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ON THE PROBLEM OF HEAT AND MASS TRANSFER IN THERMAL NON-ISOLATED RESERVOIR

DUONG NGOC HAI
Institute of Mechanics, NCNST of Vietnam

§1. INTRODUCTION

The thermal method is one of the major methods used to enhance oil recovery. In accordance to O.G.J. 69% of the enhanced oil recovery (EOR) production in the United States is due to the thermal methods and, today, EOR accounts for more than 9% of the total oil production of North America [1, 2]. In the paper the method using volume thermal source to act upon the reservoir is investigated. The presented model takes into account also the possible thermal exchange of reservoir with surrounding medium.

§2. GOVERNING EQUATIONS

Thermo - and hydrodynamics of the process of saturated porous medium heating is assessed with regard to possible phase transfer of the first mode (melting or solidification of the saturating component). Then subscripts $i = 1, 2, 3$ mark parameters of liquid (melted) phase, solid (unmelted) phase and solid porous matrix, accordingly. Subscripts f and 0 characterize media at the phase transition front and on the well boundary; α_i is volumetric fraction of the i-th phase; T is temperature, m is porosity; x is space coordinate; $x_0 = |\vec{x}_0|$ is well radius; $\vec{x}_f(t)$ is the coordinate of the mobile melting front; t is time.

According to the mentioned designations, melting front $\vec{x}_f(t)$ will be a boundary between the zone (which will be characterized by subscript ℓ) of porous solid body - matrix (third phase) filled with the melted second component (first phase): $\alpha_1 = m$; $\alpha_2 = 0$; $\alpha_3 = 1 - m$; $T > T_f$ and the zone (which will be characterized by subscript s) of porous solid body filled with the solid second component (second phase): $\alpha_1 = 0$; $\alpha_2 = m$; $\alpha_3 = 1 - m$. Note that $x_f \to +\infty$ formally corresponds to the case when initially the saturating component is in liquid ($T_\infty > T_f$) state with high viscosity, and melting surface is totally absent.

With these assumptions outside the surface of a strong break (of phase transition front $\vec{x}_f(t)$), equations of continuity, phase filtration and also equations of heat inflow (heat conduction) of the mixture in Euler coordinate system can be introduced in the following way:

$$
\begin{aligned}
&\frac{\partial \rho_i}{\partial t} + \vec{\nabla}\rho_i\vec{v}_i = 0, \quad (i = 1, 2, 3),\\
&\vec{u}_1 = \alpha_1\vec{v}_1 = -\frac{k}{\mu_1}\vec{\nabla}p, \quad \vec{v}_2 = \vec{v}_3 = 0,\\
&\rho c\frac{\partial T}{\partial t} + \alpha_1\rho_1 c_1(\vec{v}_1\vec{\nabla})T = \vec{\nabla}(\lambda\vec{\nabla}T) + Q + q,\\
&\alpha_1 + \alpha_2 + \alpha_3 = 1, \quad \alpha_1 + \alpha_2 = m, \quad \alpha_1\alpha_2 = 0,
\end{aligned}
\tag{2.1}
$$

where the main notations are the same as above, Q is intensity of a volumetric heat source and q describes heat losses to the top and bottom of the bed.

To close equation set (2.1), the relation of viscosity vs temperature (power law) and linear relation of the melted liquid density to pressure and temperature are used.

Distribution of heat sources Q appearing due to electromagnetic energy absorption is defined by Poynting equation and Bouguer - Lambert law:

$$Q = -\vec{\nabla} \vec{R}, \quad \vec{\nabla} \vec{R} = \frac{|\vec{R}|}{L}, \tag{2.2}$$

where $\vec{R}$ is radiation intensity vector, and L is the medium high-frequency electromagnetic wave (HFEW) energy length.

Neglecting pressure and temperature influence on the absorption length L, for homogeneous and isotropic medium in the case of propagation of one-dimensional (flat, $\nu = 0$; cylindric; $\nu = 1$, and spherical, $\nu = 2$) monochromatic wave, volumetric heat sources for the mixture on the whole can be represented in the following way [3]:

$$\begin{aligned} &Q = \frac{R_0}{L}\left(\frac{x_0}{x}\right)^{\nu} \exp\left(\frac{x - x_0}{L}\right), \quad R_0 = \frac{N^{(e)}}{S_0}, \\ &S_0 = \xi(\nu) x_0^{\nu}, \quad \xi(0) = 1, \quad \xi(1) = 2\pi, \quad \xi(2) = 4\pi, \end{aligned} \tag{2.3}$$

where R_0 is radiation intensity on the well border ($x = x_0$) defined by power $N^{(e)}$ and radiator surface area S_0.

