Tạp chí Cơ Học Journal of Mechanics, NCNST of Vietnam T. XVI, 1994, No 1 (2 - 8)

ELASTO-PLASTIC STABILITY OF THIN PLATES SUBJECTED TO COMPLEX LOADING PROCESS

DAO HUY BICH

Hanoi National University

SUMMARY. Analysing an elastio-plastic stability problem, the complex loading process acted on the body has an essential influence on the values of critical loads. For clearing up this effect, in the present paper the general elasto - plastic process theory, theory of process with average curvature and the simple loading process theory are applied into the consideration of the mentioned problem. A numerical comparison is given.

§1. RELATIONS DEFINING THE VALUES OF CRITICAL LOADS

In [2] a new approach to the stability problem of plates subjected to arbitrary complex loading was given. Now consider the more general case, when there exists a pre-buckling plane stress state in the plate as follows.

$$\sigma_{11} = -p, \quad \sigma_{22} = -q, \quad \sigma_{12} = -\tau, \quad \sigma_{33} = \sigma_{23} = \sigma_{13} = 0$$
$$\sigma_u = (p^2 - pq + q^2 + 3\tau^2)^{1/2}.$$

Respectively, the arc - length of the deformation trajectory is evaluated from

$$\frac{ds}{dt} = \frac{2}{\sqrt{3}}\left(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{12} + \dot{\varepsilon}_{12}^2\right)^{1/2} = F(s, p, q, \tau) \tag{1.1}$$

where $\dot{\varepsilon}_{ij}$ are defined through p, q, τ according to the elasto - plastic process theory.

The post - buckling deformation processes may be arbitrary complicated, so that the possible stress - strain increments are defined by the same theory

$$\delta\sigma_{ij} = \frac{2}{3}A(\delta\varepsilon_{ij} + \delta_{ij}\delta\varepsilon_{kk}) + (P - A)\frac{\sigma_{k\ell}\delta\varepsilon_{k\ell}}{\sigma_u^2}\sigma_{ij} \tag{1.2}$$

where

$$\begin{aligned} A &= -\frac{\sigma_u f}{\sin\theta} = \frac{\sigma_u}{s}\left[1 + \left(\frac{3Gs}{\sigma_u} - 1\right)\frac{(1-\cos\theta)}{2}\right] = \\ &= \frac{1}{2}\left(3G + \frac{\sigma_u}{s}\right) - \frac{1}{2}\left(3G - \frac{\sigma_u}{s}\right)\cos\theta, \\ P &= \frac{\psi}{\cos\theta} = \phi'(s) - \frac{(3G - \phi')(1-\cos\theta)}{2\cos\theta} = \\ &= \frac{1}{2}(3G + \phi') - \frac{3G - \phi'}{2\cos\theta}\ ; \end{aligned} \tag{1.3}$$

with

$$\cos\theta = \frac{\sigma_{ij}\delta\varepsilon_{ij}}{\sigma_u \delta s}, \quad \delta s = \frac{2}{\sqrt{3}}\left(\delta\varepsilon_{11}^2 + \delta\varepsilon_{22}^2 + \delta\varepsilon_{11}\delta\varepsilon_{22} + \delta\varepsilon_{12}^2\right)^{1/2},$$
$$\delta\varepsilon_{ij} = -z\frac{\partial^2 \delta W}{\partial x_i \partial x_j}, \qquad (i, j, k, \ell = 1, 2).$$

In the same approach to the problem [2] the kinematic boundary conditions with edges simply supported are satisfied in the sense of Saint-Venant by chosen

$$\delta W = C\cos\left(\frac{m\pi x_1}{a} + \frac{n\pi x_2}{b}\right) \tag{1.4}$$

then

$$\cos\theta = \frac{\sqrt{3}}{2}\,\frac{\left(p\dfrac{m^2}{a^2} + 2\tau\dfrac{mn}{ab} + q\dfrac{n^2}{b^2}\right)}{(p^2 - pq + q^2 + 3\tau^2)^{1/2}\left(\dfrac{m^2}{a^2} + \dfrac{n^2}{b^2}\right)}$$

the quantities A and P are not depending on z, x_1, x_2, the stability equation reduces into the following

