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DYNAMICAL SIMULATION OF A VIBRATING CRUSH MACHINE Part I

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§1. INTRODUTION

In this paper dynamics of a vibrating crush machine is considered. The main subjected of the present work is to build a mechanical model of a vibrating equipement for crushing industrial materials.

§2. MECHANICAL MODEL

Let us consider a vibrating crush machine composed of following parts: A platform is moved in a fixed horizontal plane. The platform can be regarded as a rigid body of mass m_0 and is driven by a rotating eccentric vibrator. The debalanc of mass m rotates uniformly at angular velocity ω. A pestle of mass m_2 moves on the vibrating platform and is put the inside of a motar of mass m_1. The motar moves the inside of a cylinder rigidly connected the vibrating platform (fig. 1).

The described system may be considered to represent a system of nine degrees of freedom. The generalized coordinates can be chosen as follows:

$$q_1 = x, \quad q_2 = y, \quad q_3 = \theta, \quad q_4 = s_1, \quad q_5 = \varphi_1, \quad q_6 = \theta_1,$$
$$q_7 = s_2, \quad q_8 = \varphi_2, \quad q_9 = \theta_2,$$

where

x, y - are the coordinates of the mass centre of the vibrating platform with respect to a fixed system of axes C_0xy, where C_0 is the position of the mass centre of the vibrating platform at initial time.

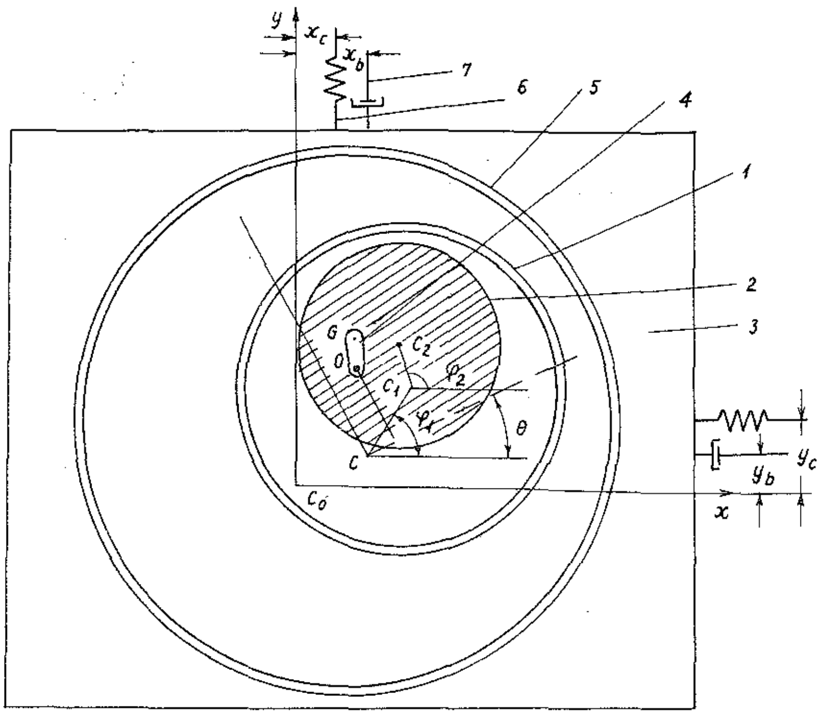
θ - the angular coordinate of the vibrating platform with respect to an inertia reference system.

s_1, s_2 - the distances from the mass centres of the vibrating respectively $(s_1 = CC_1, s_2 = CC_2)$

φ_1 - the angle between the straight line jointed two mass centres of the vibrating platform and the motar and the fixed axis C_0x.

φ_2 - the angle between the straight line jointed two mass centres of the pestle and of the motar and the fixed axis C_0x.

θ_1, θ_2 - the angular coordinates of the pestle and of the motar with respect to an inertia reference system respectively.



1. The motor
2. The pestle
3. The vibrating platform
4. The delalanc
5. The cylinder
6. The springs
7. The dampers

Fig. 1

§3. DERIVATION OF EQUATIONS OF MOTION

To write the equations of motion of the considered system we can apply the Lagrange's equations of second kind [3]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i + R_i. \tag{3.1}$$

where:

Q_i - are the generalixed forces of active forces.

R_i - the generalixed forces of reaction forces of constraints due to the connections in moving between parts of the system.