Equation set (2.1) with regard to (2.2) or (2.3) is closed. It can be used to study general behaviour of the medium heating process due to heat conductivity (surface heat source q_0) and HFEW energy absorption (volumetric heat source Q). Corresponding mathematical task consists of finding solutions of the received equation set (2.1) at the following initial and boundary conditions:

$$\begin{array}{lll} t = 0: & T = T_{\infty} < T_f & \text{or: } p = p_{\infty}, \quad T = T_{\infty} > T_f; \\ |\vec{x}| = x_0: & T = T_0, & \text{or: } \lambda_0 S_0 \vec{\nabla} T \vec{n}_0 = -q_0; \\ & p = p_0 & \text{or: } m\rho_{10} S_0 \vec{v}\, \vec{n}_0 = g_0; \\ x \to +\infty: & T \to T_{\infty} & \text{or: } p \to p_{\infty}, \quad T \to T_{\infty} \end{array} \tag{2.4}$$

and at the following condition at the phase transition front $\vec{x}_f(t)$:

$$\begin{aligned} &F(\vec{x}_f(t), t) = 0; \quad T = T_f = \text{const}, \\ &\vec{v}_{1f} \vec{n}_f = \left(1 - \frac{\rho_2}{\rho_1}\right) \frac{j}{m\rho_2}; \quad \frac{j}{m\rho_2} = \frac{1}{|\vec{\nabla} F|}\left(\frac{d\vec{x}_f}{dt} \vec{\nabla} F\right); \\ &j\ell = q_\ell^n + q_S^n; \quad q_\ell^n = -\lambda_\ell (\vec{\nabla} T \vec{n}_f)\big|_{\vec{x} = \vec{x}_f - 0}; \\ &q_S^n = -\lambda_S (\vec{\nabla} T \vec{n}_f)\big|_{\vec{x} = \vec{x}_f + 0}. \end{aligned} \tag{2.5}$$

Here g_0 is total mass consumption of the liquid (first) phase; j, ℓ are intensiveness and specific heat of phase transfer; q_ℓ^n, q_S^n are heat flows coming to interphase surface from mobile and immobile phases; $q_0 = q(\vec{x}_0, t)$ is intensiveness of total heat flows through the border $\vec{x} = \vec{x}_0$ ($q_0 > 0$ corresponds to the case of heat supply; $q_0 < 0$ corresponds to the case of heat removal; $q_0 = 0$ corresponds to absence of heat conduction on the well border); $\vec{n}$ is normal vector.

§3. DIMENSIONLESS VARIABLES AND PARAMETERS. THE PARTICULAR CASES

In order to analyse the equations given, it is best to introduce the following dimensionless variables and parameters which, together with the coefficient of porosity m, determine the solution set of the investigated problem:

$$\tau = \frac{u_* t}{L_*}, \quad X = \frac{x}{L_*}, \quad X_f = \frac{x_f}{L_*}, \quad L^{(e)} = \frac{L_*}{L_s}, \quad \theta = \frac{T}{T_f}, \quad P = \frac{p_1}{p_*},$$

$$\phi_i = \frac{\rho_i}{\rho_*}, \quad U_i = \frac{u_i}{u_*}, \quad \delta_f = \frac{\rho_2 - \rho_{1f}}{\rho_{1f}}, \quad M_1 = \frac{\mu_1(T)}{\mu_{1f}(T_f)}, \quad G_0 = \frac{g_0}{u_* \rho_{10} S_0},$$

$$Pe_i = \frac{u_* L_* \rho_* c_*}{\lambda_i} \text{(Peclet number)}, \quad N = \frac{N^{(e)}}{u_* \rho_* c_* S_0 T_f}, \quad K_i = Pe_i N X_0^{\nu}, \tag{3.1}$$

$$Q_0 = \frac{q_0 L_*}{\lambda_0 S_0 T_f}, \quad (c_* = c_1, \quad L_* = L_\ell, \quad u_* = \frac{k_1}{\mu_{1f}} \frac{p_*}{L_*}, \quad i = \ell, s).$$

Here c is heat capacity. The subscript $*$ refers to certain characteristic parameters of the medium.

