Tạp chí Cơ học Journal of Mechanics, NCNST of Vietnam T. XVI, 1994, No 2 (13 - 19)

FORCED OSCILLATION OF THE RECTANGULAR THIN PLATE ON THE ELASTIC FOUNDATION WITH TWO COEFFICIENTS

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§0. INTRODUCTION

Parametric oscillation of the rectangular thin plate on the elastic foundation with two coefficients, when making mention of the creep of material, has been investigated in earlier publications (see for example [2, 3]). However, forced oscillation of the rectangular thin plate, to the author's knowledge, has not been hitherto examined.

This problem is studied here by means of an asymptotic method for high-order systems [1] and boundary value problem [4].

§1. FORMULATION OF THE PROBLEM. THE EQUATION OF MOTION

Now, let's determine forced oscillation of a rectangular thin plate, having thickness h, Young's modulus E, specific mass M and lengths of edges b, c, which is supported on four edges and lying on the elastic foundation with two coefficients as shown in Fig. 1.

Its motion is loaded by direction force, equally distributed $q = q(t)$

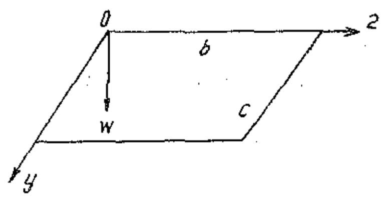

Fig. 1

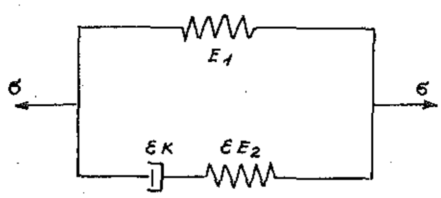

Fig. 2

The mechanical properties of material, when being straining, has been discribed by the model of the standard linear body [5] (Fig. 2). Because of the state equation in operator is written by the following form

$$\sigma = Ee \tag{1.1}$$

$$E = \frac{E_1 + K\left(1 + \dfrac{E_1}{E_2}\right)\dfrac{\partial}{\partial t}}{\left(1 + \dfrac{K}{E_2}\right)\dfrac{\partial}{\partial t}}\,. \tag{1.2}$$

Using the classical bending equation of plate with regarding the initial strain and the non-linear foundation, superseding the elastic modulus E by the analogous operator (1.2) into the expression for the bending hardness

$$D = \frac{Eh^3}{12(1-\nu^2)},$$

we get the equation of the problem

$$\frac{\partial^3 W}{\partial t^3} + \xi \frac{\partial^2 W}{\partial t^2} + \frac{1}{M}\Big[\frac{\partial}{\partial t}(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W) + \xi(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W)\Big] =$$
$$= \frac{1}{M}\Big[\varepsilon\Big(\xi f + \frac{\partial}{\partial t} - \frac{E_2 D_1}{E_1}\frac{\partial}{\partial t}\nabla^4 W\Big) + \xi q + \frac{\partial}{\partial t} q\Big]. \tag{1.3}$$

The relevant homogeneous boundary conditions are as follow

$$\begin{aligned} W\Big|_{x=0,b} &= 0, \quad \frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\Big|_{x=0,b} = 0, \\ W\Big|_{y=0,c} &= 0, \quad \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\Big|_{y=0,c} = 0. \end{aligned} \tag{1.4}$$

Here $W = W(x, y, t)$ - sagging of cross-section, $\xi = \frac{E_2}{K}$, ν - Poisson's ratio, K_1, K_2 - coefficients of the elastic foundation, f - elastic reaction, which is a non - linear function of $\Big(W, \frac{\partial W}{\partial x}, \dots\Big)$, ∇^2 - Laplace's operator.

$$D_1 = \frac{E_1 h^3}{12(1-\nu^2)}, \qquad \xi = \frac{E_2}{K}.$$

For simplicity, it is supposed that $M = 1$, $q = \varepsilon q_0 \sin \gamma t$, when the equation (1.3) is possibly written in the form

$$\frac{\partial^3 W}{\partial t^3} + \xi \frac{\partial^2 W}{\partial t^2} + \frac{\partial}{\partial t}(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W) + \xi(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W) = \varepsilon F(\theta, x, y, W, \dots), \tag{1.5}$$

where $d\theta/dt = \gamma$, F is a periodic function with period 2π relatively θ.

