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INFLUENCE OF HEAT SOURCES TO FINITE CHANNEL CONJUGATE NATURAL CONVECTION OF A POWER FLUID

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ABSTRACT. The paper extended the studies of Thomas F. Irvine et al. [1] and V. D. Quang and D. H. Chung [2] by considering the influence of heat transfer into the wall of vertical channel with natural convection motion of a power law fluid as well as the influence of heat sources. The results obtained in some test cases showed that the influence of wall will be clear and impossible to neglect when thermal conductivity of wall is small enough, so the effect of heat source distributed in the fluid is more considerable than in the wall and the heat source from $50kW/m^3$ will make an effect more considerably.

1. INTRODUCTION

The study of heat exchange in the natural convection motion of non-Newtonial fluid in a channel or pipe is one of the basic mechanics problems of the technologies of oil exploit and transport. In particular, the model of power law fluid has interested the studiers in the world more and more because of its suitability in the oil motion.

The problem of the free convection of power law fluid (Ostwald-de-Waele fluid) in finite vertical channel with the walls of thickness has been studied in [2]. In this paper the expansion is made by introducing different heat sources in both the fluid and channel walls. Such a problem maybe appears due to the requirement of heat increase in the process of oil transport.

The natural convection equations with boundary conditions were solved with finite difference method. The fields of velocity and temperature as well as dimensionless characteristics of heat exchange and the averaged velocity were determined.

2. THE GOVERNING EQUATIONS

We consider the steady 2-D convection of the power law fluid in a finite vertical channel with the heat sources in the fluid and channel walls, and suppose that the heat transfer into the walls only occurs in horizontal direction due to the channel thickness being very small in comparison with the channel height.

Introducing the following dimensionless variables

$$\tilde{x} = x/H,\ \tilde{y} = y/b,\ \tilde{u} = bu/Hu^*,\ \tilde{v} = v/u^*,\ \tilde{\delta} = \delta/b$$
$$\tilde{T} = \frac{T - T_\infty}{T_w - T_\infty},\ \tilde{T}_1 = \frac{T_1 - T_\infty}{T_w - T_\infty},\ \tilde{p}' = \frac{p'b^2}{\rho u^{*2} H^2},$$
$$Pr_g = \frac{\rho C_p}{k} u^* b,\ Gr_g = \frac{g\beta (T_w - T_\infty) b^2}{H u^{*2}},\ u^* = \frac{\nu_k^{\frac{1}{2-n}} b^{\frac{1-2n}{2-n}}}{H^{\frac{1-n}{2-n}}}$$
$$S_1 = \frac{Q_1 b^2}{k_1 (T_w - T_\infty)},\ S_2 = \frac{bQ_2}{\rho C_p u^* (T_w - T_\infty)} \tag{2.1}$$

and dropping tilde signs for convenience we obtain the basic equations in dimensionless form as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.2}$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{dp'}{dx} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n + Gr_g T \tag{2.3}$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr_g}\frac{\partial^2 T}{\partial y^2} + S_2, \qquad -\frac{1}{2} \le y \le \frac{1}{2}, \quad 0 \le x \le 1 \tag{2.4}$$
$$\frac{d^2 T_1}{dy^2} = -S_1, \qquad -\frac{1}{2} - \delta \le y \le -\frac{1}{2} \ or\ \frac{1}{2} \le y \le \frac{1}{2} + \delta, \quad 0 \le x \le 1 \tag{2.5}$$

in which u, v are the velocities in the x and y directions, respectively, x axis along the channel, ρ fluid density, p' the pressure imbalance, ν_k kinematic consistency, n flow behaviour index for power law fluid, g acceleration of gravity, β thermal expansion coefficient, T temperature of fluid, T_1 temperature inside the walls, T_∞ temperature of surrounding, k thermal conductivity, C_p specific heat, b interplace spacing, δ thickness of walls, Q_1 and Q_2 heat sources, T_w is the temperature on external sides of walls. The pressure imbalance, p', is defined by

$$p'(x) = p(x) - p_0 + \rho_\infty g x \tag{2.6}$$

where p_0 and ρ_∞ are outside ambient pressure and density, respectively.

