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A FORM OF EQUATION OF MOTION OF A MECHANICAL SYSTEM

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The one of important problems of dynamics of a multibody system is to establish automatically the equations of motion. In the present work it is constructed a form of equations of motion, which is useful for programming the problem of a multibody system, especially for applying the symbolic method in the automatical establishment of equations of motion of a multibody system.

§1. A FORM OF EQUATIONS OF MOTION OF A MECHANICAL SYSTEM

Let us consider a holonomic mechanical system of n degrees of freedom. Denote the generalized coordinates of the considered system by q_i $(i = \overline{1, n})$. Suppose that the considered system has the matrix of inertia $\underline{A}$, which is an $n \times n$ positive define symmetric matrix. The elements of this matrix depend on the generalized coordinates, i.e.

$$\underline{A} = \underline{A}(\underline{q}) \tag{1.1}$$

where $\underline{q}$ is an $n \times 1$ column matrix, which has the elements to be the generalized coordinates. The expression of the kinetic energy can be written in the form:

$$T = \frac{1}{2}\underline{\dot{q}}^T \cdot \underline{A} \cdot \underline{\dot{q}} \tag{1.2}$$

where $\underline{\dot{q}}$ is the $n \times 1$ column matrix of generalized velocities; $\underline{\dot{q}}^T$ is the transpose of the matrix of the $\underline{\dot{q}}$, i.e.

$$\underline{\dot{q}}^T = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dots, \dot{q}_n] \tag{1.3}$$

Denote the generalized forces of applied forces by $Q_i(t, q_i, \dot{q}_i)$ $(i = \overline{1, n})$ and $\underline{Q}$ is the column matrix:

$$\underline{Q}^T = [Q_1, Q_2, Q_3, \dots, Q_n] \tag{1.4}$$

To write the equations of motion we can use the equations of Lagrange of second kind:

$$\frac{d}{dt}\frac{\partial T}{\partial \underline{\dot{q}}} - \frac{\partial T}{\partial \underline{q}} = \underline{Q} \tag{1.5}$$

From these equations we have found the form of equations of motion, that is,

$$\underline{A} \cdot \underline{\ddot{q}} + \underline{\dot{q}}^* \cdot \underline{D} \cdot \underline{\dot{q}}_0 = \underline{Q} \tag{1.6}$$

where $\underline{\dot{q}}^*$ is the $n \times n^2$ matrix,

$$\underline{\dot{q}}^* = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dots & \dot{q}_n & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dots & \dot{q}_n & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & \dots & \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dots & \dot{q}_n \end{bmatrix} \tag{1.7}$$

$\underline{\dot q}_0^T$ is the $1 \times n^2$ matrix,

$$\underline{\dot q}_0^T = \left[\dot q_1 \ \dot q_2 \ \dot q_3 \ \ldots \ \dot q_n \ \ldots \ \dot q_1 \ \dot q_2 \ \dot q_3 \ \ldots \ \dot q_n \ \ldots \ \dot q_1 \ \dot q_2 \ \dot q_3 \ \ldots \ \dot q_n\right] \tag{1.8}$$

Note that $\underline{\dot q}^*$ is an $n \times n^2$ line diagonal matrix, that is the diagonal matrix, the elements of the principal diagonal are the line matrix $\dot q^T$, the remains are equal to zero, $\underline{D}(\underline{q})$ is an $n^2 \times n^2$ matrix, which is defined directly from the matrix of inertia $\underline{A}$.

By this way, the equations of motion of a mechanical system are established by the help of only the matrix of inertia $\underline{A}$ and the matrix $\underline{Q}$ of generalized forces. It is evident that the obtained form of equations of motion will be usefully applied for the finding automatically the equations of motion of a multibody system by using the programming such as the Maple or Maxima programming, etc...

§2. EXAMPLES

For the aim of illustration let us consider some following examples.

Example 1.

The physical pendulum is suspended from a slider as shown in Fig. 1. The slider has the mass of m_1 and is jointed with the frame of the ground by the spring of stiffness c. The physical pendulum has the mass of m and the moment of inertia about its mass center C of I (OC $= a$). Write the equations of motion of the pendulum.