§2. CONSTRUCTION OF THE ASYMPTOTIC SOLUTION

When $\varepsilon = 0$, we have the boundary problem

$$\frac{\partial^3 W}{\partial t^3} + \xi \frac{\partial^2 W}{\partial t^2} + \frac{\partial}{\partial t}(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W) + \xi(D_1 \nabla^4 W - K_2 \nabla^2 W + K_1 W) = 0, \tag{2.1}$$

with the boundary conditions

$$\begin{aligned} W\Big|_{x=0,b} &= 0, \quad \frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\Big|_{x=0,b} = 0, \\ W\Big|_{y=0,c} &= 0, \quad \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\Big|_{y=0,c} = 0. \end{aligned} \tag{2.2}$$

The solution of this problem can be found in the form

$$W_0(x, y, t) = Z(x, y) T(t) \tag{2.3}$$

Substituting (2.3) into (2.1) and (2.2) we obtain

$$\frac{d^3T}{dt^3} + \xi\frac{d^2T}{dt^2} + \beta^2 D_1\frac{dT}{dt} + \xi\beta^2 D_1 T = 0, \tag{2.4}$$

$$\nabla^4 Z - \frac{K_2}{D_1}\nabla^2 Z + \frac{K_1}{D_1}Z - \beta^2 Z = 0, \tag{2.5}$$

$$\begin{aligned} Z\Big|_{x=0,b} &= 0, \quad \frac{\partial^2 Z}{\partial x^2} + \nu\frac{\partial^2 Z}{\partial y^2}\Big|_{x=0,b} = 0, \\ Z\Big|_{y=0,c} &= 0, \quad \frac{\partial^2 Z}{\partial y^2} + \nu\frac{\partial^2 Z}{\partial x^2}\Big|_{y=0,c} = 0. \end{aligned} \tag{2.6}$$

It is easy seen that the solution (2.3) takes form

$$W_0(x,y,t) = \sum_{r,s=1}^{\infty} A_{rs} Z_{r,s}(x,y)\cos\phi_{rs} + \sum_{r,s=1}^{\infty} D_{rs} Z_{r,s}(x,y)e^{-\xi t}. \tag{2.7}$$

$$\Omega^2 = D_1\beta^2, \quad Zrs(x,y) = \sin\frac{r\pi x}{b}\sin\frac{s\pi y}{c}, \quad \phi_{r,s} = (\Omega_{rs}t + \psi_{rs}), \tag{2.8}$$

where A_{rs}, D_{rs}, ψ_{rs} are positive constants determined from the initial conditions

$$\Omega_{rs}^2 = \left\{D_1\left[\left(\frac{\pi r}{b}\right)^2 + \left(\frac{s\pi}{c}\right)^2\right]^2 + K_2\left[\left(\frac{r\pi}{b}\right)^2 + \left(\frac{s\pi}{c}\right)^2\right] + K_1\right\}. \tag{2.9}$$

It is supposed that when $\varepsilon = 0$ there exists a periodic solution with frequency Ω_{11}

$$\Omega_{11}^2 = \left\{D_1\left[\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2\right]^2 + K_2\left[\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2\right] + K_1\right\}, \tag{2.10}$$

and there is a resonance relation

$$\Omega_{11}^2 = \gamma^2 + \varepsilon\delta, \tag{2.11}$$

δ is the detuning coefficient.

With these assumptions, we are going to find the partial solution of the boundary value problem (1.4), (1.5) in the asymptotic form

$$W(x,y,z) = aZ_{11}(x,y)\cos\phi + \varepsilon U_1(x,y,a,\phi,\theta) + \varepsilon^2 U_2(x,y,a,\phi,\theta) + \dots \tag{2.12}$$

where the functions U_1, U_2, ... are periodic with period 2π relatively ϕ, θ, the quantities a, ψ are determined from the equations

$$\begin{aligned} \frac{da}{dt} &= \varepsilon A_1(a,\psi) + \varepsilon^2 A_2(a,\psi) + \varepsilon^3 \dots, \\ \frac{d\psi}{dt} &= (\Omega_{11} - \gamma) + \varepsilon B_1(a,\psi) + \varepsilon^2 B_2(a,\psi) + \varepsilon^3 \dots \end{aligned} \tag{2.13}$$