$$\alpha_1\frac{\partial^4\delta W}{\partial x_1^4} + \alpha_2\frac{\partial^4\delta W}{\partial x_1^3\partial x_2} + \alpha_3\frac{\partial^4\delta W}{\partial x_1^2\partial x_2^2} + \alpha_4\frac{\partial^4\delta W}{\partial x_1\partial x_2^3} + \alpha_5\frac{\partial^4\delta W}{\partial x_2^4} + \\ + \frac{9}{Ah^2}\left(p\frac{\partial^2\delta W}{\partial x_1^2} + 2\tau\frac{\partial^2\delta W}{\partial x_1\partial x_2} + q\frac{\partial^2\delta W}{\partial x_2^2}\right) = 0 \tag{1.5}$$

where

$$\alpha_1 = 1 - \frac{3}{4}\left(1 - \frac{P}{A}\right)\frac{p^2}{\sigma_u^2}, \quad \alpha_2 = -3\left(1 - \frac{P}{A}\right)\frac{p\tau}{\sigma_u^2},$$
$$\alpha_3 = 2 - 3\left(1 - \frac{P}{A}\right)\frac{\tau^2}{\sigma_u^2} - \frac{3}{2}\left(1 - \frac{P}{A}\right)\frac{pq}{\sigma_u^2},$$
$$\alpha_4 = -3\left(1 - \frac{P}{A}\right)\frac{q\tau}{\sigma_u^2}, \quad \alpha_5 = 1 - \frac{3}{4}\left(1 - \frac{P}{A}\right)\frac{q^2}{\sigma_u^2}.$$

Since there exists non - trivial solution $\delta W \neq 0$, i.e. $C \neq 0$, from the equations (1.4), (1.5) it follows a relation for defining critical loads

$$i^2 \equiv 9\frac{b^2}{h^2} = \frac{\pi^2 A\left(\alpha_1\dfrac{m^4}{a^4} + \alpha_2\dfrac{m^3 n}{a^3 b} + \alpha_3\dfrac{m^2n^2}{a^2b^2} + \alpha_4\dfrac{mn^3}{ab^3} + \alpha_5\dfrac{n^4}{b^4}\right)}{p\dfrac{m^2}{a^2} + 2\tau\dfrac{mn}{ab} + q\dfrac{n^2}{b^2}} \tag{1.6}$$

For determining a combination of critical loads p, q, τ we apply the loading parameter method [1], suppose that the complex loading process is given, i.e. $p \equiv p(t)$, $q \equiv q(t)$, $\tau \equiv \tau(t)$ are known as functions of a loading parameter t. Therefore, a solution $s \equiv s(t)$ of the equation (1.1) with the initial condition $t = 0$, $s = s_0$ can be found by Euler method or Runge - Kutta method. Using obtained result and solving the equation (1.6) gives a critical value of loading parameter t_*. Thus the critical loads are of the form

$$q_* = q(t_*), \quad p_* = p(t_*), \quad \tau_* = \tau(t_*).$$

Remarks

- If the elasto-plastic deformation process at the buckling moment is arbitrary complicated, the values A, P are taken in the form (1.3).

- If this process is less complicated, i.e. process with average curvature, replace A, P by $\dfrac{\sigma_u}{s}$ and $\phi'(s)$.

- If this process is simple, replace A, P by $E_c(s)$ and $E_t(s)$, where E_c - secant modulus, E_t - tangential modulus of stress versus strain characteristic, moreover $q = \beta_1 p$, $\tau = \beta_2 p$, $\sigma_u = \gamma p$, $\gamma = (1 - \beta_1 + \beta_1^2 + 3\beta_2^2)^{1/2}$.

For illustration consider a square plate ($a = b$) subjected to biaxial compressions and shear force, the minimal values of critical loads are obtained from (1.6) by putting $m = n = 1$.