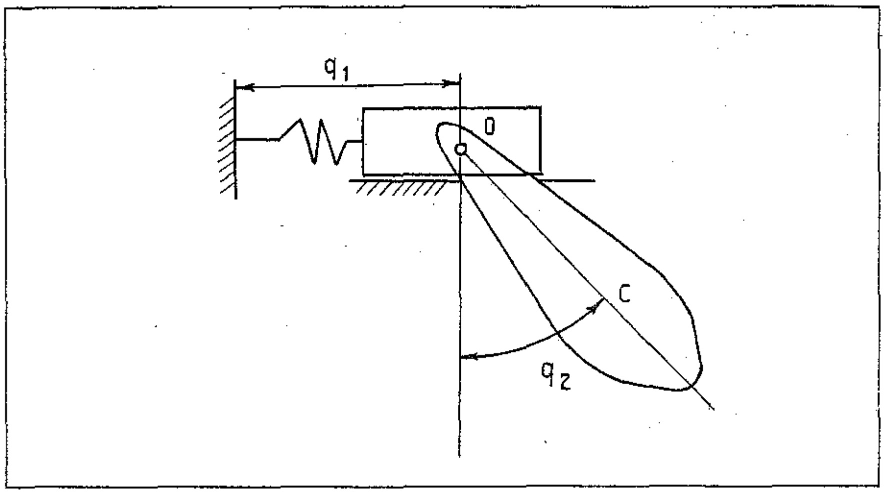

Fig. 1

Let us denote the generalized coordinates of the system by q_1 and q_2. The matrix of inertia of the considered system will be:

$$\underline{A} = \begin{bmatrix} m_1 + m_2 & m_2 a \cos q_2 \\ m_2 a \cos q_2 & 1 + m_2 a^2 \end{bmatrix}$$

$$\underline{Q}^T = \left[-cq_1, \ -m_2 g a \sin q_2 \right]$$

$$\underline{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -m_2 a \sin q_2 & 0 & 0 \\ 0 & 0 & 0 & 0.5 m_2 a \sin q_2 \\ 0 & 0 & -0.5 m_2 a \sin q_2 & 0 \end{bmatrix}$$

Applying (1.6) we obtain the equations of motion

$$(m_1 + m_2)\ddot q_1 + m_2 a \cos q_2 \ddot q_2 - m_2 a \sin q_2 \dot q_2^2 = -cq_1$$

$$m_2 a \cos q_2 \ddot{q}_1 + (I + m_2 a^2)\ddot{q}_2 = -m_2 g a \sin q_2$$

Example 2

Write the equation of motion of a hammer crush machine, shown in Fig. 2. It consists of a disk and a physical pendulum. The disk is of the radius R and the moment of inertia I_1 about the rotation axis O. The physical pendulum has the mass of m_2 and the moment of inertia about the mass center C of I_2 (AC = e).