Because of the symmetry we only consider a half channel flow field. In this case the boundary conditions are as follows

$$\begin{aligned}
&u(0,y) = u_0,\ v(0,y) = 0,\\
&T(0,y) = 0,\ p'(0,y) = 0,\\
&u(x,\tfrac{1}{2}) = 0,\ v(x,\tfrac{1}{2}) = 0,\\
&\frac{\partial u}{\partial y}(x,0) = 0,\ v(x,0) = 0,\\
&\frac{\partial T}{\partial y}(x,0) = 0,\ p'(1,y) = 0,\\
&T(x,y)\Big|_{y=\frac{1}{2}} = T_1\Big|_{y=\frac{1}{2}}\\
&T_1(y = \tfrac{1}{2} + \delta) = 1,\ k\frac{dT}{dy}\Big|_{y=\frac{1}{2}} = k_1\frac{dT_1}{dy}\Big|_{y=\frac{1}{2}}
\end{aligned} \tag{2.7}$$

where k_1 is thermal conductivity for walls.

3. NUMERICAL SOLUTIONS AND DISCUSSIONS

Let Ω be the continuous region of consideration for the fluid

$$\Omega = \{(x, y) : 0 \le x \le 1,\ 0 \le y \le 1/2\}$$

and Ω_* be corresponding grid region

$$\Omega_* = \{(i, j) : 1 \le i \le N,\ 1 \le j \le J\}$$

in which steps Δy is constant and Δx is variable. Using the three-point difference formulae we easily discretise the equations (2.2)–(2.5) as well as the boundary conditions (2.7). Due to the feature of difference formulae the equations (2.4) and (2.5) are solved first. Difference equation corresponding to equation (2.4) is

$$a_i T_{i-1}^{j+1} + b_i T_i^{j+1} + c_i T_{i+1}^{j+1} = d_i,\ (i = 2, N-1) \tag{3.1}$$

in which

$$a_i = \frac{\lambda}{Pr_g} + \tau v_i^j,\ b_i = -\frac{2\lambda}{Pr_g} - u_i^j,$$
$$c_i = \frac{\lambda}{Pr_g} - \tau v_i^j,\ d_i = -u_i^j T_i^j - \Delta x S_2,$$
$$\tau = \frac{\Delta x}{2\Delta y},\ \lambda = \frac{\Delta x}{\Delta y^2},\ (i = 2, N-1)$$

In order to get the boundary condition for equations (3.1) at $y = \frac{1}{2}$ corresponding to $i = N$ we have to integrate equation (2.5) and then use the above conjugate condition, finally we have

$$T_N^{j+1}\left(1 + \frac{k_1}{k}\frac{\Delta y}{\delta}\right) - T_{N-1}^{j+1} = \frac{k_1 \Delta y}{k\delta}\left[-\frac{S_1\delta}{2} + \frac{S_1}{2}\left(\frac{1}{2} + \delta\right)^2 - \frac{S_1}{8} + 1\right] \tag{3.2}$$

Applying the finite difference method to the boundary condition at $y = 0$ for T we obtain

$$T_1^{j+1} - T_2^{j+1} = 0 \tag{3.3}$$

It is clear that the equations (3.1)–(3.3) represent a tridiagonal linear equation system and easily to solve. Next the difference equations corresponding to equation (2.3) are solved for u and p'. However, because of the appearance of p' in equation (2.3) so a supplement equation is required. In a similar fashion in Thomas F. Irvine et al.[1] we introduce the condition representing the conservation of fluid mass in the channel:

$$\int_0^{\frac{1}{2}} u\,dy = \frac{1}{2}u_0 \tag{3.4}$$

For the equation (2.2) we can directly integrate on the base of the given solution of u. And thus with a guessing of u_0 the iteration is done until $p' = 0$ at $x = 1$.