§2. PLATE SUBJECTED TO BIAXIAL COMPRESSIONS

In this case $\sigma_{11} = -p$, $\sigma_{22} = -q$, $\sigma_{12} = -\tau = 0$. Analysis of the problem was given in [1, 3]. For clearing up the influence of the complex loading process, we observe the case when it happens outside the yield stress surface. Critical loads are determined from the relation (1.6) according to

a) the elasto - plastic process theory

$$i^2 \equiv 9\frac{a^2}{h^2} = \frac{\pi^2}{2(p+q)}\Big[\Big(3G + \frac{\sigma_u}{s}\Big) - \frac{\sqrt{3}}{4}\Big(3G - \frac{\sigma_u}{s}\Big)\frac{p+q}{\sigma_u}\Big] \times$$

$$\times \Big[4 - \frac{3}{4}\Big(1 - \frac{(3G+\phi') - \frac{4}{\sqrt{3}}(3G-\phi')\frac{\sigma_u}{p+q}}{\Big(3G + \frac{\sigma_u}{s}\Big) - \frac{\sqrt{3}}{4}\Big(3G - \frac{\sigma_u}{s}\Big)\frac{p+q}{\sigma_u}}\Big)\frac{(p+q)^2}{\sigma_u^2}\Big], \quad (2.1)$$

where $\sigma_u = (p - pq + q^2)^{1/2}$

b) the theory of process with average curvature [1]

$$i^2 = \frac{\pi^2(p^2 - pq + q^2)^{1/2}}{s(p+q)}\Big[4 - \frac{3}{4}\Big(1 - \frac{s\phi'(s)}{(p^2 - pq + q^2)^{1/2}}\Big)\frac{(p+q)^2}{p^2 - pq + q^2}\Big] \quad (2.2)$$

c) the simple process theory

$$i^2 = \frac{\gamma_1 \pi^2}{s(1+\beta_1)}\Big[4 - \frac{3}{4}\Big(1 - \frac{E_t(s)}{E_c(s)}\Big)\frac{(1+\beta_1)^2}{\gamma_1^2}\Big] \quad (2.3)$$

with $\gamma_1 = (1 - \beta_1 + \beta_1^2)^{1/2}$, then $p_* = \dfrac{E_c(s^*)s^*}{\gamma_1}$, $q_* = \beta_1 p_*$.

A numerical comparison is established on the square plate made of steel 30ХГСА and subjected to complex loading as follows (see fig. 1)

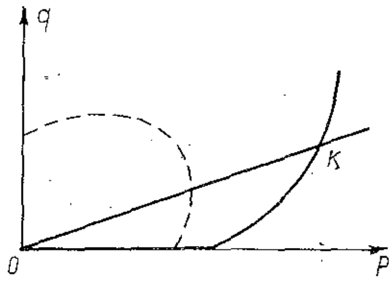

Fig. 1

$$p(t) = p_0 t + p_1 t, \quad q(t) = q_0 t^2, \quad p_0 \geq \sigma_s, \ t \geq 0$$

with $p_0 = 4260 kG/cm^2$, $p_1 = 10 kG/cm^2$, $q_0 = 200 kG/cm^2$, $\sigma_s = 4000 kG/cm^2$, $3G = 2.6 \cdot 10^6 kG/cm^2$; $t = 0$, $s_0 = 1.75 \cdot 10^{-3}$

The ratio a/h varies from 22 to 43. All calculations in solving problem on PC are fulfilled by Dao Thi Bich Hanh. Obtained results are presented on the tables: table 1 - critical loading parameter, critical loads calculated by the elasto plastic process theory (in solving equations (1.1) and (2.1)), table 2 - mentioned values by the theory of process with average curvature ((1.1) and (2.2)) and table 3 - by the simple process theory (2.3).

Table 1

a/h	$s.10^3$	t_*	p_*	q_*	σ_u^*
22	3.52	2.871	4288	1648	3747
25	3.136	2.563	4285	1313	3803
28	2.8	2.291	4282	1049	3866
31	2.53	2.038	4280	831	3931
34	2.309	1.781	4277	634	3998
37	2.108	1.479	4274	437	4073
40	1.94	1.157	4271	267	4144
43	1.82	0,754	4267	113	4211

Table 2

a/h	$s.10^3$	t_*	p_*	q_*	σ_u^*
22	3.54	2.890	4288	1670	3744
25	3.16	2.584	4285	1335	3798
28	2.838	2.320	4283	1076	3859
31	2.57	2.082	4280	867	3920
34	2.35	1.844	4278	680	3982
37	2.17	1.584	4275	501	4048
40	2.07	1.294	4272	335	4115
43	1.88	0.985	4269	194	4176