Neglecting the thermal expansion of liquid and assumming that the liquid phase is incompressible, the considered problem is simplified to the problem on the heating of porous media taking into account convective thermal conductivity in the fluid and the existence of a volume thermal source and heat loss. In this case the velocity and pressure fields in the liquid phase can be expressed through the temperature field and the phase transition front dynamic in the following way:

$$\begin{aligned} & X \in [X_0, X_f), \\ & U_1 = -m\delta_f \left(\frac{X_f}{X}\right)^{\nu} \frac{dX_f}{d\tau}, \\ & P = 1 + m\delta_f X_f \frac{dX_f}{d\tau} \int_{X_0}^{X} \frac{M_1(\theta)}{\varsigma^{\nu}} d\varsigma. \end{aligned} \tag{3.2}$$

Neglecting the influence of temperature on the liquid phase viscosity the pressure field is found in an elementary manner. For example, when $\nu = 0$:

$$P = 1 + m\delta_f X \frac{dX_f}{d\tau}, \quad X \in [X_0, X_f). \tag{3.3}$$

It should be noted that on the case of (3.2) and (3.3) the influence of the thermal effect is expressed only through dynamic of interface surface X_f and viscosity of liquid phase $M_1(\theta)$.

§4. ONE-DIMENTIONAL FLAT PROBLEM. THE EXISTENCE OF STATIONARY SOLUTION WITH A PHASE TRANSFER FRONT

Consider the case of $\nu = 0$, $x_f = \text{const}$ and the heat loss q has the following form:

$$q = \frac{2h_\lambda}{L_0}(T - T_\infty), \tag{4.1}$$

where h_λ is the heat transfer coefficient, L_0 is the thickness of heating medium layer. In this case for region $X \in [X_0, X_f)$ the general form of the temperature field is the following:

$$\begin{aligned} \theta(x) = & \frac{K_\ell}{(1 + \sqrt{H_{\lambda\ell}})(1 - \sqrt{H_{\lambda\ell}})} \{ \exp[-(X - X_0)] + H \exp[-(X_f - X_0) - (X_f - X)] \} + \\ & + \frac{C_{\ell 1}}{2\sqrt{H_{\lambda\ell}}} \exp(\sqrt{H_{\lambda\ell}} X) - \frac{C_{\ell 2}}{2\sqrt{H_{\lambda\ell}}} \exp(-\sqrt{H_{\lambda\ell}} X). \end{aligned} \tag{4.2}$$

where

$$H_{\lambda i} = \frac{2h_{\lambda i} L_*^2}{\lambda_i L_0} \quad (i = \ell \text{ or } s),$$

H is the HFEW reflection coefficient [4].

Using the boundary conditions the integral constants $C_{\ell 1}$, $C_{\ell 2}$ can be determined:

$$\begin{aligned} C_{\ell 1} &= -\frac{2A_\ell \exp\left(\sqrt{H_{\lambda\ell}}X_0\right) + 2B_\ell\sqrt{H_{\lambda\ell}}\,\exp\left(\sqrt{H_{\lambda\ell}}X_f\right)}{\exp\left(2\sqrt{H_{\lambda\ell}}X_0\right) + \exp\left(2\sqrt{H_{\lambda\ell}}X_f\right)}\ ; \\ C_{\ell 2} &= \frac{-2A_\ell \exp\left(-\sqrt{H_{\lambda\ell}}X_0\right) + 2B_\ell\sqrt{H_{\lambda\ell}}\,\exp\left(-\sqrt{H_{\lambda\ell}}X_f\right)}{\exp\left(-2\sqrt{H_{\lambda\ell}}X_0\right) + \exp\left(-\sqrt{H_{\lambda\ell}}X_f\right)}\ , \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} A_\ell &= Q_0 + \frac{K_\ell}{\left(1+\sqrt{H_{\lambda\ell}}\right)\left(1-\sqrt{H_{\lambda\ell}}\right)}\{1 - H\exp[-2(X_f - X_0)]\}; \\ B_\ell &= -1 - \frac{K_\ell(1+H)\exp[-(X_f - X_0)]}{\left(1+\sqrt{H_{\lambda\ell}}\right)\left(1-\sqrt{H_{\lambda\ell}}\right)}\ . \end{aligned}$$

For the region $X > X_f$ we have:

$$\begin{aligned} \theta(x) = &-\frac{K_S L^{(e)}(1-H)}{\left(L^{(e)}+\sqrt{H_{\lambda S}}\right)\left(L^{(e)}-\sqrt{H_{\lambda S}}\right)}\exp[-(X_f - X_0) - L^{(e)}(X - X_f)] + \\ &+ \frac{C_{S1}}{2\sqrt{H_{\lambda S}}}\exp\left(\sqrt{H_{\lambda S}}X\right) - \frac{C_{S2}}{2\sqrt{H_{\lambda S}}}\exp\left(-\sqrt{H_{\lambda S}}X\right). \end{aligned} \tag{4.4}$$