In order to illustrate the influences of the wall thickness as well as the heat sources in the paper several concrete cases have been computed as examples. The common input data for all the cases is shown in Table 1, excepting the case 7 with $\delta = 0$, which has been studied before [1]. However, it should be noted that only the channel walls of thermal conductivity equal to four or ten times larger than the one of fluid have been considered here. The algorithm has been coded in FORTRAN 77 language (FTN77/386) to run under graphics mode, so the computation results are presented by curves on the screen, facilitating our follow on the behaviour of solutions to adjust the value of u_0 at the entrance.

Table 1. Input data

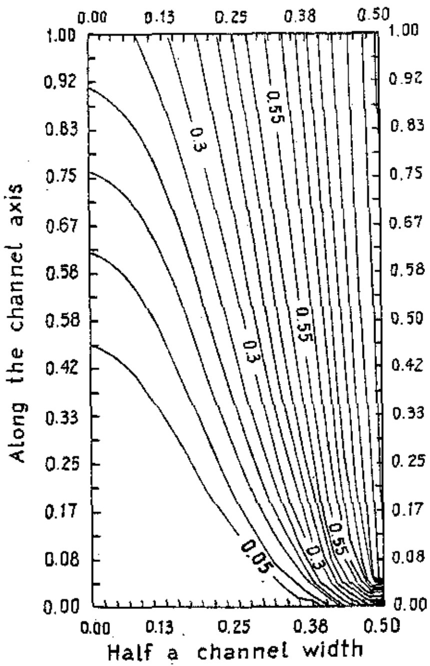
T_w	$= 25°C$	T_∞	$= 15°C$
b	$= 2cm$	H	$= 20cm$
ρ	$= 1000kg/m^3$	C_p	$= 4.18 \cdot 10^{-3} J/kg.K$
k	$= 0.597W/m.K$	ν_k	$= 7.35 \cdot 10^{-5} m^2/s^{2-n}$
β	$= 1.8 \cdot 10^{-4} 1/K$	n	$= 0.66$
δ	$= b/8$		

The computational results for cases 1–7 are shown in Table 2 and in Figures 1–6, the case 7 is the one of Thomas F. Irvine et al. [1]. The final results of interest for different cases are the characteristic parameters: the velocity at channel entry u_0 and the average heat transfer Q (see Table 2).

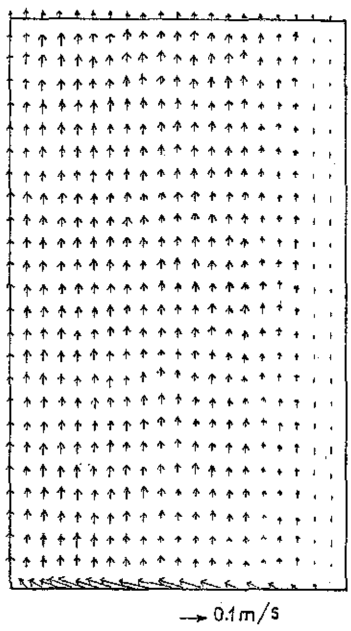
Table 2. Computation results

Cases	k_1/k	$Q_1(KW/m^3)$	$Q_2(KW/m^3)$	$u_0(cm0/s)$	$Q(W/m)$
1	10	3	3	1.3	475.03
2	4	3	3	1.22	460.5
3	10	50	0	1.29	475.52
4	10	0	50	1.55	475.95
5	4	0	0	1.21	460.23
6	10	0	0	1.27	474.96
7	∞	0	0	1.33	486.

Figures 1–6 present isothermal lines and velocity vector fields in the plane of half channel.

