

# COMPUTER AIDED SIMULATION FOR NON-LINEAR MODEL OF SHIP MOTION

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**SUMMARY.** The ship motion is simulated on computer by non-linear model of small vibration and strong non-linear model. The results are compared between two models. The influences of different parameters to the ship motion are considered.

## §1. INTRODUCTION

The non-linear model simulation of ship motion has been studied by Nguyen Van Dao [1]. The proposed model consists of a mass  $M$  restrained by a non-linear elastic spring and a mass  $m$  attached to a hinged, weightless rod (Fig. 1). This system has the vertical and angular oscillations.

## §2. SIMULATION FOR NON-LINEAR MODEL OF SMALL VIBRATION

The differential equations of motion for the system represented in Fig. 1 are written as follows

$$\begin{aligned} (M + m)(\ddot{Z} + \ddot{u}) + k_0 Z + \beta_0 Z^3 + h_0 \dot{Z} + ml(\ddot{\varphi} \sin \varphi + \dot{\varphi}^2 \cos \varphi) &= 0 \\ ml^2 \ddot{\varphi} + C_0 \dot{\varphi} + ml(g + \ddot{Z} + \ddot{u}) \sin \varphi &= 0 \end{aligned} \quad (2.1)$$

where  $Z = x - u$  is the relative vertical displacement of the mass  $M$ ;  $x$  is the vertical displacement of the mass  $M$  from its static position of equilibrium;  $u = q \cos \omega t$  is the vertical displacement of the base of the system;  $\varphi$  is the angular displacement of the pendulum;  $k_0$  and  $\beta_0$  are the linear and non-linear characteristics of the spring, respectively;  $h_0$  and  $C_0$  are the damping coefficients for the vertical and angular motions, respectively.

Supposing that the damping forces and the ratios  $\sigma = q/l$ ;  $\mu = m/(M + m)$  are small, the differential equations of small vibrations are written as follows [1]:

$$\begin{aligned} W'' + k^2 W &= -\varepsilon \left[ -\sigma \eta^2 \cos \eta \tau + h W' + \beta W^3 + \mu (\varphi \varphi'' + \varphi'^2) \right] + \varepsilon^2 \dots \\ \varphi'' + \varphi &= \varepsilon \left[ -C \varphi' + \frac{\varphi^3}{6} - \varphi W'' + \sigma \eta^2 \varphi \cos \eta \tau \right] + \varepsilon^2 \dots \end{aligned} \quad (2.2)$$

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$$W = \frac{Z}{\ell}; \quad \eta = \frac{\omega}{\omega_0}; \quad \omega_0 = \sqrt{\frac{g}{\ell}}; \quad C = \frac{C_0}{\omega_0 m \ell^2}$$

$$h = \frac{h_0}{\omega_0(M+m)}; \quad k^2 = \frac{k_0}{\omega_0^2(M+m)}; \quad \beta = \frac{\beta_0 \ell^2}{\omega_0^2(M+m)}$$

$$\mu = \frac{m}{(M+m)}; \quad \sigma = \frac{g}{\ell}; \quad \tau = \omega_0 t.$$

Computer aided simulation for differential equations of small vibrations (2.2) with the different data in [1] with:

$$\mu = 0.05; \quad k = 2; \quad h = C = 0.10; \quad \sigma = 4.5 \cdot 10^{-2}; \quad \beta = 0 \text{ and } \beta = 1$$

we obtain the behaviours of  $\varphi(t)$  and  $W(t)$  as shown in Fig. 2. Where  $\varphi(\tau)$ : damped vibration,  $W(\tau)$ : periodic vibration after a time.

The change of coefficient  $\beta$  ( $\beta = 0$  or  $\beta = 1$ ) and the change of coefficient  $\eta$  have a very small influence to the motions.

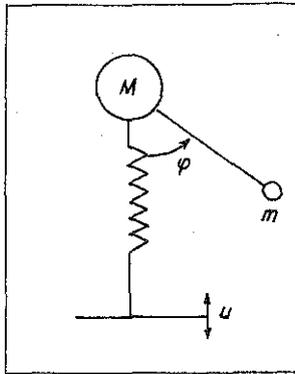


Fig. 1

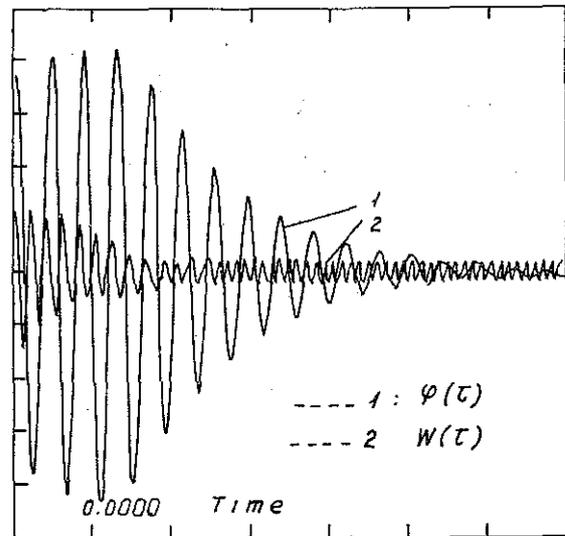


Fig. 2

### §3. SIMULATION FOR STRONG NON-LINEAR MODEL

For general strong non-linear model of ship motion, the differential equations of motion has the form (2.1). This form has been studied by means of the asymptotic method of non-linear mechanics [1]. It is can be simulated on computer as shown in Fig. 3:

With this computer aided simulation we can observe all motions corresponding to the different real parameters.

