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THEORY OF TWO-PHASE FLOW OF FLUID WITH RIGID ELLIPSOIDAL PARTICLES

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SUMMARY. In the paper [1] the general continuum theory has been developed for two-phase of fluid with deformable particles of arbitrary form, where the microdeformation of the particles and the relative motion between phases are taken into account. The extended theory of irrotational flow of fluid caused by a moving deformable body has been used to obtain the general expressions for the generalized induced mass tensors.

The simplest case, when the particles have a spherical form during the micro-deformation, has been considered in the paper [2].

This paper is devoted to the theory of two-phase flow of fluid with rigid ellipsoidal particles. The obtained equation system can be used to determine the characteristic mean velocity, the particle rotation, the generalized diffusion flux of particles, the mass densities, the volume concentration of particles, the inertia tensor and generalized induced mass tensors of particle.

1. DETERMINATION OF THE GENERALIZED INDUCED MASS TENSORS AND INERTIA TENSOR OF RIGID ELLIPSOIDAL PARTICLES

We consider the motion of incompressible fluid with rigid ellipsoidal particles. In this case there is no particle micro-deformation and the particles can only rotate with rolation velocity $\overline{\omega}$ and translate.

Then it can be shown [1] that the generalized induced mass tensors can be determined by following expressions.

$$\begin{aligned}
\overline{M} &= -\frac{1}{2}\rho_2 \int\limits_{ds_1} (\overline{n}\overline{\Phi}^u + \overline{\Phi}^u\overline{n})ds_1' \\
\overline{L} &= -\rho_2 \int\limits_{ds_1} \left\{[\overline{n}\overline{\Phi}^\omega - \overline{\Phi}^u(\overline{n}\times\overline{\xi})] + [\overline{\Phi}^\omega\overline{n} - (\overline{n}\times\overline{\xi})\overline{\Phi}^u]\right\}ds_1', \qquad (1.1)\\
\overline{N} &= \rho_2 \int\limits_{ds_1} [\overline{\Phi}^\omega(\overline{n}\times\overline{\xi}) + (\overline{n}\times\overline{\xi})\overline{\Phi}^\omega]ds_1'
\end{aligned}$$

The inertia tensor of particles has a form

$$\overline{I} = \rho_1 \int\limits_{dv_1} \overline{\xi}_1\overline{\xi}_1 dv_1' \qquad (1.2)$$

It is easy to show that $\overline{M}$, $\overline{N}$, $\overline{I}$ are polar tensors of 2nd order, $\overline{L}$ - axial tensor of 2nd order.

In the expressions (1.1) and (1.2) ds_1' and dv_1' are the surface and volume elements of particle, ds_1 and dv_1 - surface and volume of particle [1], $\overline{n}$ - a unit vector normal to a paticle surface, ρ_1 and ρ_2 - mass densities of particles and fluid, the operator $(\times)$ - vector product. The potential vectors $\overline{\Phi}_u$ and $\overline{\Phi}_\omega$ satisfy the Laplace equations inside the fluid micro-volume element and the following conditions on the particle surface

$$\begin{aligned} (\overline{n}\cdot\overline{\nabla})\overline{\Phi}^u &= \overline{n} \\ (\overline{n}\cdot\overline{\nabla})\overline{\Phi}^\omega &= -\overline{n}\times\overline{\xi} \end{aligned} \tag{1.3}$$

In other words, the potential of fluid motion caused by transtating and rotating motion of particle has a form

$$\Phi = (\overline{u}_1 - \overline{u}_2)\cdot\overline{\Phi}^u + \overline{\omega}\cdot\overline{\Phi}^\omega \tag{1.4}$$

where $\overline{u}_1$ and $\overline{u}_2$ are translating velocity of particles and fluid.

In the coordinate system connected with the principal axes of ellipsoidal particle the particle surface is determined by an equation

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1 \tag{1.5}$$

where a, b and c are principal radii of ellipsoid.

