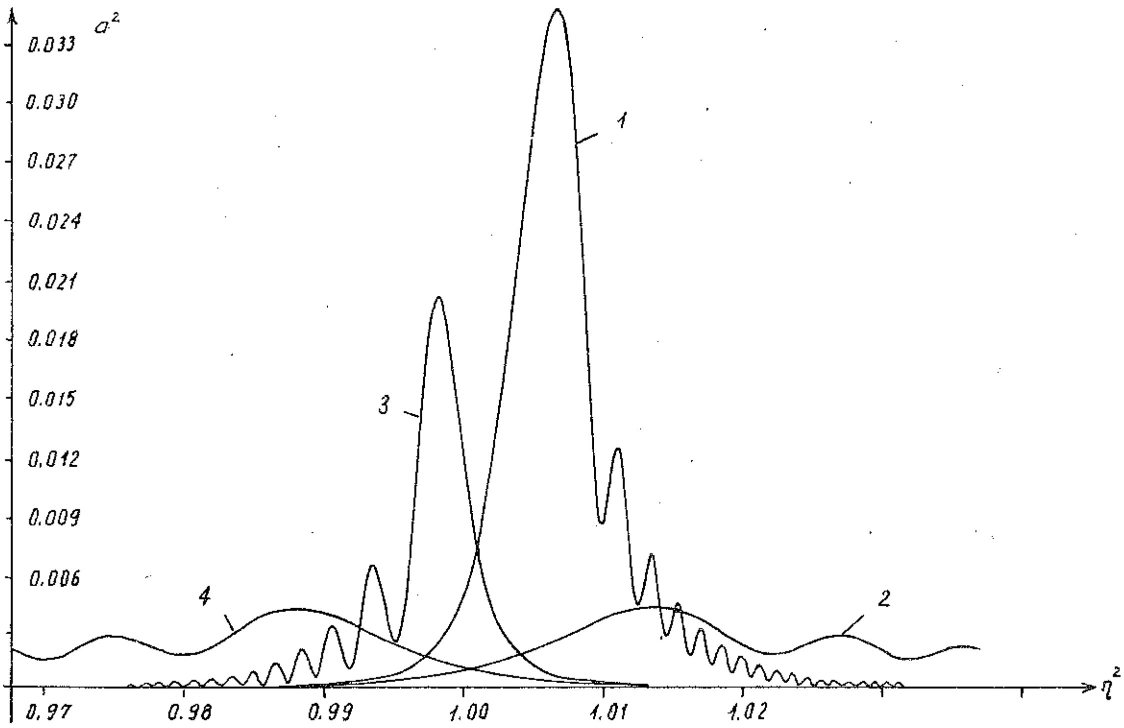

Fig. 2

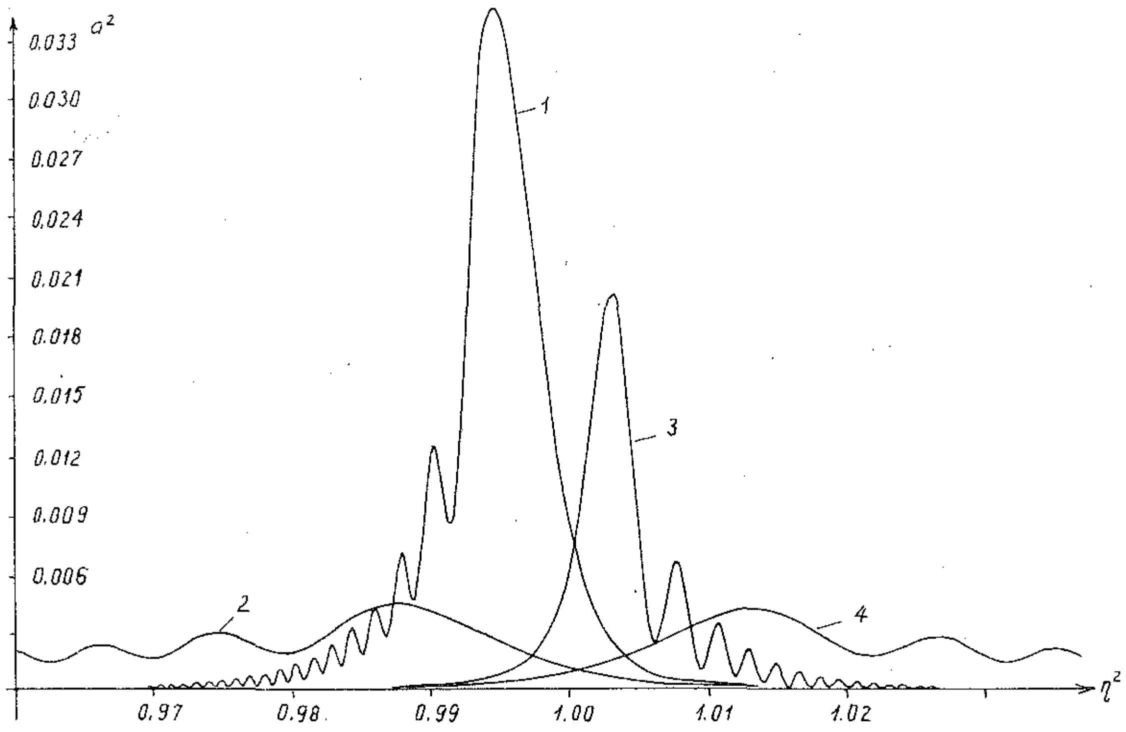

Fig. 3

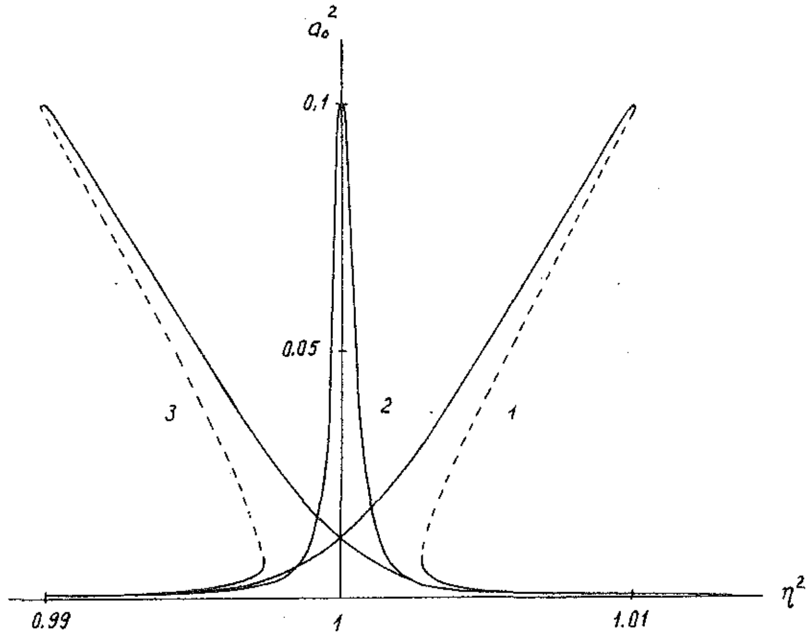

Fig. 4

Comparing the Figs 2, 3 and Fig. 4 it is seen that increasing the velocity of passing through the resonance, the maximum of the amplitudes decreases and less peaks appear after the resonance peak. The maximum of the amplitudes of stationary oscillations is biggest.

Case 2. Passing of the system through the parametric resonance
Assuming that the function $F(\tau, \varphi, x, \dot{x})$ has the form

$$F(\tau, \varphi, x, \dot{x}) = -h\dot{x} - \beta x^3 + ex\cos\varphi(t); \quad p = 1,\ q = 2, \tag{3.12}$$

where e is a constant. In this case the equation for determination of A_2, B_2 and u_2 is

$$\nu^2(\tau)\frac{\partial^2 u_2}{\partial\varphi^2} + 2\omega\nu(\tau)\frac{\partial^2 u_2}{\partial\varphi\partial\theta} + \omega^2\frac{\partial^2 u_2}{\partial\theta^2} + \omega^2 u_2 = 2a\gamma u_1\cos\theta + ha\omega\sin\theta - \beta a^3\cos^3\theta$$
$$+\sigma a\cos\theta\cos\varphi - \Big[\big(\omega - \frac{1}{2}\nu(\tau)\big)\frac{\partial A_2}{\partial\psi} - 2a\omega B_2\Big]\cos\theta + \Big[\big(\omega - \frac{1}{2}\nu(\tau)\big)a\frac{\partial B_2}{\partial\psi} + 2\omega A_2\Big]\sin\theta. \tag{3.13}$$