And from the boundary conditions at infinity: $\theta \to \theta_\infty$ when $X \to +\infty$ and $\theta_f = 1$ we have:

$$\begin{aligned} C_{S1} &= 0; \\ C_{S2} &= 2A_S\sqrt{H_{\lambda S}}\exp\left(\sqrt{H_{\lambda S}}X_f\right). \end{aligned} \tag{4.5}$$

where

$$A_S = -1 - \frac{K_S L^{(e)}(1-H)\exp[-(X_f - X_0)]}{\left(L^{(e)}+\sqrt{H_{\lambda S}}\right)\left(L^{(e)}-\sqrt{H_{\lambda S}}\right)}\ .$$

In order to determine X_f using the energy conservation condition on the phase transition front we have:

$$\begin{aligned} &\frac{K_\ell(1-H)\exp[-(X_f - X_0)]}{\left(1+\sqrt{H_{\lambda\ell}}\right)\left(1-\sqrt{H_{\lambda\ell}}\right)} + \frac{C_{\ell 1}}{2}\exp\left(\sqrt{H_{\lambda\ell}}X_f\right) + \frac{C_{\ell 2}}{2}\exp\left(-\sqrt{H_{\lambda\ell}}X_f\right) = \\ &= \frac{K_\ell}{K_S}\left\{\frac{K_S L^{(e)^2}(1-H)\exp[-(X_f - X_0)]}{\left(L^{(e)}+\sqrt{H_{\lambda S}}\right)\left(L^{(e)}-\sqrt{H_{\lambda S}}\right)} + \frac{C_{S1}}{2}\exp\left(\sqrt{H_{\lambda S}}X_f\right) + \frac{C_{S2}}{2}\exp\left(-\sqrt{H_{\lambda S}}X_f\right)\right\}' \end{aligned} \tag{4.6}$$

or

$$F(X_f) = 0, \tag{4.7}$$

where

$$\begin{aligned} F(X_f) = &\frac{(1-H)\exp[-(X_f - X_0)]}{\left(1+\sqrt{H_{\lambda\ell}}\right)\left(1-\sqrt{H_{\lambda\ell}}\right)} - \\ &- \frac{\left[A_\ell\exp\left(\sqrt{H_{\lambda\ell}}X_0\right) + B_\ell\sqrt{H_{\lambda\ell}}\,\exp\left(\sqrt{H_{\lambda\ell}}X_f\right)\right]\exp\left(\sqrt{H_{\lambda\ell}}X_f\right)}{K_\ell\left[\exp\left(2\sqrt{H_{\lambda\ell}}X_0\right) + \exp\left(2\sqrt{H_{\lambda\ell}}X_f\right)\right]} - \\ &- \frac{\left[A_\ell\exp\left(-\sqrt{H_{\lambda\ell}}X_0\right) - B_\ell\sqrt{H_{\lambda\ell}}\,\exp\left(-\sqrt{H_{\lambda\ell}}X_f\right)\right]\exp\left(-\sqrt{H_{\lambda\ell}}X_f\right)}{K_\ell\left[\exp\left(-2\sqrt{H_{\lambda\ell}}X_0\right) + \exp\left(-2\sqrt{H_{\lambda\ell}}X_f\right)\right]} - \\ &- \frac{(1-H)L^2\exp[-(X_f - X_0)]}{\left(L^{(e)}+\sqrt{H_{\lambda S}}\right)\left(L^{(e)}-\sqrt{H_{\lambda S}}\right)} - \frac{A_S\sqrt{H_{\lambda S}}}{K_S} \end{aligned} \tag{4.8}$$

Consider the function $F(X_f)$. When $X_f \to X_0$ we have:

$$F \to \frac{(1-H)\big(LH_{\lambda\ell}+\sqrt{H_{\lambda S}}\big)}{(1-H_{\lambda\ell})\big(L^{(e)}+\sqrt{H_{\lambda S}}\big)}+\frac{\sqrt{H_{\lambda S}}}{K_S}-\frac{Q_0}{2K_\ell}\big[\exp\big(2\sqrt{H_{\lambda\ell}}X_0\big)+\exp\big(-2\sqrt{H_{\lambda\ell}}X_0\big)\big]. \tag{4.9}$$

And when $X_f \to +\infty$ we have:

$$F \to \frac{\sqrt{H_{\lambda\ell}}}{K_\ell}+\frac{\sqrt{H_{\lambda S}}}{K_S}>0. \tag{4.10}$$

From (4.9) and (4.10) it follows that when $F(X_0)<0$ the equation (4.7) has a solution. This always can be reached by increasing heat inflow Q_0.