By substituting (2.12) into the equation (1.5) and paying attention to (2.13), (1.4), in the better first approximation we get

$$\begin{aligned} &L_3[U_1] + \xi L_2[U_1] + L_1[D_1\nabla^4 U_1 + K_1\nabla^2 U_1 + K_1 U_1] + \xi[D_1\nabla^4 U_1 - K_2\nabla^2 U_1 + K_1 U_1] + \\ &+ [(2\Omega_{11}^2 A_1 + 2\xi a\Omega_{11}B_1)\cos\phi + (2\xi\Omega_1 A_1 - 2a\Omega_{11}^2 B_1)\sin\phi]Z_{11} = F_1, \end{aligned} \tag{2.14}$$

with the boundary conditions

$$U_1\Big|_{x=0,b}=0,\quad \frac{\partial^2 U_1}{\partial x^2}+\nu\frac{\partial^2 U_1}{\partial y^2}\Big|_{x=0,b}=0,$$
$$U_1\Big|_{y=0,c}=0,\quad \frac{\partial^2 U_1}{\partial y^2}+\nu\frac{\partial^2 U_1}{\partial x^2}\Big|_{y=0,c}=0. \tag{2.15}$$

The operators

$$L_1[U_1]=\Big(\Omega_{11}\frac{\partial}{\partial\phi}+\gamma\frac{\partial}{\partial\theta}\Big)U_1,\quad L_2[U_1]=\Big(\Omega_{11}\frac{\partial}{\partial\phi}+\gamma\frac{\partial}{\partial\theta}\Big)^2U_1,$$
$$L_3[U_1]=\Big(\Omega_{11}\frac{\partial}{\partial\phi}+\gamma\frac{\partial}{\partial\theta}\Big)^3,\quad F_1=F(\theta,x,y,aZ_1,\cos\phi,\dots).$$

To find the function U_1, we shall expand F_1 and U_1 relatively the foundation functions $\{Z_{rs}(x,y)\}$

$$U_1=\sum_{rs}U_{1rs}(a,\phi,\theta)Z_{rs}(x,y), \tag{2.16}$$

$$F_1=\sum_{rs}F_{1rs}(a,\phi,\theta)Z_{rs}(x,y), \tag{2.17}$$

where $F_{1rs}\int_0^b\int_0^c F_1Z_{rs}dxdy/\int_0^b\int_0^c Z_{rs}^2dxdy$ are defined, still U_{1rs} need be determined. Writing U_1 in the form (2.16), the boundary conditions (2.15) are selves satisfied.

Putting (2.16), (2.17) into the equation (2.14) and then comparing the coefficients of the functions $Z_{rs}(x,y)$, we have the following equations for determining U_{1rs}, A_1, B_1

$$L_3[U_{111}]+\xi L_2[U_{111}]+\Omega_{11}^2L_1[U_{111}]+\xi\Omega_{11}^2U_{111}=F_{111}+$$
$$+(2\Omega_{11}^2A_1+2\xi a\Omega_{11}B_1)\cos\phi+(2\xi\Omega_{11}A_1-2a\Omega_{11}^2B_1)\sin\phi, \tag{2.18}$$

$$L_3[U_{1rs}]+\xi L_2[U_{1rs}]+\Omega_{11}^2L_1[U_{1rs}]+\xi\Omega_{11}^2U_{1rs}=F_{1rs}. \tag{2.19}$$
$$(r,s=1,2,\dots,r=s\neq 1)$$

Now, we first expand F_{1rs} and U_{1rs} in the Fourier series

$$F_{1rs}=\sum_{n,m}F_{1nm}^{rs}(a)e^{i(n\theta+m\phi)}, \tag{2.20}$$

$$U_{1rs}=\sum_{n,m}U_{1nm}^{rs}(a)e^{i(n\theta+m\phi)}. \tag{2.21}$$

Here $F_{1nm}^{rs}=\dfrac{1}{4\pi^2}\displaystyle\int_0^{2\pi}\int_0^{2\pi}F_{1rs}e^{-i(n\theta+m\phi)}d\theta d\phi$ are known, yet U_{1nm}^{rs} need be determined.