In view of the plane motion of every part of the system the expression of their kinetic energy is calculated by the formulae:

$$T = \frac{1}{2}M(\dot{x}_c^2 + \dot{y}_c^2) + \frac{1}{2}J_c\Omega^2. \tag{3.2}$$

where

M, J_c - are the mass and the moment of inertia about the mass centre of parts of the system respectively.

x_c, y_c - the coordinates of their mass centres with respect to an inertia reference system.

Ω - the angular velocity of every part about the mass centre with respect to an inertia reference system.

Then it is necessary to write the expression (3.2) for every part of the system, i.e. for the vibrator, the motar, the pestle and the vibrating platform.

For this aim we will express the coordinates of the mass centre of every part of the system in function of chosen generalized coordinates.

In the first let us write the expressions of the coordinates of the mass centre of the debalance in function of generalized coordinates x, y, θ, s_1, φ_1, θ_1, s_2, φ_2, θ_2. We have:

$$x_d = x + x_0 \cos\theta - y_0 \sin\theta + e \cos\omega t,$$
$$y_d = y + x_0 \sin\theta - y_0 \cos\theta + e \sin\omega t,$$

where: x_0, y_0 - are the coordinates of axis of rotation of the debalanc. Hence:

$$\dot{x}_d = \dot{x} - x_0 \sin\theta\ \dot{\theta} - y_0 \cos\theta\ \dot{\theta} + e\omega \cos\omega t,$$
$$\dot{y}_d = \dot{y} - x_0 \cos\theta\ \dot{\theta} - y_0 \sin\theta\ \dot{\theta} + e\omega \sin\omega t.$$

Let us write expression (3.2) for the debalanc. Taking $\sin\theta \approx 0$; $\cos\theta \approx 1$ and neglecting infinitesimals of higher orders than second one, the kinetic energy of the debalanc will have the form:

$$T_0 = \frac{1}{2}m_0(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m_0 e^2\dot{\theta}^2 + \frac{1}{2}J_0\omega^2 + m_0 x_0 \dot{y}\dot{\theta} - m y_0 \dot{x}\dot{\theta} -$$
$$- m_0 e\omega \sin\omega t\ \dot{x} + m_0 e\omega \cos\omega t\ \dot{y} + m_0 e\omega(y_0 \sin\omega t + x_0 \cos\omega t)\dot{\theta}.$$

where e is the eccentricity of the debalanc ($e = OG$), J_0 - the moment of inertia of the debalanc about its mass centre.

Denoting by x_1, y_1 the coordinates of the mass centre of the motar, we have:

$$x_1 = x + s_1 \cos\varphi_1, \quad y_1 = y + s_1 \sin\varphi_1.$$

Therefore:

$$\dot{x}_1 = \dot{x} + \dot{s}_1 \cos\varphi_1 - s_1 \sin\varphi_1\ \dot{\varphi}_1;$$
$$\dot{y}_1 = \dot{y} + \dot{s}_1 \sin\varphi_1 + s_1 \cos\varphi_1\ \dot{\varphi}_1;$$

Writing the expression (3.2) for the motar, we obtain:

$$T_1 = \frac{1}{2}m_1(\dot{x}^2 + \dot{y}^2 + \dot{s}_1^2 + s_1^2\dot{\varphi}_1^2 + 2\cos\varphi_1\ \dot{x}\dot{s}_1 - 2s_1 \sin\varphi_1\ \dot{x}\dot{\varphi}_1 +$$
$$+ 2s_1 \sin^2\varphi_1\ \dot{y}\dot{s}_1 + 2s_1 \cos\varphi_1\ \dot{y}\dot{\varphi}_1) + \frac{1}{2}J_1\dot{\theta}_1^2,$$

where J_1 is the moment of inertia of the motar about its mass centre.