§5. THE CASE OF PHASE TRANSITION SURFACE ASBSENCE ($T_\infty > T_f$)

Consider the case of one-dimensional symmetric (flat $\nu=0$ and cylindrical $\nu=1$) motion. In this case the equation set and boundary conditions have following dimensionless form:

$$\begin{aligned}
&-G_0\frac{d\theta}{dx}=\frac{1}{Pe_\ell}\frac{d}{dX}\Big(X^\nu\frac{d\theta}{dX}\Big)+NX_0^\nu\exp[-(X-X_0)]-2H_{\lambda\ell}X^\nu(\theta-\theta_\infty)\\
&\frac{dP}{dX}=-M_1U,\quad U=-\frac{G_0}{\phi_1X^\nu},\quad G_0=-(\phi_1UX_0^\nu)\Big|_{x=x_0}=\text{const}>0,\\
&\phi_1=1+B_p(P-1)-B_T(\theta-\theta_\infty),
\end{aligned}\tag{5.1}$$

$$x=x_0:\ \frac{d\theta}{dX}\Big|_{x=x_0+0}=-Q_0,\quad X\to\infty:\quad \theta\to\theta_\infty<+\infty,\quad P\to P_\infty<+\infty, \tag{5.2}$$

where B_p and B_T are dimensionless compresibility and thermal expansion coefficient, respectively.

It is easily to show that for the considered cases ($\nu=0$ and 1) no solution of the system of equations (5.1) exists which satisfies condition (5.2). Indeed, from second equation of (5.1) it follows that when $X\to+\infty$ the pressure should increase without limit:

$$\begin{aligned}
&\nu=0;\quad p^2\sim\text{const}\quad X\to+\infty;\\
&\nu=1;\quad p^2\sim\text{const}\quad \ln X\to+\infty;
\end{aligned}\tag{5.3}$$

where contradicts the last condition of (5.2).

It can be shown, however, that a solution of equations (5.1) with the boundary conditions

$$\begin{aligned}
&X=X_0:\quad \frac{d\theta}{dX}\Big|_{x=x_0+0}=-Q_0,\quad P=P_0,\\
&x=+\infty:\quad \theta\to\theta_\infty<+\infty,
\end{aligned}\tag{5.4}$$

i.e. without any contraint on pressure asymptotic behaviour as $X\to+\infty$, exists. In this case when $\nu=0$ the temperature field has the following distribution:

$$\theta(x)=\theta_\infty-\frac{NPe_\ell\exp[(X-X_0)]}{(1+\gamma_1)(1+\gamma_2)}+C\exp(\gamma_2X), \tag{5.5}$$

where

$$\begin{aligned}
\gamma_{1,2}&=\frac{1}{2}\Big(-Pe_\ell G_0\pm\sqrt{Pe_\ell^2G_0^2+16H_{\lambda\ell}Pe_\ell}\Big),\\
C&=-\frac{1}{\lambda_2}\Big[Q_0+\frac{NPe_\ell}{(1+\gamma_1)(1+\gamma_2)}\Big]
\end{aligned}$$

For the case $\nu = 1$ the medium temperature distributions can be determined, for example, by numerical method. In both cases ($\nu = 0$ and 1) the medium pressure distribution can be determined from the following equation:

$$\frac{dP}{dX} = f_\nu(P, X), \tag{5.6}$$

where

$$f_\nu(P, X) = \frac{M_1 G_0}{\phi_1 X^\nu}, \quad \frac{df_\nu}{dP} = -\frac{M_1 G_0 B_p}{\phi_1^2 X^\nu}. \tag{5.7}$$

Because in the region $[X_0, +\infty)$ the derivation of the function f_ν with respect to P is limited. Therefore in this region a solution of eq. (5.6) exists and unique.

CONCLUSION

In the paper the method using volume thermal source to act upon the reservoir is investigated. The model takes into account also the possible thermal exchange with surrounding medium is presented. The particular cases and the existence and uniqueness of stationary solutions are considered.

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VỀ BÀI TOÁN TRUYỀN NHIỆT VÀ TRUYỀN CHẤT TRONG RESERVOIR KHÔNG CÁCH NHIỆT

Bài báo nghiên cứu phương pháp sử dụng nguồn nhiệt khối để tác động lên reservoir. Trình bày mô hình toán học có xét đến sự trao đổi nhiệt với môi trường xung quanh. Nghiên cứu những trường hợp riêng và sự tồn tại và duy nhất của nghiệm dừng.