Table 3

a/h	$s.10^3$		p_*	q_*	σ_u^*
22	3.449		5602	2152	4895
25	3.116		5428	1663	4817
28	2.827		5260	1288	4750
31	2.583		5081	986	4667
34	2.382		4906	727	4586
37	2.222		4741	484	4548
40	2.062		4607	288	4470
43	1.936		4459	118	4401

§3. PLATE SUBJECTED TO COMPRESSION AND SHEAR FORCE

On this case $\sigma_{11} = -p$, $\sigma_{22} = 0$, $\sigma_{12} = -\tau$, $\sigma_u = (p^2 + 3\tau^2)^{1/2}$. Components of the strain velocity tensor are determined by the following equations

$$
\begin{aligned}
\dot{\varepsilon}_{11} &= -\frac{\dot{p}}{\sigma_u/s} - \Big(\frac{1}{\phi'(s)} - \frac{1}{\sigma_u/s}\Big)\frac{p(p\dot{p} + 3\tau\dot{\tau})}{p^2 + 3\tau^2}, \\
\dot{\varepsilon}_{12} &= -\frac{3\dot{\tau}}{2\sigma_u/s} - \frac{3}{2}\Big(\frac{1}{\phi'} - \frac{1}{\sigma_u/s}\Big)\frac{\tau(p\dot{p} + 3\tau\dot{\tau})}{p^2 + 3\tau^2}, \\
\dot{\varepsilon}_{22} &= -\dot{\varepsilon}_{11}/2.
\end{aligned}
\tag{3.1}
$$

The arc - length of the strain trajectory is evaluated from

$$
\frac{ds}{dt} = \big(\dot{\varepsilon}_{11}^2 + \frac{4}{3}\dot{\varepsilon}_{12}^2\big)^{1/2} = F(s, p, \tau) \tag{3.2}
$$

Finding the critical loads is suggested on the following relations respectively

a) the elasto - plastic process theory

$$
\begin{aligned}
i^2 = \frac{\pi^2}{2(p+2\tau)}\Big[\Big(3G + \frac{\sigma_u}{s}\Big) - \frac{\sqrt{3}}{4}\Big(3G - \frac{\sigma_u}{s}\Big)\frac{p+2\tau}{\sigma_u}\Big] \times \\
\times\Big[3 - \frac{3}{4}\Big(1 - \frac{(3G + \phi') - \dfrac{4}{\sqrt{3}}(3G - \phi')\dfrac{\sigma_u}{p+2\tau}}{\Big(3G + \dfrac{\sigma_u}{s}\Big) - \dfrac{\sqrt{3}}{4}\Big(3G - \dfrac{\sigma_u}{s}\Big)\dfrac{p+2\tau}{\sigma_u}}\Big)\frac{(p+2\tau)^2}{\sigma_u^2}\Big]
\end{aligned}
\tag{3.3}
$$

where $\sigma_u = (p^2 + 3\tau^2)^{1/2}$.

b) the theory of process with average curvature

$$
i^2 = \frac{\pi^2(p^2 + 3\tau^2)^{1/2}}{(p+2\tau)s}\Big[3 - \frac{3}{4}\Big(1 - \frac{s\phi'(s)}{(p^2+3\tau^2)^{1/2}}\Big)\frac{(p+2\tau)^2}{p^2+3\tau^2}\Big] \tag{3.4}
$$

c) the simple process theory

$$
i^2 = \frac{\gamma_2\pi^2}{s(1+2\beta_2)}\Big[3 - \frac{3}{4}\Big(1 - \frac{E_t(s)}{E_c(s)}\Big)\frac{(1+2\beta_2)^2}{\gamma_2^2}\Big], \tag{3.5}
$$

with $\gamma_2 = (1 + 3\beta_2^2)^{1/2}$, then $p_* = \dfrac{E_c(s_*)s_*}{\gamma_2}$, $q_* = \beta_2 p_*$.