It can be shown that in this coordinate system vectors $\overline{\Phi}^u$ and $\overline{\Phi}^\omega$ can be determined [3] and they have following components

$$\begin{aligned} \Phi_x^u &= -\frac{XA}{2-A_0}\,; & \Phi_x^\omega &= \frac{(b^2-c^2)YZ(B-C)}{2(b^2-c^2)+(B_0-C_0)(b^2+c^2)}\,; \\ \Phi_y^u &= -\frac{YA}{2-B_0}\,; & \Phi_y^\omega &= \frac{(c^2-a^2)ZX(C-A)}{2(c^2-a^2)+(C_0-A_0)(c^2+a^2)}\,; \\ \Phi_Z^u &= -\frac{ZC}{2-C_0}\,; & \Phi_Z^\omega &= \frac{(a^2-b^2)XY(A-B)}{2(a^2-b^2)+(A_0-B_0)(a^2+b^2)}\,; \end{aligned} \tag{1.6}$$

In (1.6) A, B and C have the form

$$\begin{aligned} A &= abc\int_\lambda^\infty \frac{d\alpha}{(a^2+\alpha)\sqrt{(a^2+\alpha)(b^2+\alpha)(c^2+\alpha)}} \\ B &= abc\int_\lambda^\infty \frac{d\alpha}{(b^2+\alpha)\sqrt{(a^2+\alpha)(b^2+\alpha)(c^2+\alpha)}} \\ C &= abc\int_\lambda^\infty \frac{d\alpha}{(c^2+\alpha)\sqrt{(a^2+\alpha)(b^2+\alpha)(c^2+\alpha)}} \end{aligned} \tag{1.7}$$

In the integrals (1.7) value λ is positive solutions of the equation

$$\frac{X^2}{a^2+\lambda} + \frac{Y^2}{b^2+\lambda} + \frac{Z^2}{c^2+\lambda} = 1 \tag{1.8}$$

with values X, Y, Z satified equation (1.5)

The values A_0, B_0, C_0 are values of A, B, C at $\lambda = 0$.

Taking into account (1.6) - (1.8), from (1.1) it can be proved that in the coordinate system X, Y, Z the generalized induced mass tensors have following components

$$M_{xx} = \frac{4}{3}\pi abc\rho_2 \frac{A_0}{2 - A_0}\ ; \quad M_{xy} = M_{yx} = 0;$$
$$M_{yy} = \frac{4}{3}\pi abc\rho_2 \frac{B_0}{2 - B_0}\ ; \quad M_{yz} = M_{zy} = 0;$$
$$M_{zz} = \frac{4}{3}\pi abc\rho_2 \frac{C_0}{2 - C_0}\ ; \quad M_{xz} = M_{zx} = 0;$$
$$L_{xx} = L_{yy} = L_{zz} = L_{xy} = L_{yx} = L_{xz} = L_{zx} = L_{yz} = L_{zy} = 0; \tag{1.9}$$
$$N_{xx} = -\frac{4}{15}\pi abc\rho_2 \frac{(b^2 - c^2)^2 (B_0 - C_0)}{2(b^2 - c^2) + (B_0 - C_0)(b^2 + c^2)}\ ; \quad N_{xy} = N_{yx} = 0;$$
$$N_{yy} = -\frac{4}{15}\pi abc\rho_2 \frac{(a^2 - b^2)^2 (C_0 - A_0)}{2(c^2 - a^2) + (C_0 - A_0)(c^2 + a^2)}\ ; \quad N_{yz} = N_{zy} = 0;$$
$$N_{zz} = -\frac{4}{15}\pi abc\rho_2 \frac{(a^2 - b^2)^2 (A_0 - B_0)}{2(a^2 - b^2) + (A_0 - B_0)(a^2 + b^2)}\ ; \quad N_{zx} = N_{xz} = 0;$$

It is obvious that the values $M_{xx}, \ldots, N_{zz}$ in (1.9) will be determined only when the integrals (1.7) can be calculated. In the paper the explicit expressions of the integrals (1.7) are obtained in following cases

a. Case $a = b > c$

$$M_{xx} = M_{yy} = \rho_2 V_0 \frac{A_1}{-A_1 + 2A_3}\ ; \quad M_{zz} = \rho_2 V_0 \frac{-A_2}{A_2 + A_3}\ ;$$
$$N_{xx} = N_{yy} = -\frac{1}{5}\rho_2 V_0 \frac{(a^2 - c^2)(A_1 + 2A_2)}{2A_3 + \frac{a^2 + c^2}{a^2 - c^2}(A_1 + 2A_2)}\ ; \quad N_{zz} = 0. \tag{1.10}$$