Suppose that U_{111} does not contain $\cos\phi$, $\sin\phi$, putting the expressions (2.20) and (2.21) into the equation (2.18) and (2.19) and then comparing the coefficients of the functions $e^{i(n\theta+m\phi)}$, we receive

$$U_{1nm}^{rs}(a)=\frac{\int_0^{2\pi}\int_0^{2\pi}F_{1rs}(a,\phi,\theta)e^{-i(n\theta+m\phi)}d\phi d\theta}{4\pi^2\big[\xi+i(m\Omega_{11}+n\gamma)\big]\big[\Omega_{rs}^2-(m\Omega_{11}+n\gamma)^2\big]\int_0^b\int_0^c Z_{rs}^2dxdy}, \tag{2.22}$$

Consequently the function U_1 is determined by the following expression

$$U_1 = \sum_{n,m} \sum_{r,s=1}^{\infty} \frac{\int_0^{2\pi}\int_0^{2\pi}\int_0^{b}\int_0^{c} F_1 Z_{rs} e^{-i(n\theta+m\phi)} dxdyd\phi d\theta . Z_{rs} e^{i(n\theta+m\phi)}}{4\pi^2\big[\xi + i(m\Omega_{11}+n\gamma)\big]\big[\Omega_{rs}^2 - (m\Omega_{11}+n\gamma)^2\big]\int_0^b\int_0^c Z_{rs}^2 dxdy}, \tag{2.23}$$

$$r, s = 1, \quad [\Omega_{rs}^2 - (m\Omega_{11}+n\gamma)^2] \neq 0.$$

By comparing the coefficients of the function $\cos\phi$, $\sin\phi$ in (2.18), we get the following equational system to determine the quantities A_1, B_1

$$\begin{aligned} 2\Omega_{11}^2 A_1 + 2\xi a\Omega_{11}B_1 &= -\frac{1}{4\pi^2}\sum_{\sigma} e^{i\sigma\psi}\int_0^{2\pi}\int_0^{2\pi} F_{111}e^{-i(\phi-\theta)}\cos\phi d\phi d\theta = -G(a,\psi), \\ 2\xi\Omega_{11}A_1 - 2a\Omega_{11}^2 B_1 &= -\frac{1}{4\pi^2}\sum_{\sigma} e^{i\sigma\psi}\int_0^{2\pi}\int_0^{2\pi} F_{111}e^{-i(\phi-\theta)}\sin\phi d\phi d\theta = -H(a,\phi). \end{aligned} \tag{2.24}$$

From here, we have

$$A_1 = -\frac{(\Omega_{11}G + \xi H)}{\Omega_{11}(\Omega_{11}^2+\xi^2)}, \quad B_1 = -\frac{(\xi G - \Omega_{11}H)}{a\Omega_{11}(\Omega_{11}^2+\xi^2)}. \tag{2.25}$$

Thus, in the better first approximation the solution of the given boundary problem (2.12) is determined.

§3. CONCRETE CASE

Suppose that the right hand side of the equation (1.5) is of the form

$$\begin{aligned} F = \xi\Big\{ &-K_1W^3 - \frac{K_2}{2}\Big[\Big(\frac{\partial W}{\partial x}\Big)^2\frac{\partial^2 W}{\partial x^2} + \Big(\frac{\partial W}{\partial y}\Big)^2\frac{\partial^2 W}{\partial y^2}\Big]\Big\} + \\ &+ \frac{\partial}{\partial t}\Big\{-K_1W^3 - \frac{K_2}{2}\Big[\Big(\frac{\partial W}{\partial x}\Big)^2\frac{\partial^2 W}{\partial x^2} + \Big(\frac{\partial W}{\partial y}\Big)^2\frac{\partial^2 W}{\partial y^2}\Big]\Big\} + \\ &+ \xi q_0\sin\gamma t + q_0\gamma\cos\gamma t - \frac{E_2}{E_1}D_1\frac{\partial}{\partial t}(\nabla^4 W). \end{aligned} \tag{3.1}$$