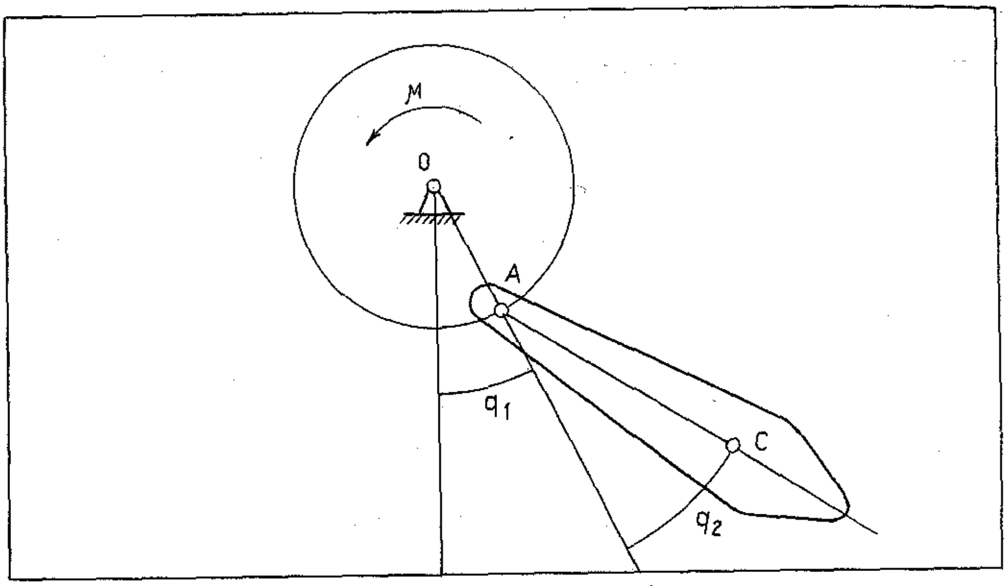

Fig. 2

The moment acts on the disk M. Let us denote the generalized coordinates of the system by q_1 and q_2, where the q_1 - is the rotation angle of the disk and the q_2 is the rotation angle of the pendulum with respect to the disk. The matrix of inertia $\underline{A}$ of the considered system will have the form:

$$A = \begin{bmatrix} I_1 + I_2 + m_2(e^2 + r^2 + 2er\cos q_2) & I_2 + m_2(e^2 + er\cos q_2) \\ I_2 + m_2(e^2 + er\cos q_2) & I_2 + m_2 e^2 \end{bmatrix}$$

The 2×1 matrix $\underline{Q}$ of generalized forces is:

$$\underline{Q}^T = [M - m_2 g(r\sin q_1 + e\sin(q_1 - q_2)),\ -m_2 g e \sin(q_1 + q_2)]$$

And the 4×4 matrix $\underline{D}$ will take the form:

$$\underline{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2m_2 er\sin q_2 & -m_2 er\sin q_2 & 0 & 0 \\ 0 & 0 & m_2 er\sin q_2 & 0.5m_2 er\sin q_2 \\ 0 & 0 & -0.5m_2 er\sin q_2 & 0 \end{bmatrix}$$

Hence, we are obtained the equations of motion:

$$\begin{aligned} &[I_1 + I_2 + m_2(e^2 + r^2 + 2er\cos q_2)]\ddot{q}_1 + [I_2 + m_2(e^2 + er\cos q_2)]\ddot{q}_2 \\ &- 2m_2 er\sin q_2 \dot{q}_1\dot{q}_2 - m_2 er\sin q_2 \dot{q}_2^2 = M - m_2 g[r\sin q_1 + e\sin(q_1 - q_2)] \end{aligned}$$

$$[I_2 + m_2(e^2 + er\cos q_2)]\ddot{q}_1 + (I_2 + m_2 e^2)\ddot{q}_2 + m_2 er\sin q_2 \dot{q}_1^2 = -m_2 g e\sin(q_1 + q_2)$$

CONCLUSIONS

The discovered form of equations of motion allows us to write the equations of motion of a mechanical system with the help of only the matrix of inertia of mechanical system and the matrix of generalized forces. It is important for writing the equations of motion of a multibody system.

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MỘT DẠNG CỦA PHƯƠNG TRÌNH CHUYỂN ĐỘNG CỦA HỆ CƠ HỌC

Trong bài này đã đưa ra một dạng phương trình chuyển động thích hợp cho việc thiết lập tự động các phương trình chuyển động của một hệ cơ học nói chung và cơ hệ nhiều vật nói riêng trên máy tính cá nhân. Để viết phương trình chuyển động của một cơ hệ chỉ cần biết ma trận quán tính của cơ hệ và các lực suy rộng của cơ hệ. Đặc biệt dạng phương trình nêu trên được sử dụng rất có hiệu quả khi sử dụng các phần mềm như Chương trình Maple, Chương trình Maxima để thiết lập phương trình chuyển động của cơ hệ một cách trực tiếp bằng các biểu thức chữ.

FREE CONVECTION FLOW IN A VERTICAL THIN CYLINDER...

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CHUYỂN ĐỘNG ĐỐI LƯU NHIỆT CỦA CHẤT LỎNG QUY LUẬT MŨ TRONG KÊNH TRỤ MỎNG THẲNG ĐỨNG VỚI CHIỀU CAO HỮU HẠN

Xét chuyển động đối lưu nhiệt của chất lỏng quy luật mũ trong kênh trụ mỏng thẳng đứng có chiều cao hữu hạn với nhiệt độ ở thành ngoài cho trước. Bài toán được giải bằng phương pháp sai phân hữu hạn. Kết quả tính toán được so sánh với nghiệm tiệm cận. Đã đưa ra điều kiện để bỏ qua bề dày thành kênh.