Where we used the symbols

$$A_1 = \arcsin e - e\sqrt{1 - e^2}\ ; \quad A_2 = \arcsin e - \frac{e}{\sqrt{1 - e^2}}\ ;$$
$$A_3 = \frac{e^3}{\sqrt{1 - e^2}}\ ; \quad e = \frac{\sqrt{a^2 - c^2}}{a}\ ; \quad V_0 = \frac{4}{3}\pi a^2 c.$$

b. Case $a = b < c$

$$M_{xx} = M_{yy} = \rho_2 V_0 \frac{B_2}{B_3 - B_1}\ ; \quad M_{zz} = -\rho_2 V_0 \frac{B_1}{B_1 + B_3}\ ;$$
$$N_{xx} = N_{yy} = -\frac{1}{5}\rho_2 V_0 (c^2 - a^2) \frac{B_1 + \frac{1}{2}B_2}{\frac{a^2 + c^2}{c^2 - a^2}\left(B_1 + \frac{1}{2}B_2\right) - B_3}\ ; \quad N_{zz} = 0; \tag{1.11}$$

In (1.11) the symbols B_1, B_2, B_3 are

$$B_1 = \ln\left|\frac{1 - e}{1 + e}\right| + 2e; \quad B_2 = \ln\left|\frac{1 - e}{1 + e}\right| + \frac{2e}{1 - e^2}\ ; \quad B_3 = \frac{2e^3}{1 - e^2}\ ;$$

It is easy to determine the inertia tensor of rigid ellipsoidal particle in the coordinate system XYZ. Its components have following forms

$$I_{xx} = I_{yy} = \frac{4}{15}\pi a^2 c\rho_1 (a^2 + c^2);$$
$$I_{zz} = \frac{8}{15}\pi a^4 c\rho_1; \quad I_{xy} = I_{yx} = I_{zx} = I_{xz} = I_{yz} = I_{zy} = 0; \tag{1.12}$$

In conclusions it has to emphasize that the expressions (1.10) - (1.12) determine explicitly the components of generalized induced mass tensors and inertia moment tensor of ellipsoidal particle in the coordinate sytem connected with principal axes of particle.

2. BASIC EQUATIONS SYSTEM

Suppose that the considered fluid is incompressible ($\rho_2 = \text{const}$), particles are absolutely rigid ($\rho_1 = \text{const}$) and have a ellipsoidal form.

In this case it can be shown that the components of tensors $\overline{I}$, $\overline{M}$, $\overline{L}$, $\overline{N}$ in the fixed corrdinate system with satisfy the following change equations

$$\begin{aligned}
&\frac{d^{(a)}\overline{I}}{dt} + \Big(\frac{\overline{J}^a}{\rho_1^*} \cdot \overline{\nabla}\Big)\overline{I} = \overline{\omega} \times \overline{I} - \overline{I} \times \overline{\omega};\\
&\frac{d^{(a)}\overline{M}}{dt} + \Big(\frac{\overline{J}^a}{\rho_1^*} \cdot \overline{\nabla}\Big)\overline{M} = \overline{\omega} \times \overline{M} - \overline{M} \times \overline{\omega};\\
&\frac{d^{(a)}\overline{L}}{dt} + \Big(\frac{\overline{J}^a}{\rho_1^*} \cdot \overline{\nabla}\Big)\overline{L} = \overline{\omega} \times \overline{L} - \overline{L} \times \overline{\omega};\\
&\frac{d^{(a)}\overline{N}}{dt} + \Big(\frac{\overline{J}^a}{\rho_1^*} \cdot \overline{\nabla}\Big)\overline{N} = \overline{\omega} \times \overline{N} - \overline{N} \times \overline{\omega};
\end{aligned} \tag{2.1}$$

If at the initial moment of time the particle principal axes coincide with the fixed coordinate system, then the expressions (1.9) - (1.12) can be considered as the initial conditions for solwing the equations system (2.1). Here we can see that $\overline{L} \equiv 0$.

The general equations system describing the motion of fluid with rigid ellipsoidal particles has the following form.