The coordinates of the mass centre of the pestle can be expressed in function of generalized coordinate as follows:

$$x_2 = x + s_1 \cos\varphi_1 + s_2 \cos\varphi_2$$
$$y_2 = y + s_1 \sin\varphi_1 + s_2 \sin\varphi_2$$

Differentiating with respect to time gives:

$$\dot{x}_2 = \dot{x} + \dot{s}_1 \cos\varphi_1 - s_1 \sin\varphi_1\ \dot{\varphi}_1 + \dot{s}_2 \cos\varphi_2 - s_2 \sin\varphi_2\ \dot{\varphi}_2$$
$$\dot{y}_2 = \dot{y} \sin\varphi_1 + s_1 \cos\varphi_1\ \dot{\varphi}_1 + \dot{s}_2 \sin\varphi_2 + s_2 \cos\varphi_2 \dot{\varphi}_2$$

The kinetic energy of the peste will be now:

$$T_2 = \frac{1}{2}m_2\big[\dot{x}^2 + \dot{y}^2 + \dot{s}_1^2 + s_1^2\dot{\varphi}_1^2 + \dot{s}_2^2 + s_2^2\dot{\varphi}_2^2 +$$
$$+ 2\cos\varphi_1\dot{x}\dot{s}_1 - 2s_1\sin\varphi_1\ \dot{x}\dot{\varphi}_1 + 2\cos\varphi_2\dot{x}\dot{s}_2 - 2s_2\sin\varphi_2\dot{x}\dot{\varphi}_2 +$$
$$+ 2\sin\varphi_1\dot{y}\dot{s}_1 - 2s_1\cos\varphi_1\dot{y}\dot{\varphi}_1 + 2\sin\varphi_2\dot{y}\dot{s}_2 - 2s_2\sin\varphi_2\dot{y}\dot{\varphi}_2 +$$
$$+ 2s_1\sin(\varphi_2 - \varphi_1)\dot{\varphi}_1\dot{s}_2 + 2s_1s_2\cos(\varphi_2 - \varphi_1)\dot{\varphi}_1\dot{\varphi}_2\big] + \frac{1}{2}J_2\dot{\theta}_2^2.$$

where J_2 is the moment of inertia of the motar about its mass centre.

In the next let us concern to the kinetic energy of the vibrating platform, which has the form:

$$T_3 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J\dot{\theta}^2,$$

where J is the moment of inertia of the vibrating platform about its mass centre.

At last, the kinetic of energy of the considered system will be:

$$T = T_0 + T_1 + T_2 + T_3.$$

which is expressed in function of generalized coordinates and generalized velocities as follows:

$$T = \frac{1}{2}(m_0 + m + m_1 + m_2)\dot{x}^2 + \frac{1}{2}(m_0 + m + m_1 + m_2)\dot{y}^2 + \frac{1}{2}(J + m_0e^2)\dot{\theta}^2 +$$
$$+ \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 + \frac{1}{2}J_0\omega^2 - m_0y_0\dot{x}\dot{\theta} + m_0x_0\dot{y}\dot{\theta} - m_0e\omega\sin\omega t\ \dot{x} +$$
$$+ m_0e\omega\cos\omega t\dot{y} + m_0e\omega(y_0\sin\omega t + x_0\cos\omega t)\dot{\theta} + \frac{1}{2}(m_1 + m_2)\dot{s}_1^2 +$$
$$+ \frac{1}{2}(m_1 + m_2)s_1^2\dot{\varphi}_1^2 + \frac{1}{2}m_2s_2^2\dot{\varphi}_2^2 + \frac{1}{2}m_2\dot{s}_2^2 + (m_1 + m_2)\sin\varphi_1\ \dot{y}\dot{s}_1 -$$
$$- (m_1 + m_2)\sin\varphi_1s_1\dot{x}\dot{\varphi}_1 + (m_1 + m_2)\sin\varphi_1\dot{y}\dot{s}_2 + m_2\cos\varphi_2\dot{x}\dot{s}_2 -$$
$$- m_2\sin\varphi_2s_1\dot{x}\dot{\varphi}_1 + m_2\sin\varphi_2\dot{y}\dot{s}_2 + m_2\cos\varphi_2s_2\dot{y}\dot{\varphi}_2 + m_2\cos(\varphi_2 - \varphi_1)\dot{s}_1\dot{s}_2 -$$
$$- m_2s_2\sin(\varphi_2 - \varphi_1)\dot{s}_1\dot{\varphi}_2 + m_2\sin(\varphi_2 - \varphi_1)s_1\dot{\varphi}_1\dot{s}_2 + (m_1 + m_2)s_1\cos\varphi_1\ \dot{\varphi}_1\dot{y}_1 +$$
$$m_2\cos(\varphi_2 - \varphi_1)s_1s_2\dot{\varphi}_1\dot{\varphi}_2.$$

Let us now calculate the generalized forces of active forces. For this aim we will write the expression of the potential energy and the dissipative function.