#### 1. Influence of the non-linear damping coefficients:

Making change the non-linear damping coefficients, we see that the term  $\beta_0$  in the non-linear characteristic of the spring has a very small influence to the vertical motion, while the damping term  $C_0$  of the angular motion has a considerable influence to this motion.

#### 2. Influence of the length of the rod:

Taking the ratios

$$\mu = \frac{m}{(M+m)} = 0.05; \quad \sigma = \frac{q}{l} = 0.045; \quad \eta = \frac{\omega}{\omega_0} = 1.8$$

the change of  $q$  and  $l$  has a influence to the behaviours of motions.

- with  $l = 2.2$ ,  $q = 0.1$ : The response curves of displacements  $Z$  and  $\varphi$  are plotted in Fig. 4

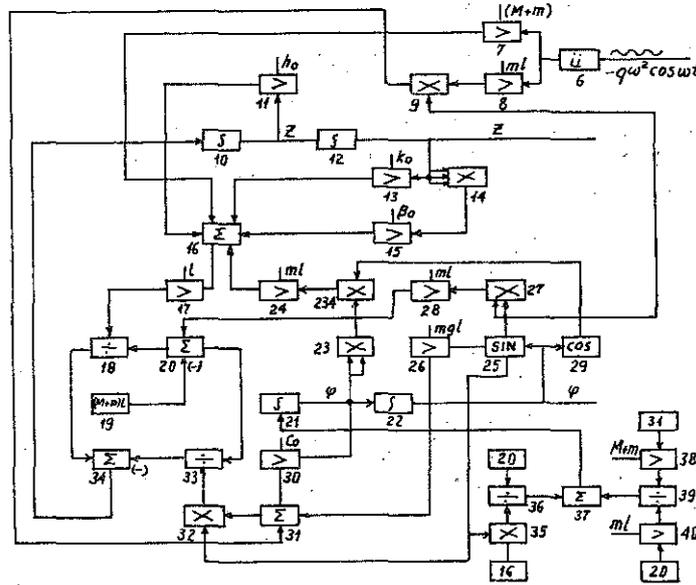


Fig. 3

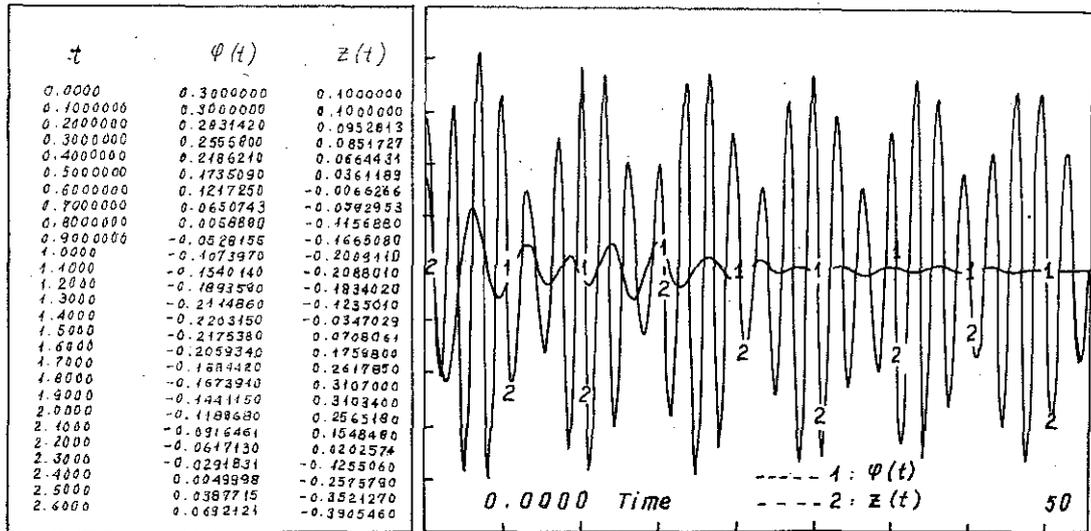


Fig. 4

Where, the angular displacement  $\varphi$  is damped, while the vertical displacement  $Z$  has a beat phenomenon.

Thus, corresponding with the observation in [1], for the case considered this is only the purely vertical motion.

- with  $\ell = 1$ ;  $q = 0.045$ : the shape of the response curves and the coupling between  $\varphi$  and  $Z$  are shown in Fig. 5. Where, both the behaviours  $\varphi(t)$  and  $Z(t)$  have the beat phenomenon.