By comparing the coefficients of $\cos\theta$ and $\sin\theta$ in (3.13) we have

$$\Big(\omega - \frac{1}{2}\nu(\tau)\Big)\frac{\partial A_2}{\partial\psi} - 2a\omega B_2 = -\alpha a^3 + \frac{ea}{2}\cos 2\psi,$$
$$\Big(\omega - \frac{1}{2}\nu(\tau)\Big)a\frac{\partial B_2}{\partial\psi} + 2\omega A_2 = -ha\omega - \frac{ea}{2}\sin 2\psi,$$

where $\alpha = \frac{3}{4}\beta - \frac{5\gamma^2}{6\omega^2}$. From these equations we obtain

$$A_2 = -\frac{h}{2}a - \frac{ea}{2\nu(\tau)}\sin 2\psi,$$
$$B_2 = \frac{\alpha a^2}{2\omega} - \frac{e}{2\nu(t)}\cos 2\psi. \tag{3.14}$$

Hence, the equations of the second approximation become

$$\frac{da}{dt} = -\frac{\varepsilon^2}{2}\Big(ha + \frac{ea}{\nu(\tau)}\sin 2\psi\Big),$$
$$\frac{d\psi}{dt} = \omega - \frac{\nu(\tau)}{2} + \frac{\varepsilon^2\alpha}{2\omega}a^2 - \frac{\varepsilon^2 e}{2\nu(\tau)}\cos 2\psi. \tag{3.15}$$

These equations are solved on the personal computer for the parameters $\frac{\varepsilon^2 e}{2\omega^2} = 8.9\cdot 10^{-3}$, $\frac{\varepsilon^2 h}{\omega} = 0.002$ and $\frac{\varepsilon^2\alpha}{\omega^2} = 0.02$ (Fig.5), $\frac{\varepsilon^2\alpha}{\omega^2} = -0.02$ (Fig.6) and with the initial condition $t = 0$, $a_0 = 0.09$, $\psi_0 = 0$. For the case of Fig.5: $\mu = \frac{\nu(t)}{2\omega} = 1 + 10^{-5}t$ (curve 1), $\mu = 1 + 2\cdot 10^{-5}t$ (curve 2) and for the case of Fig.6: $\mu = 1 - 10^{-5}t$ (curve 1), $\mu = 1 - 2\cdot 10^{-5}t$ (curve 2)

4. CONCLUSION

The nonstationary autonomous and non - autonomous systems with large static deflection of elastic elements have been examined. It is turned out that, Althrough the elastic elements are nonlinear, the systems under consideration can become either nonlinear with different characteristics or linear, depending on the relation between the parameters of the elastic elements and their static deflections. The passage of the systems through the principal and parametric resonances has been investigated. With the growth of the velocity of passing through the resonance the maximum of amplitudes decreases and less peaks appear after the resonance peak.

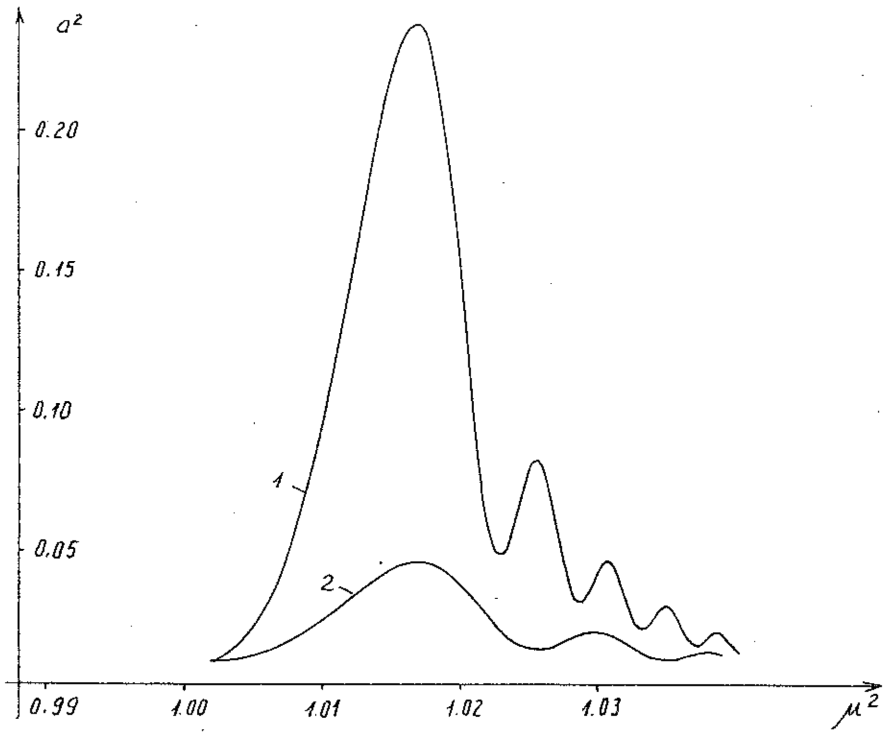
a^2
0.20
0.15
0.10
0.05
1
2
0.99
1.00
1.01
1.02
1.03
μ^2