The potential energy of the considered system is of the form:

$$U = \frac{1}{2}c_x(x - y_c\theta)^2 + \frac{1}{2}c_y(y - x_c\theta)^2$$

where c_x, c_y are the spring coefficients; x_c, y_c - the coordinates of the joint of the spring to the vibrating platform.

The dissipative function will be:

$$\phi = \frac{1}{2}b_x(\dot{x} - y_b\dot{\theta})^2 + \frac{1}{2}b_y(\dot{y} - x_b\dot{\theta})^2$$

where b_x, b_y - are the damping coefficients;

x_b, y_b - the coordinates of the joint of the damper to the vibrating platform.

It is easy to calculate the generalized forces of active forces. They are:

$$\begin{aligned}
Q_x &= -c_x(x - y_c\theta) - b_x(\dot{x} - y_b\dot{\theta}),\\
Q_y &= -c_y(y + x_c\theta) - b_y(\dot{y} + x_b\dot{\theta}),\\
Q_\theta &= -c_x(x - y_c\theta)y_c - b_x(\dot{x} - y_b\dot{\theta})y_b -\\
&\quad - c_y(y + x_c\theta)x_c - b_y(\dot{y} - x_b\dot{\theta})x_b,\\
Q_{\theta 1} &= Q_{\theta 2} = Q_{S_1} = Q_{S_2} = Q_{\varphi_1} = Q_{\varphi_2} = 0.
\end{aligned}$$

Equations of motion of the considered system in the form (3.1) will be written as follows:

$$\begin{aligned}
&M\ddot{x} - m_0 y_0\ddot{\theta} + (m_1 + m_2)\cos\varphi_1\ddot{s}_1 - (m_1 + m_2)s_1\sin\varphi_1\ddot{\varphi}_1 + m_2\cos\varphi_2\ddot{s}_2 +\\
&+ m_2\sin\varphi_2 s_2\ddot{\varphi}_2 - 2(m_1 + m_2)\sin\varphi_1\dot{s}_1\dot{\varphi}_1 - (m_1 + m_2)s_1\cos\varphi_1\dot{\varphi}_1^2 - 2m_2\sin\varphi_2\dot{s}_2\dot{\varphi}_2 -\\
&- m_2 s_2\cos\varphi_2\dot{\varphi}_2^2 + c_x(x - y_c\theta) + b_x(\dot{x} - y_b\dot{\theta}) = m_o e\omega^2\cos\omega t + Rx.
\end{aligned}$$

$$\begin{aligned}
&M\ddot{y} + m_0 x_0\ddot{\theta} + (m_1 + m_2)\sin\varphi_1\ddot{s}_1 + (m_1 + m_2)s_1\cos\varphi_1\ddot{\varphi}_1 + m_2\sin\varphi_2\ddot{s}_2 +\\
&+ m_2 s_2\cos\varphi_2\ddot{\varphi}_2 + 2(m_1 + m_2)\cos\varphi_1\dot{s}_1\dot{\varphi}_1 - (m_1 + m_2)s_1\sin\varphi_1\dot{\varphi}_1^2 + 2m_2\cos\varphi_2\dot{s}_2\dot{\varphi}_2 -\\
&- m_2 s_2\sin\varphi_2\dot{\varphi}_2^2 + c_y(y - x_c\theta) + b_y(\dot{y} - x_b\dot{\theta}) = m_0 e\omega^2\sin\omega t + R_y,
\end{aligned}$$

$$-m_0 y_0\ddot{x} + m_0 x_0\ddot{y} + (J_0 + me^2)\ddot{\theta} = m_0 e\omega^2(x_0\sin\omega t - y_0\cos\omega t) + R_\theta,$$