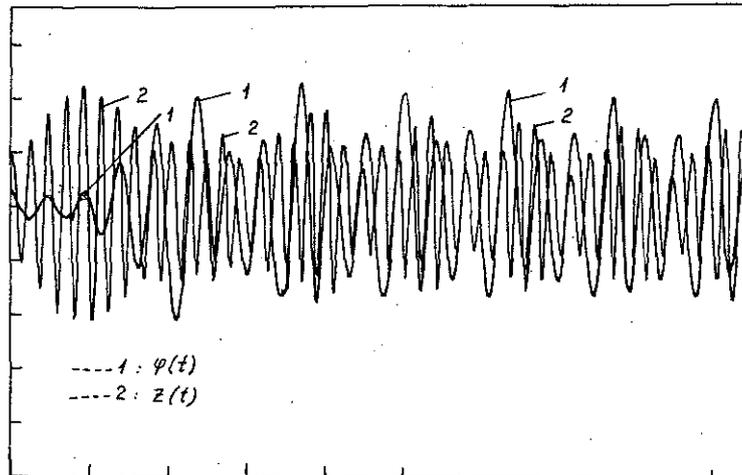


Fig. 5

### 3. Coupling between vertical and angular motions:

The coefficient  $\eta = \omega/\omega_0$  has an essential influence on the coupling between vertical and angular motions. However the coupling between two displacements  $\varphi$  and  $Z$  is small with

$$1.65 \leq \eta < 2$$

where  $\varphi(t)$  is damping oscillation,  $Z(t)$  has the beat phenomenon.

Its behaviours are analogous with those in Fig. 4.

The coupling strongly occurs in the resonant regions, i.e.

$$2 = \eta \leq 2.12$$

The coupling at the resonance ( $\eta = 2$ ) is plotted in Fig. 6.

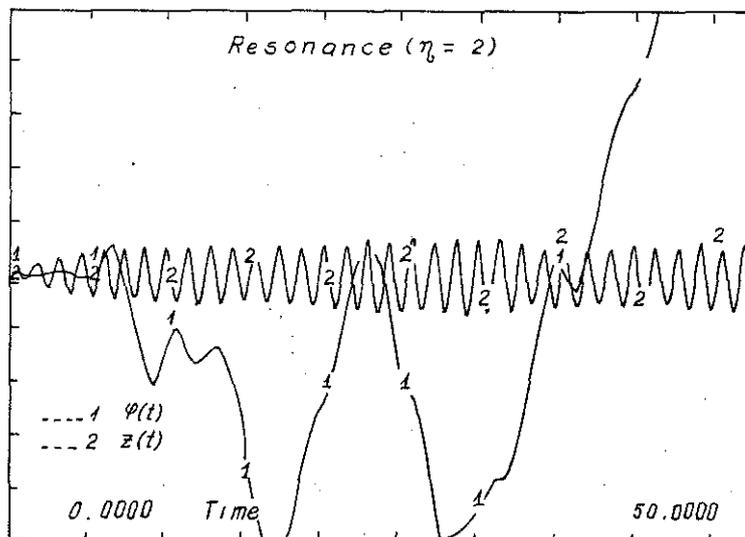


Fig. 6

With  $\eta \geq 2.5$ , the shape of the response curves is analogous with that in Fig. 5.

#### 4. Others influence of the parameters:

Making change others parameters of the different blocs in the flow diagram on Fig. 3 we can see on computer screen the direct displays of different motions considered.

Basing on the study in [1] and the computer aided simulation we can observe others phenomena occurred by the strong non-linearity of the system (2.1). Also, we can select the convenient parameters to occur the desirous motions.

### CONCLUSION

The ship motion is simulated on computer by two models of small vibration and of strong non-linearity.

The influences of different parameters are considered and compared. The experimental results are in accordance with theoretical ones [1]. With this computer aided simulation we can select the reel parameters to find the convenient motions.

This publication is completed with financial support from the National Basic Research Program in Natural Sciences.

### REFERENCES

1. Nguyen Van Dao. Non-linear model simulation of ship motion. Journal of Mechanics, T. XIV, No 2, 1992.

*Received May 8, 1993*

### MÔ PHỎNG TRÊN MÁY VI TÍNH MÔ HÌNH PHI TUYẾN CỦA CHUYỂN ĐỘNG TÀU THỦY

Mô hình phi tuyến yếu cho dao động lắc ngang và thẳng đứng của tàu thủy đã được nghiên cứu bằng phương pháp tiệm cận của dao động phi tuyến [1].

Trong bài này, mô hình phi tuyến trên đã được mô phỏng trên máy tính, với hai mô hình: dao động bé và phi tuyến mạnh. Đã xét ảnh hưởng của các thông số đến dáng điệu của chuyển động. Với sự mô phỏng này, chúng ta có thể dễ dàng thay đổi các tham số và quan sát trực tiếp trên màn hình các dạng chuyển động đồng thời thẳng đứng và lắc ngang của tàu.