The equation of mass conservation for the mixture

$$\begin{aligned}
&\frac{d^{(a)}\rho}{dt} + \rho(\overline{\nabla} \cdot \overline{u}_a) = -\overline{\nabla}\Big[\Big(1 - \frac{a_1\rho_2^*}{a_2\rho_1^*}\Big)\overline{J}^a\Big];\\
&\rho = \rho_1^* + \rho_2^*; \quad \rho_1^* = \varphi\rho_1; \quad \rho_2^* = (1-\varphi)\rho_2;
\end{aligned} \tag{2.2}$$

The particles concentration charge equation

$$\begin{aligned}
&\frac{d^{(a)}\varphi}{dt} + \varphi\overline{\nabla} \cdot \overline{u}_a = -\frac{1}{\rho_1}\overline{\nabla} \cdot \overline{J}^a\\
&\overline{u}_a = \sum_{k=1}^{2} a_k \overline{u}_k; \quad \sum_{k=1}^{2} a_k = 1; \quad \overline{J}^a = \rho_1^*(\overline{u}_1 - \overline{u}_a);\\
&\frac{d^{(a)}}{dt}(\ldots) = \frac{\partial}{\partial t}(\ldots) + (\overline{u}_a \cdot \overline{\nabla})(\ldots);
\end{aligned} \tag{2.3}$$

The balance equation for the momentum of mixture

$$\begin{aligned}
\rho\frac{d^{(a)}\overline{U}_a}{dt} &= \rho\overline{f} - \overline{\nabla}p + \overline{\nabla}\tau_0 - \overline{\nabla} \times \overline{\tau}_1^a + \overline{\nabla} \cdot \overline{\tau}_2^a -\\
&- \frac{D^{(a)}}{Dt}\Big(\frac{a_2\rho_1^* - a_1\rho_2^*}{a_2\rho_1^*}\overline{J}^a\Big) - \overline{\nabla} \cdot \Big[\frac{\overline{J}^a\overline{J}^a}{\rho_1^*}\Big(1 + \frac{a_1^2\rho_2^*}{a_1^2\rho_1^*}\Big)\Big];\\
\frac{D^{(a)}}{Dt}(\ldots) &= \frac{d^{(a)}}{dt}(\ldots) + \Big[(\ldots) \cdot \overline{\nabla}\Big]\overline{u}_a + (\ldots)(\overline{\nabla} \cdot \overline{u}_a),
\end{aligned} \tag{2.4}$$

The equation for determining the generalized diffusion flux of particles

$$
\begin{aligned}
&\frac{a_2^2\rho_1^* + a_1^2\rho_2^*}{a_2^2\rho_1^*}\,\frac{D^{(a)}\overline{J}^{a}}{Dt} + \frac{a_2^3\rho_1^* - a_1^3\rho_2^*}{a_2^3\rho_1^*}\overline{\nabla}\cdot\Big(\frac{\overline{J}^{a}\overline{J}^{a}}{\rho_1^*}\Big) + \frac{a_1}{a_2}\overline{J}^{a}\frac{d^{(a)}}{dt}\Big(\frac{a_1\rho_2^*}{a_2\rho_1^*}\Big) -\\
&-\frac{a_1}{a_2}\frac{\overline{J}^{a}}{\rho_1^*}(\overline{J}^{a}\cdot\overline{\nabla})\Big(\frac{a_1^2\rho_2^*}{a_2^2\rho_1^*}\Big) + n\Big(1+\frac{a_1}{a_2}\Big)\overline{M}\cdot\Big\{\frac{d^{(a)}}{dt}\Big[\Big(1+\frac{a_1}{a_2}\Big)\frac{\overline{J}^{a}}{\rho_1^*}\Big]+\\
&+\Big(\frac{\overline{J}^{a}}{\rho_1^*}\cdot\overline{\nabla}\Big)\Big[\Big(1+\frac{a_1}{a_2}\Big)\frac{\overline{J}^{a}}{\rho_1^*}\Big]\Big\} + \frac{M}{2}\Big[\frac{d^{(a)}\overline{M}}{dt} + \Big(\frac{\overline{J}^{a}}{\rho_1^*}\cdot\overline{\nabla}\Big)\overline{M}\Big]\cdot\overline{J}^{a} =\\
&=\rho_1^*\Big(\overline{f}_1 - \frac{a_1\rho_2^*}{a_2\rho_1^*}\overline{f}_2\Big) - \rho_1^*\Big(1-\frac{a_1\rho_2^*}{a_2\rho_1^*}\Big)\frac{d^{(a)}\overline{u}_a}{dt} - \varphi\Big[1-\frac{a_1(1-\varphi)}{a_2\varphi}\Big]\overline{\nabla}p-\\
&-\rho_1^*\Big(1+\frac{a_1}{a_2}\Big)\big(\overline{\nabla}\mu\big)_{p,T} + \overline{\nabla}R_0^a - \overline{\nabla}\times\overline{R}_1^a + \overline{\nabla}\cdot\overline{R}_2^a;
\end{aligned}
\qquad (2.5)
$$