A numerical analysis is considered on the square plate made of steel 30XГCA and subjected to complex loading process:

$$
\begin{aligned}
p(t) &= p_0 + p_1 t \\
\tau(t) &= \tau_0 t^2, \quad p_0 \geq \sigma_s,\ t \geq 0
\end{aligned}
$$

with $p_0 = 4260kG/cm^2$, $p_1 = 10kG/cm^2$, $\tau_0 = 200kG/cm^2$. The ratio a/h varies from 22 to 37. Dao Thi Bich Hanh has fulfilled all calculations in solving problem on PC. Obtained results are shown on the tables: table 4 - critical loading parameter, critical loads calculated by the elasto - plastic process theory (in solving (3.2) and (3.3)); table 5 - by the theory of process with average curvature ((3.2) and (3.4)); table 6 - by the simple process theory (3.5).

Table 4

a/h	$s.10^3$	t_*	p_*	q_*	σ_u^*
22	2.6	1.955	4279	764	4479
25	2.329	1.736	4277	602	4402
28	2.11	1.462	4274	427	4338
31	1.96	1.159	4271	268	4296
34	1.85	0.828	4268	137	4274
37	1.75	0.18	4261	6	4261

Table 5

a/h	$s.10^3$	t_*	p_*	q_*	σ_u^*
22	3.095	2.19	4281	959	4592
25	2.62	1.97	4279	776	4485
28	2.33	1.737	4277	604	4403
31	2.11	1.456	4274	423	4337
34	1.95	1.144	4271	261	4295
37	1.839	0.776	4276	120	4272

Table 6

a/h	$s.10^3$		p_*	q_*	σ_u^*
22	3.379		4667	833	4885
25	2.834		4615	649	4756
28	2.522		4568	456	4636
31	2.297		4521	283	4547
34	2.085		4472	143	4479
37	1.944		4407	6	4407

§4. DISCUSSION

From the above results we can lead some conclusions

a) Critical loads of the plate subjected to complex loading are always smaller than critical one when the simple loading happens. The workable possibility of structures with complex loading is less than that with simple loading.

b) The influence of the complex loading process on the critical loads is shown apparently when the process happens outside the yield stress surface. In the paper [3] the difference is not obvious, because the complex loading process happens inside the yield surface.

c) In the stability problem of plate subjected to compression and shear force results obtained by theory of process with average curvature differ obviously from that one by elasto plastic process theory, since in this case the deformation process is more complicated.

d) In [4, 5] there are experimental data, the comparison can't be established, because the materials are different. But in the qualitative side this analysis gives an accordance with experimental data.

e) Elasto plastic deformation process plays an important role in the stability of structures, it requires to apply the theory of elasto plastic processes in solving the problem. This analysis may be highly reliable and may describe the real state of the structures.

This publication is completed with financial support from the National Basic Research Program in Natural Sciences.

REFERENCES

1. Dao Huy Bich. Influence of complex loading on the stability outside elastic limit of thin plates and shells. Journal of Mechanics, vol. 11, No 3, 1989 (in Vietnamese).
2. Dao Huy Bich. A new approach to the stability problem of plates subjected to arbitrary complex loading. Journal of Mechanics, vol. 14, No 2, 1992.
3. Dao Van Dung. On the stability of thin plates subjected to biaxial compressions. Journal of Mechanics, vol. 15, No 2, 1993 (in Vietnamese).
4. Zubtsaninov V. H. On the contemporary problems of inelastic stability. Selection: Stability in solid Mechanics. Kalinin 1981 (in Russian)
5. Garanikov V. V. Lotov V. N. Experimental research on buckling process of plates. Selection: Stability in solid Mechanics. Kalinin 1982 (in Russian).

Received September 28, 1993

ỔN ĐỊNH ĐÀN DẺO CỦA BẢN DƯỚI TÁC DỤNG CỦA QUÁ TRÌNH ĐẶT TẢI PHỨC TẠP

Trong bài toán ổn định ngoài giới hạn đàn hồi quá trình đặt tải phức tạp lên vật thể có ảnh hưởng đáng kể đến giá trị tải tới hạn. Để làm sáng tỏ hiệu quả đó, trong bài này đã sử dụng lý thuyết quá trình đàn dẻo, lý thuyết quá trình độ cong trung bình và lý thuyết quá trình đặt tải đơn giản để xét bài toán nêu trên, khi độ phức tạp của quá trình xảy ra ngoài mặt giới hạn chảy. Đã trình bày so sánh kết quả bằng số và nêu lên một số nhận xét.