Fig. 5

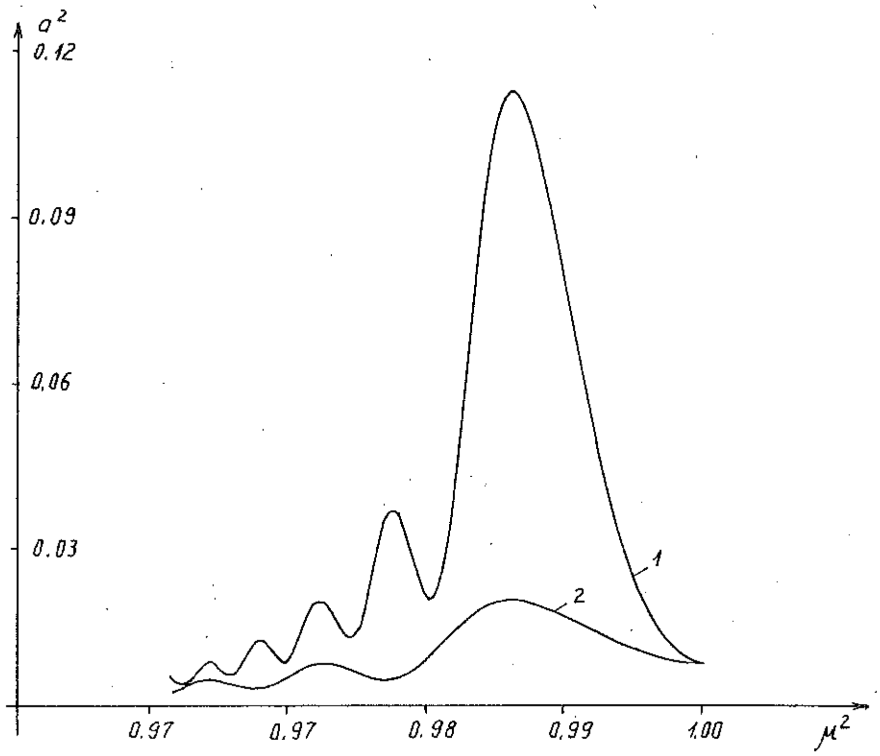
a^2
0.12
0.09
0.06
0.03
1
2
0.97
0.97
0.98
0.99
1.00
μ^2

Fig. 6

The most interesting phenomenon in the systems under consideration is that their nonlinearity depends not only on the nonlinear characteristic of the spring as in the classical theory but also on the static deflection Δ. Namely,

If the spring has soft chasracteristic ($\beta < 0$) then the system under consider action also belongs to the soft type with more soft characteristic, because

$$\alpha = \frac{3}{4}\beta - \frac{5\gamma^2}{6\omega^2} < 0.$$

When the spring has hard characteristic ($\beta > 0$), the system under consideration belongs to the hard type if

$$\alpha > 0 \quad \text{or} \quad c_0 > 7\beta_0\Delta^2 \tag{4.1}$$

and to the soft type if

$$c_0 < 7\beta_0\Delta^2 \tag{4.2}$$

and to the neutral type if

$$c_0 = 7\beta_0\Delta^2 \tag{4.3}$$

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DAO ĐỘNG PHI TUYẾN TRONG CÁC HỆ CÓ ĐỘ DÃN TĨNH LỚN

Việc kể đến độ dãn tĩnh lớn trong hệ dao động phi tuyến nhỏ dẫn tới phương trình chuyển động có dạng đặc biệt, trong đó các số hạng phi tuyến xuất hiện với các độ nhỏ khác nhau $0(\varepsilon)$ và $0(\varepsilon^2)$. Kết quả là độ cứng của hệ khảo sát thay đổi, tùy thuộc vào tương quan giữa các thông số của yếu tố đàn hồi và độ dãn tĩnh của nó. Chẳng hạn, yếu tố đàn hồi có đặc trưng cứng, đường cộng hưởng có thể mang đặc trưng mềm nếu giữa các thông số của hệ có sự phụ thuộc (4.2).

Bài toán cụ thể trình bày ở đây gợi lên sự cần thiết phải xem xét chi tiết hơn vấn đề dao động của các kết cấu đàn hồi, các dầm, tấm có độ võng tĩnh lớn.