Tạp chí Cơ học Journal of Mechanics, NCNST of Vietnam T. XVI, 1994, No 3 (47 - 49)

A NOTE ON NONLINEAR SYSTEMS WITH DIFFERENT DEGREES OF EXCITATIONS

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In this note an approximate solution of a differential equation of special form which is close to the Liapunoff type is given. This solution is proportional to the small parameter ε.

1. DIFFERENTIAL EQUATION AND ITS APPROXIMATE SOLUTION

Let us consider a nonlinear system described by the differential equation

$$\ddot{x} + \omega^2 x = \gamma x^2 - \beta x^3 - \varepsilon^2 h\dot{x} + \varepsilon^3 f(\varphi, x, \dot{x}), \quad \varphi = \nu t \tag{1.1}$$

where ω, γ, β, h, ν are constants, ε is a small positive parameter, $f(\varphi, x, \dot{x})$ is a periodic function relatively to t and analytic to x, $\dot{x}$. We are intersted in finding the periodic solution of the equation (1.1). It is noted that when $\varepsilon = 0$ the equation (1.1) is degenerated into

$$\ddot{x} + \omega^2 x = \gamma x^2 - \beta x^3 \tag{1.2}$$

which has a trivial stable solution $x = 0$. Hence, the solution of the equation (1.1) is found in the form:

$$x = \varepsilon a \cos\left(\frac{p}{q}\varphi + \psi\right) + \varepsilon^2 u_1(a, \varphi, \theta) + \varepsilon^3 u_2(a, \varphi, \theta) + \ldots, \tag{1.3}$$

where $\theta = \frac{p}{q}\varphi + \psi$, $\frac{p}{q}\nu \simeq \omega$, p and q are integers, $u_i(a, \varphi, \theta)$ are periodic functions of φ, θ and do not contain the first harmonics $\sin\theta$, $\cos\theta$. The amplitude a and phase ψ are determined from the equations of form

$$\begin{aligned} \frac{da}{dt} &= \varepsilon A_1(a, \psi) + \varepsilon^2 A_2(a, \psi) + \ldots, \\ \frac{d\psi}{dt} &= \omega - \frac{p}{q}\nu + \varepsilon B_1(a, \psi) + \varepsilon^2 B_2(a, \psi) + \ldots \end{aligned} \tag{1.4}$$

To find the unknown functions u_i, A_i, B_i we calculate the derivatives $\dot{x}$, $\ddot{x}$ from (1.3) and substitute them and (1.3), (1.4) into (1.1). By comparing the coefficients of ε^2 we obtain

$$\begin{aligned} &\left(\nu\frac{\partial}{\partial\varphi} + \omega\frac{\partial}{\partial\theta}\right)^2 u_1 + \omega^2 u_1 = \gamma a^2 \cos^2\theta - \\ &- \left[\left(\omega - \frac{p}{q}\nu\right)\frac{\partial A_1}{\partial\psi} - 2a\omega B_1\right]\cos\theta + \left[\left(\omega - \frac{p}{q}\nu\right) a\frac{\partial B_1}{\partial\psi} + 2\omega A_1\right]\sin\theta. \end{aligned} \tag{1.5}$$

Comparing the coefficients of the harmonics in (1.5) we have

$$A_1 = 0, \quad B_1 = 0, \quad , u_1 = \frac{\gamma a^2}{2\omega^2}\left(1 - \frac{1}{3}\cos 2\theta\right). \tag{1.6}$$

If one compares the coefficients of ε^3 in (1.1) one has

$$\left(\nu\frac{\partial}{\partial\varphi} + \omega\frac{\partial}{\partial\theta}\right)^2 u_2 + \omega^2 u_2 = 2a\gamma u_1 \cos\theta + f(\varphi, a\cos\theta, -a\omega\sin\theta) -$$
$$- \left[\left(\omega - \frac{p}{q}\nu\right)\frac{\partial A_2}{\partial\psi} - 2a\omega B_2\right]\cos\theta + \left[\left(\omega - \frac{p}{q}\nu\right)a\frac{\partial B_2}{\partial\psi} + 2\omega A_2\right]\sin\theta. \tag{1.7}$$

The form of functions A_2, B_2 and u_2 depends on the concrete form of the function $f(\varphi, x, \dot{x})$.