$$J_1\ddot{\theta}_1 = R_{\theta 1};\quad J_2\ddot{\theta}_2 = R_{\theta 2};$$

$$\begin{aligned}
&(m_1 + m_2)\cos\varphi_1\ddot{x} + (m_1 + m_2)\sin\varphi_1\ddot{y} + (m_1 + m_2)\ddot{s}_1 + m_2\cos(\varphi_2 - \varphi_1)\ddot{s}_2 -\\
&- (m_1 + m_2)s_1\dot{\varphi}_1^2 - m_2 s_2\sin(\varphi_2 - \varphi_1)\ddot{\varphi}_2 + (m_1 + m_2)\cos\varphi_1\dot{y}\dot{\varphi}_1 -\\
&- m_2 s_2\cos(\varphi_2 - \varphi_1)\dot{\varphi}_2^2 - 2m_2\sin(\varphi_2 - \varphi_1)\dot{\varphi}_2\dot{s}_2 = R_{s1}.
\end{aligned}$$

$$\begin{aligned}
&m_2\cos\varphi_2\ddot{x} + m_2\sin\varphi_2\ddot{y} + m_2\cos(\varphi_2 - \varphi_1)\ddot{s}_1 + m_2\sin(\varphi_2 - \varphi_1)s_1\ddot{\varphi}_1 +\\
&+ 2m_2\sin(\varphi_2 - \varphi_1)\dot{s}_1\dot{\varphi}_1 - m_2\cos(\varphi_2 - \varphi_1)s_1\dot{\varphi}_1^2 - m_2 s_2\dot{\varphi}_2^2 = R_{S2},
\end{aligned}$$

$$\begin{aligned}
&- (m_1 + m_2)s_1\sin\varphi_1\ddot{x} + (m_1 + m_2)s_1\cos\varphi_1\ddot{y} + (m_1 + m_2)s_1^2\ddot{\varphi}_1 +\\
&+ m_2\sin(\varphi_2 - \varphi_1)s_1\ddot{s}_2 + 2(m_1 + m_2)s_1\dot{s}_1\dot{\varphi}_1 + m_2 s_1 s_2\cos(\varphi_2 - \varphi_1)\ddot{\varphi}_2 +\\
&+ 2m_2\cos(\varphi_2 - \varphi_1)s_1\dot{s}_2\dot{\varphi}_2 - m_2 s_2\sin(\varphi_2 - \varphi_1)s_1\dot{\varphi}_2^2 = R_{\varphi 1},
\end{aligned}$$

$$\begin{aligned}
&- m_2\sin\varphi_2\ddot{x} + m_2 s_2\cos\varphi_2\ddot{y} - m_2 s_2\sin(\varphi_2 - \varphi_1)\ddot{s}_1 + m_2\cos(\varphi_2 - \varphi_1)s_1\dot{s}_2\ddot{\varphi}_1 +\\
&+ m_2 s_2^2\ddot{\varphi}_2 + 2m_2 s_2\cos(\varphi_2 - \varphi_1)\dot{s}_1\dot{\varphi}_1 - m_2\sin(\varphi_2 - \varphi_1)s_1 s_2\dot{\varphi}_1^2 + 2m_2 s_2\dot{s}_2\dot{\varphi}_2 = R_{\varphi 2},
\end{aligned}$$

where: $M = m_0 + m_1 + m_2 + m$.

The written equations will describe the motion of the considered system if the generalized forces of reaction forces are determined.

However, the last quantities will be calculated when the shape of constraints between the parts of the system is described. In the other words, it is necessary to simulate the connections between parts of the system in moving process, example, if the parts of the system doesn't make the contact of each other all constraint reactions are equal to zero. Due to the aim of crush technology the parts of the system must make the contact of each other.

CONCLUSIONS

The contact of the parts in crush process can be to realize in different ways. For describing the contact conditions we can notice two of following situations:

1. The crush process.- In such a case the parts of the system (the pestle, the motar and the crush materials) roll nosliding one another (the case of bilateral constraints).

2. The collision process. In this process the parts of the system will collide each other.

In accordance with the occurred processes, the constraints will be described in different ways, in which we will solve a problem of usual mechanical motion or the one of impact (the case of unilateral constraints).

The simulation of above mentioned processess will be discussed in the next paper.

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MÔ PHỎNG ĐỘNG LỰC CỦA MÁY NGHIỀN RUNG
(Phần 1)

Trong bài báo này tác giả khảo sát động lực của máy nghiền rung phẳng, xây dựng mô hình cơ học, phương trình chuyển động và các nhận xét để mô phỏng động lực của các quá trình nghiền rung: quá trình chuyển động cơ học thông thường (khi đó các liên kết là liên kết hai phía) và quá trình va đập (khi đó các liên kết là liên kết một phía).