The equation for determining the particles rotation

$$
\rho_1^*\overline{\sigma} = \rho_1^*\overline{\ell} + \overline{\tau}_1^a + \overline{\nabla}\lambda_0 - \overline{\nabla}\times\overline{\lambda}_1 + \overline{\nabla}\cdot\overline{\lambda}_2; \qquad (2.6)
$$

where

$$
\begin{aligned}
\rho_1^*\overline{\sigma} = n(\overline{N}+\overline{I})\cdot\Big[\frac{d^{(a)}\overline{\omega}}{dt} + \Big(\frac{\overline{J}_1^{a}}{\rho_1^*}\cdot\overline{\nabla}\Big)\overline{\omega}\Big]+\\
+\frac{n}{2}\Big[\frac{d^{(a)}}{dt}(\overline{N}+\overline{I}) + \Big(\frac{\overline{J}_1^{a}}{\rho_1^*}\cdot\overline{\nabla}\Big)(\overline{N}+\overline{I})\Big]\cdot\overline{\omega}
\end{aligned}
\qquad (2.7)
$$

In the equations system (2.1) - (2.7) we use following symbols: ρ - the mean mass density of the two-phase medium $\overline{u}_a$ - characteristic mean velocity, $\overline{J}^{a}$ - generalized duffusion flux of particles; $\overline{f}$ - density of the external body forces acting on an unit mass of mixture ; p - thermodynamical pressure; $\overline{\tau}_1^a$ and $\overline{\tau}_2^a$ - antisymmetric and symmetric parts of stress tensor; R_0^a, $\overline{R}_1^{a}$ and $\overline{R}_2^{a}$ trace, antisymmetric and symmetric parts of diffusion stress; $\overline{\ell}$ - density of the external body moments, λ_0, $\overline{\lambda}_1$ and $\overline{\lambda}_2$ - trace, antisymmetric and symmetric parts of moment stress tensor, n - number of particles in an unit volume of the flow.

The obtained equation system (2.1) - (2.7) will be sufficient to determine all unknowns if the constitutive equations will be constructed. For this purpose one can use the results [1].

CONCLUSION

It has been constructed a full equations system, sufficient to determine all parameters of the flow of incompressible fluid with rigid ellipsoidal particles.

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LÝ THUYẾT DÒNG CHẢY HAI PHA CHẤT LỎNG MANG CÁC HẠT CỨNG DẠNG ELLIPSOID

Trong công trình này, chuyển động của chất lỏng mang các hạt cứng dạng ellipsoid đã được khảo sát. Đã xác định được các ten xơ khối lượng nước kèm mở rộng về các phương trình biến đổi của chúng. Hệ phương trình thu nhận được đủ để xác định các tham số cần thiết của chuyển động của chất lỏng mang các hạt cứng dạng ellipsoit.

VỀ HỘI NGHỊ KỸ THUẬT BIỂN VÀ ĐỊA CỰC QUỐC TẾ LẦN THỨ III

(On the Third International Offshore and Polar Engineering Conference)

Hội nghị quốc tế về kỹ thuật biển và địa cực lần thứ III (ICOPE-93) đã được tổ chức tại Singapor từ ngày 06 đến 11 tháng 6 năm 1993. So với các lần trước (Lần thứ I: Edinburgh - 1991 và lần thứ II: San Fransisco - 1992) Hội nghị lần này đã tập hợp được rộng rãi hơn đội ngũ cán bộ khoa học và công nghệ của thế giới (35 nước), đặc biệt là ở khu vực châu Á - Thái bình dương.

Gần 450 báo cáo khoa học và công nghệ được trình bày và trao đổi tại Hội nghị. Các báo cáo khoa học và công nghệ được công bố trong bộ tuyển tập gần 3000 trang bao gồm 04 tập với các nội dung như sau:

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Hội nghị lần thứ IV sẽ được tổ chức tại Osaka (Nhật) từ ngày 10 đến 15 tháng 4 năm 1994.

Có thể tham khảo tài liệu của các Hội nghị I - III và thể thức tham dự Hội nghị tại Viện Cơ học, 224 Đội Cấn, Hà Nội.