2. FORCED OSCILLATION

Let us consider the case when the function $f(\varphi, x, \dot{x})$ is of the form

$$f(\varphi, x, \dot{x}) = E\sin\nu t \tag{2.1}$$

and consider the resonant case $p = q = 1$:

$$\nu \simeq \omega. \tag{2.2}$$

Comparing the coefficients of harmonics in (1.7) we have

$$\begin{aligned}
&(\omega - \nu)\frac{\partial A_2}{\partial\psi} - 2a\omega B_2 = \frac{5}{6}\frac{\gamma^2 a^3}{\omega^2} - E\sin\psi, \\
&(\omega - \nu)a\frac{\partial B_2}{\partial\psi} + 2\omega A_2 = -E\cos\psi, \\
&\left(\nu\frac{\partial}{\partial\varphi} + \omega\frac{\partial}{\partial\theta}\right)^2 u_2 + \omega^2 u_2 = -\frac{1}{6}\frac{\gamma^2 a^3}{\omega^2}\cos 3\theta.
\end{aligned} \tag{2.3}$$

From the equations (2.3) one obtaines

$$\begin{aligned}
A_2 &= \frac{-E\cos\psi}{\nu + \omega}, \\
B_2 &= -\frac{5\gamma^2 a^2}{12\omega^3} + \frac{E\sin\psi}{(\nu + \omega)a}, \\
u_2 &= \frac{\gamma^2 a^3}{48\omega^4}\cos 3\theta.
\end{aligned} \tag{2.4}$$

Hence, the solution of the equation (1.1) in the second approximation is

$$x = \varepsilon a\cos(\nu t + \psi) + \varepsilon^2\frac{\gamma a^2}{2\omega^2}\left(1 - \frac{1}{3}\cos 2\theta\right) + \varepsilon^3\frac{\gamma^2 a^3}{48\omega^4}\cos 3\theta, \tag{2.5}$$

where a and ψ are determined from the system:

$$\begin{aligned}
\frac{da}{dt} &= -\varepsilon^2\frac{E\cos\psi}{\nu + \omega}, \\
\frac{d\psi}{dt} &= \omega - \nu - \frac{5\varepsilon^2}{12}\frac{\gamma^2 a^2}{\omega^3} + \frac{\varepsilon^2 E\sin\psi}{a(\nu + \omega)}.
\end{aligned} \tag{2.6}$$

These equations can be easily solved.

3. PARAMETRIC OSCILLATION

We suppose that $f(\varphi, x, \dot{x})$ has the form

$$f(\varphi, x, \dot{x}) = ex\cos\nu t \tag{3.1}$$

and consider the resonant case $p = 1$, $q = 2$:

$$\nu \simeq 2\omega. \tag{3.2}$$

The equations for A_2, B_2 and u_2 are

$$\begin{aligned}
&\left(\omega - \frac{\nu}{2}\right)\frac{\partial A_2}{\partial\psi} - 2a\omega B_2 = \frac{5}{6\omega^2}\gamma^2 a^3 + \frac{1}{2}ea\cos 2\psi,\\
&\left(\omega - \frac{\nu}{2}\right)a\frac{\partial B_2}{\partial\psi} + 2\omega A_2 = -\frac{1}{2}ea\sin 2\psi,\\
&\left(\nu\frac{\partial}{\partial\varphi} + \omega\frac{\partial}{\partial\theta}\right)^2 u_2 + \omega^2 u_2 = -\frac{1}{6\omega^2}\gamma^2 a^3\cos 3\theta + \frac{1}{2}ea\cos(\theta + \varphi).
\end{aligned} \tag{3.3}$$

Solving these equations gives:

$$\begin{aligned}
&A_2 = -\frac{ea}{2\nu}\sin 2\psi, \quad B_2 = -\frac{5}{12\omega^3}\gamma^2 a^2 - \frac{e}{2\nu}\cos 2\psi,\\
&u_2 = \frac{-ea}{2\nu(\nu + 2\omega)}\cos(\theta + \varphi) + \frac{\gamma^2 a^3}{48\omega^4}\cos 3\theta.
\end{aligned} \tag{3.4}$$

The solution of the equation (1.1) in the second approximation is then

$$x = \varepsilon a\cos\left(\frac{1}{2}\nu t + \psi\right) + \varepsilon^2\frac{\gamma a^2}{2\omega^2}\left(1 - \frac{1}{3}\cos 2\theta\right) - \frac{\varepsilon^3 ea}{2\nu(\nu + 2\omega)}\cos(\theta + \varphi) + \frac{\varepsilon^3\gamma^2 a^3}{48\omega^4}\cos 3\theta, \tag{3.5}$$

where a and ψ satisfy the equations:

$$\begin{aligned}
&\frac{da}{dt} = -\frac{\varepsilon^2 ea}{2\nu}\sin 2\psi,\\
&\frac{d\psi}{dt} = \omega - \frac{1}{2}\nu - \frac{5\varepsilon^2}{12\omega^3}\gamma^2 a^2 - \frac{\varepsilon^2 e}{2\nu}\cos 2\psi.
\end{aligned} \tag{3.6}$$

MỘT NHẬN XÉT VỀ HỆ PHI TUYẾN VỚI CÁC BẬC KÍCH ĐỘNG KHÁC NHAU

Trong bài đã đưa ra một lời giải xấp xỉ có dạng đặc biệt cho phương trình vi phân gần với Liapunôp. Hai trường hợp của dao động cưỡng bức và dao động thông số đã được khảo sát.