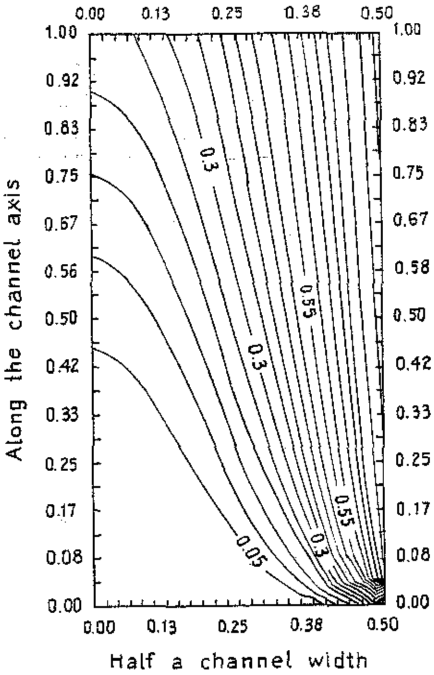


Isothermal lines

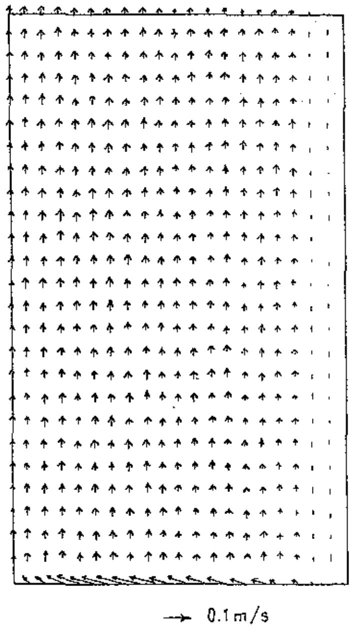


Velocity vectors

Fig. 1 Case 1

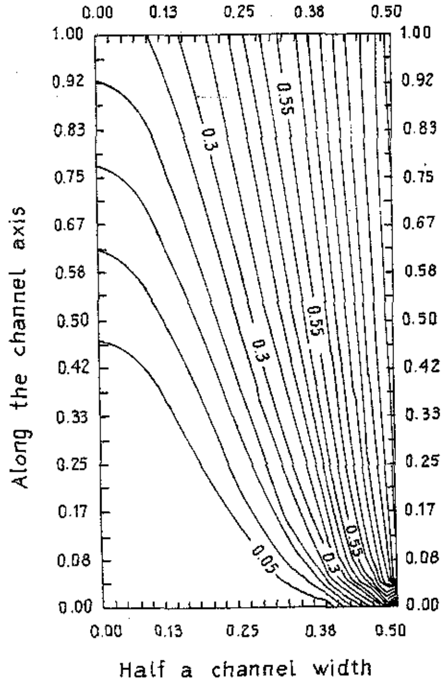


Isothermal lines

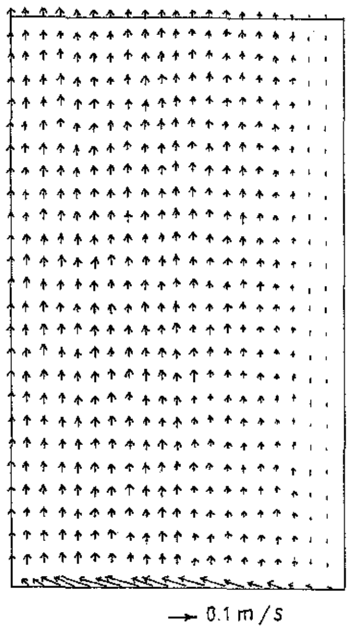


Velocity vectors

Fig. 2 Case 2

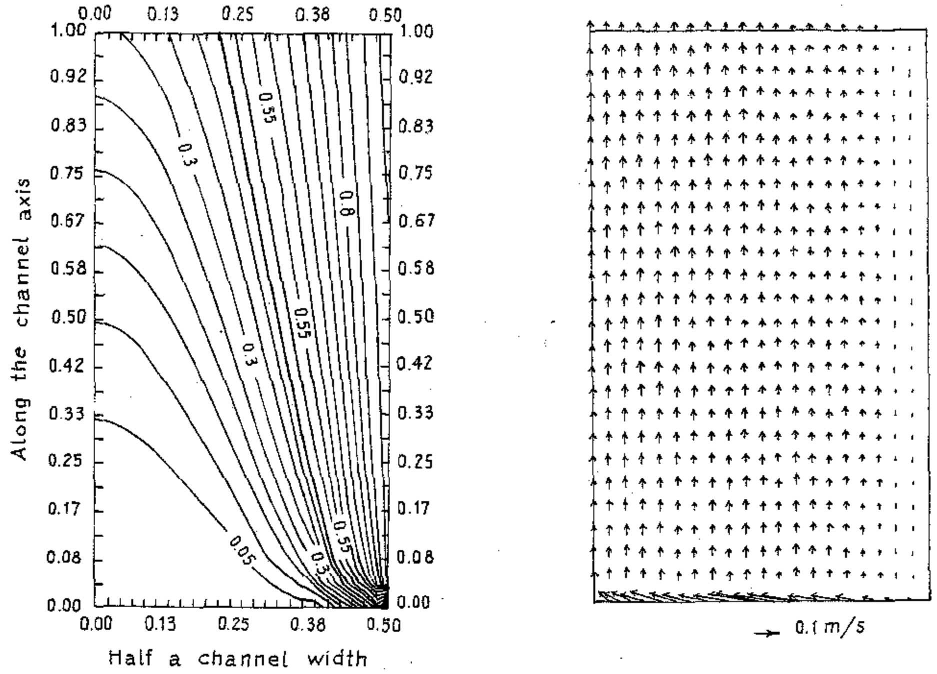


Isothermal lines



Velocity vectors

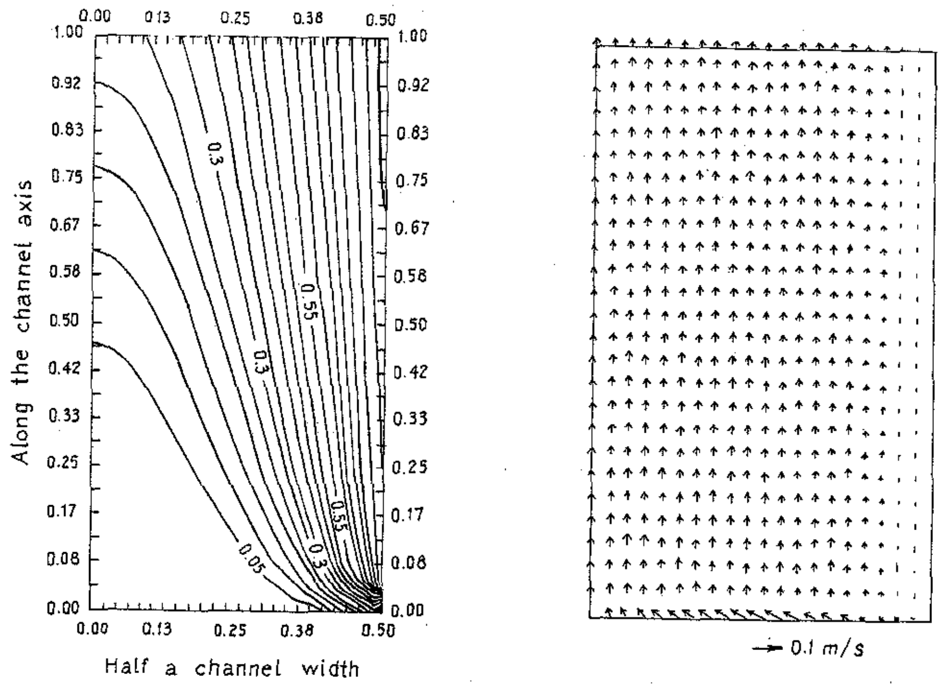
Fig. 3 Case 3



Isothermal lines

Velocity vectors

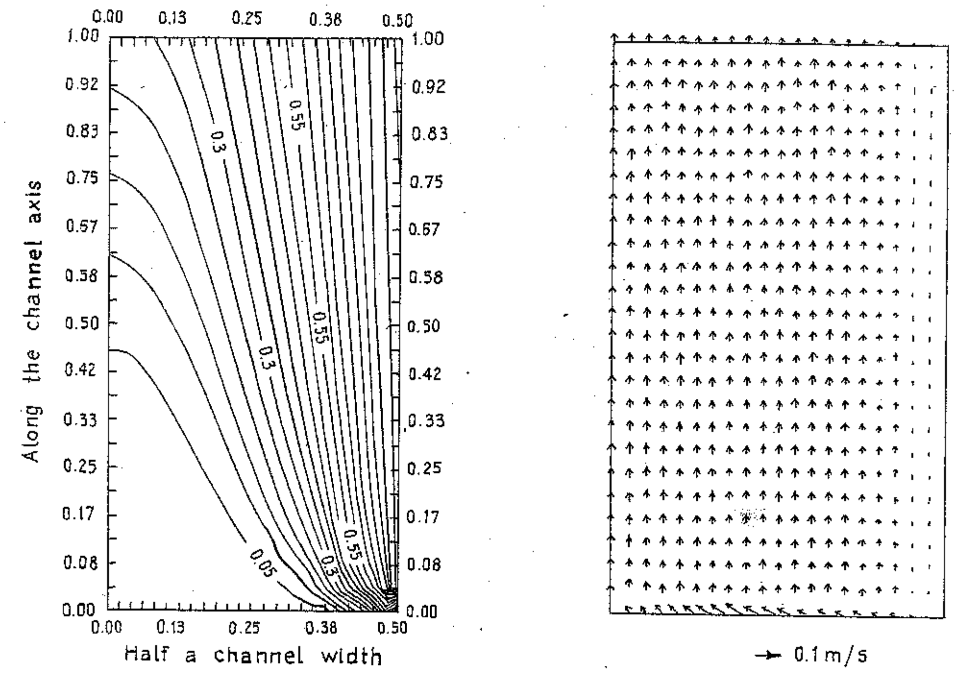
Fig. 4 **Case 4**



Isothermal lines

Velocity vectors

Fig. 5 **Case 5**



Isothermal lines Velocity vectors

Fig. 6 Case 6

By comparing the results obtained, we have got some remarks as follows:

- Cases 5–6 [2] in Table 2 and Figures 5, 6 reflect the influence of heat transfer into the channel wall , the velocity u_0 at entrance and the average heat transfer Q decreased considerably in comparison with the case 7. At the same time it is also shown that this influence will be ignored when $k_1/k \gg 1$, i.e., it turn to the case of the channel without thickness(case 7).
- Cases 1–4 present the influence of both the heat sources and the thickness of channel at different levels which are illustrated in Table 2 and Figures 1–4. The existence of heat sources under $50kW/m^3$ maybe does not change the distribution of heat very much in the channel. However, this effect will be large when the heat source is from $50kW/m^3$ and placed in the fluid (case 4, Figures 4).
- In all the cases it is seen that next to entry the flow almost is directed to the axis of channel, and the influences are almost only concentrated here, while it goes along the axis in the upper.

4. CONCLUSION

With a finite difference method the governing equations for the flow of a power law fluid in a finite vertical channel of thick walls with heat sources uniformly distributed have been solved completely. The fields of temperature and fluid velocities as well as average heat transfer illustrating numerical results have been obtained for different cases. The results showed that the influences of wall thickness and heat sources will become considerable when the substance of channel wall has the small thermal conductivity and the heat sources are large enough.

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