Using the above expressions, in the first approximation we have

$$W = a\sin\frac{\pi x}{b}\sin\frac{\pi y}{c}\cos(\gamma t + \psi). \tag{3.2}$$

Here a, ψ are determined from the following equational system

$$\begin{aligned} 2\gamma\frac{da}{dt} &= -h\xi\gamma a - P_0\cos\psi, \\ 2a\gamma\frac{d\psi}{dt} &= a(\Omega_{11}^2 - \gamma^2) - Qa^3 + h\Omega_{11}^2 a + P_0\sin\psi, \end{aligned} \tag{3.3}$$

where

$$h = \frac{D_1E_2\Big[\frac{\pi^2}{b^2}+\frac{\pi^2}{c^2}\Big]^2}{E_1(\Omega_{11}^2+\xi^2)}\varepsilon, \quad P_0 = \frac{16q_0}{\pi^2}\varepsilon, \tag{3.4}$$

$$Q = \frac{6}{64}\Big[-3K_1 + \frac{K_2}{2}\Big(\frac{\pi^4}{b^4}+\frac{\pi^4}{c^4}\Big)\Big]\varepsilon. \tag{3.5}$$

Vanishing the right part of the equation (3.3), we obtain the stationary solution a_0, ψ_0 related to the frequency γ and amplitude q_0 of the force q

$$f(a_0,\gamma^2) = \left[a_0(\Omega_{11}^2 - \gamma^2) - Qa_0^3 + h\Omega_{11}^2 a_0\right]^2 - P_0^2 + h^2\xi^2\gamma^2 a_0^2 = 0, \tag{3.6}$$

$$\gamma^2 = -Qa_0^2 + (1+h)\Omega_{11}^2 \pm \sqrt{\frac{P_0^2}{a_0^2} - h^2\xi^2\gamma^2}\,. \tag{3.7}$$

The relation (3.7) is plotted in Fig. 3 for the case

$$Q = -1; \quad \Omega_{11}^2 = 1; \quad h^2 = 0.09; \quad P_0^2 = 0.045; \quad \xi^2 = 2,\ 3.$$

In Fig. 4 for the case

$$Q = +1; \quad \Omega_{11}^2 = 1; \quad h^2 = 0.09; \quad P_0^2 = 0.045; \quad \xi^2 = 2,\ 3.$$

In Fig. 5 for the case

$$Q = 0; \quad \Omega_{11}^2 = 1; \quad h^2 = 0.09; \quad P_0^2 = 0.045; \quad \xi^2 = 2,\ 3.$$

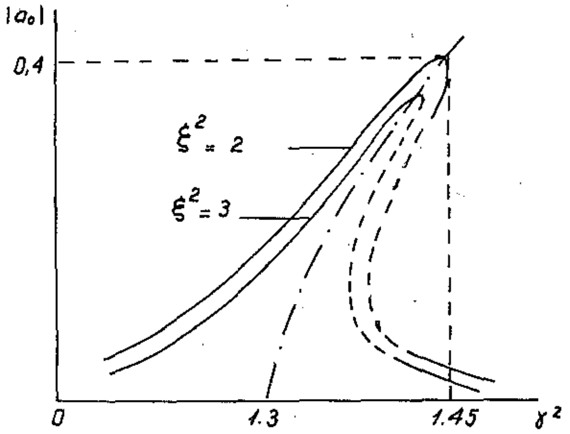

Fig. 3

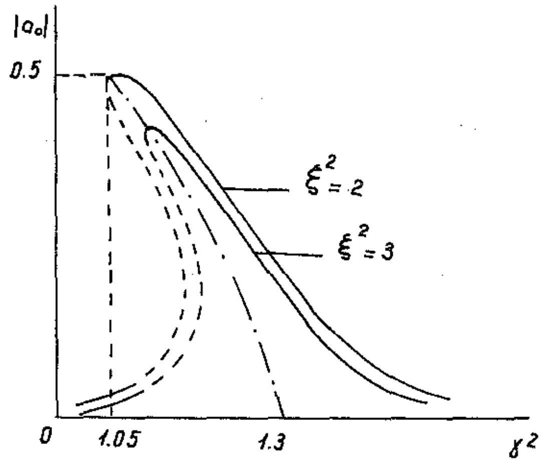

Fig. 4

To study the stability of stationary oscillations, we set into the equation (3.3) for $a = a_0 + \delta a$ and $\psi = \psi_0 + \delta\psi$, where δa, $\delta\psi$ are small perturbations. Neglecting the small quantities of higher order, we receive following variational equations for $a_0 \neq 0$

$$\begin{aligned} 2\gamma\frac{d}{dt}(\delta a) &= -h\xi\gamma\delta a + P_0\sin\psi_0\delta\psi, \\ 2a_0\gamma\frac{d}{dt}(\delta\psi) &= \left[(\Omega_{11}^2 - \gamma^2) - 3Qa_0^2 + h\Omega_{11}^2\right]\delta a + P_0\cos\psi_0\delta\psi. \end{aligned} \tag{3.8}$$

The characteristic equation of (3.8) is

$$4\gamma^2\lambda^2 + \left[(\Omega_{11}^2 - \gamma^2) - Qa_0^2 + h\Omega_{11}^2\right]\left[(\Omega_{11}^2 - \gamma^2) - 3Qa_0^2 + h\Omega_{11}^2\right] + h^2\xi^2\gamma^2 = 0 \tag{3.9}$$

From here it is easy to see that the condition for stability stationary oscillation is

$$\left[(\Omega_{11}^2 - \gamma^2) - Qa_0^2 + h\Omega_{11}\right]\left[(\Omega_{11}^2 - \gamma^2) - 3Qa_0^2 + h\Omega_{11}^2\right] + h^2\xi^2\gamma^2 > 0. \tag{3.10}$$

The inequality (3.10) will be satisfied, when

$$\frac{\partial f(a_0, \gamma^2)}{2a_0 \partial a_0} > 0. \tag{3.11}$$

In Fig. (3, 4), the solid curves correspond to stable states of vibration where the stability condition (3.11) is beeing valid

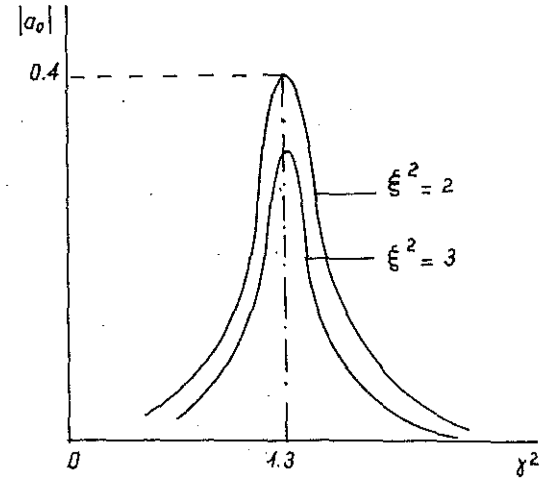

Fig. 5

CONCLUSION

1. The equation of motion for a rectangular thin plate on an elastic foundation with two coefficients was set up. Its solution has been found by means of an asymptotic method for high-order-systems, further the stability condition of the stationary oscillation has been investigated.

2. Taking into account two coefficients of the elastic foundation, the partial frequency increases. Thus, the problem is changed on the peculiarity.

3. From the presented Fig. 3, 4, 5 we can see the parameters b, c, K_1, K_2 may be chosen so that the system observed has hard or soft character.

I would like to express my gratitude and warm thanks to Prof. Dr. Nguyen Van Dao for his keen interest in this work, now completed.

This publication is completed with financial support from the National Basic Research Program in Natural Sciences.

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Received December 5, 1993

DAO ĐỘNG CƯỠNG BỨC CỦA BẢN MỎNG CHỮ NHẬT TRÊN NỀN ĐÀN HỒI HAI HỆ SỐ NỀN

Trong bài báo, bài toán dao động cưỡng bức của bản mỏng chữ nhật trên nền đàn hồi với hai hệ số nền, được nghiên cứu bằng phương pháp tiệm cận đối với hệ cấp cao. Đã chỉ ra các đặc trưng của dao động mà các tài liệu trước đó chưa đề cập đến. Dễ dàng thấy rằng có thể chọn các thông số b, c, K_1, K_2 để hệ dao động có đặc trưng cứng, đặc trưng mềm hoặc dao động tuyến tính.