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INTERACTION BETWEEN PARAMETRIC AND FORCED OSCILLATIONS IN FUNDAMENTAL RESONANCE

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In nonlinear systems, the interaction between different oscillations is complicated and has attracted the attention of a lot of researches [1, 2]. Interesting results have been obtained, some aspects of this phenomenon can be found in a recent work [3].

The present paper is devoted to examine the interaction between parametric and forced oscillations in fundamental resonances. Some remark about the resolution of the equations determining the stationary oscillations will be given, some particularities of the resonance curve will be described.

§1. SYSTEM UNDER CONSIDERATION AND THE AVERAGING METHOD

Let us consider a quasi-linear oscillating system governed by the differential equation

$$\ddot{x} + \omega^2 x = \varepsilon\{ -h\dot{x} - \gamma x^3 + \Delta x + 2px\cos 2\omega t + q\cos(\omega t + \sigma)\} \tag{1.1}$$

where x - an oscillatory variable, $\varepsilon > 0$ - a small parameter, $h \geq 0$ - the damping viscous coefficient, γ - the cubic nonlinearity coefficient, $2p > 0$, $q > 0$ and 2ω, ω - intensities and frequencies of the parametric and external excitations, respectively, $\Delta = (\omega^2 - 1)$ - the detuning parameter (1 - the natural frequency), $\sigma (0 \leq \sigma < 2\pi)$ - the dephase angle between two excitations.

Introducing slowly varying variables a, θ (amplitude and dephase of the oscillatory regime).

$$x = a\cos\varphi, \quad \dot{x} = -a\omega\sin\varphi, \quad \varphi = \omega t + \theta \tag{1.2}$$

we establish the averaged equations:

$$\begin{aligned} \dot{a} &= -\frac{\varepsilon}{2\omega}\{h\omega a + pa\sin 2\theta + q\sin(\theta - \sigma)\} \\ \dot{\theta} &= -\frac{\varepsilon}{2\omega a}\Big\{\Big(\Delta - \frac{3\gamma}{4}a^2\Big)a + pa\cos 2\theta + q\cos(\theta - \sigma)\Big\} \end{aligned} \tag{1.3}$$

Let (a_0, θ_0) be the amplitude and the dephase of the stationary oscillation. By vanishing the right hand sides of (1.3), we obtain two algebraic - trigonometrical equations for determining (a_0, θ_0):

$$\begin{aligned} &ha\omega + pa\sin 2\theta + q\sin(\theta - \sigma) = 0 \\ &\Big(\Delta - \frac{3\gamma}{4}a^2\Big)a + pa\cos 2\theta + q\cos(\theta - \sigma) = 0 \end{aligned} \tag{1.4}$$

or, in equivalent form:

$$ha\omega\sin\theta - \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) - p\Big]a\cos\theta = -q\cos\sigma \tag{1.5a}$$

$$\Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]a\sin\theta + ha\omega\cos\theta = q\sin\sigma \tag{1.5b}$$

As usual, first, (1.5) will be considered as two linear algebraic equations of two unknowns $u = \sin\theta$, $v = \cos\theta$. Then using trigonometrical formulae (for instance $\sin^2\theta + \cos^2\theta = 1$) the amplitude - frequency relationship will be obtained

Two cases must be distinguished

1. The "ordinary" case where the determinant (of the algebraic linear equations (1.5a)) is different to zero

$$\overline{D} = \begin{vmatrix} ha\omega & -\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) - p\right]a \\ \left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) + p\right]a & ha\omega \end{vmatrix} = a^2 D \neq 0$$

or (since $q \neq 0$, the system considered does not admit the equilibrium regime $a = 0$):

$$D = \left(\frac{3\gamma}{4}a^2 - \Delta\right)^2 + h^2\omega^2 - p^2 \neq 0$$

2. The "critical" case where $\overline{D} = 0$ or $D = 0$

To illustrate this remark, we shall examine in detail the oscillating system without damping.

§2. RESONANCE CURVE OF THE OSCILLATING SYSTEM WITHOUT DAMPING

For the system without damping, $h = 0$, the equations (1.5) as the determinant (1.6) become more simple

$$\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) - p\right]a\cos\theta = q\cos\sigma \tag{2.1a}$$

$$\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) + p\right]a\sin\theta = q\sin\sigma \tag{2.1b}$$

$$D = \left(\frac{3\gamma}{4}a^2 - \Delta\right)^2 - p^2 = \left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) - p\right]\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) + p\right] \tag{2.2}$$

It is noted that, in the plane (Δ, a^2), $D = 0$ is just the resonance curve of the pure - parametrically - excited system ($q = 0$) which degenerates into two straight lines:

$$D_1 : \frac{3\gamma}{4}a^2 = \Delta + p \quad \text{and} \quad D_2 : \frac{3\gamma}{4}a^2 = \Delta - p \tag{2.3}$$

1. If $D \neq 0$ (the plane (Δ, a^2) after excluding D_1 and D_2), we have:

$$\cos\theta = \frac{q\cos\sigma}{\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) - p\right]a}\,; \quad \sin\theta = \frac{q\sin\sigma}{\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) + p\right]a} \tag{2.4}$$

and the amplitude - frequency relationship is of the form:

$$W_1 = \frac{q^2\cos^2\sigma}{\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) - p\right]^2 a^2} + \frac{q^2\sin^2\sigma}{\left[\left(\frac{3\gamma}{4}a^2 - \Delta\right) + p\right]^2 a^2} - 1 = 0 \tag{2.5}$$

(2.5) give only some "parts" of the resonance curve, the "parts" lying out of the straight lines D_1 and D_2.

2. If $D = 0$ (either in D_1 or in D_2) the two algebraic equation (2.1) can be resolved when $\sigma = 0, \pi/2, \pi, 3\pi/2$ Indeed

2a. In D_1 : $\frac{3\gamma}{4}a^2 = \Delta + p$. from (2.1a) we deduce $\sigma = \pi/2,\ 3\pi/2$ and $\nu_1 = \cos\theta$ arbitrary, then, from (2.1b)

$$u_1 = \sin\theta = \frac{\pm q}{2pa} \tag{2.6}$$

The corresponding algebraic - trigonometrical equations (2.1) admit the solution:

$$\frac{3\gamma}{4}a^2 = \Delta + p, \quad a^2 \geq \frac{q^2}{4p^2}, \quad \sin\theta = \frac{\pm q}{2pa}, \quad \cos\theta = \pm\sqrt{1-\sin^2\theta} \tag{2.7}$$

2b. In D_2 : $\frac{3\gamma}{4}a^2 = \Delta - p$, from (2.1b), we deduce $\sigma = 0,\ \pi$ and $u_2 = \sin\theta$ arbitrary, then, from (2.1a):

$$\nu_2 = \cos\theta = \frac{\pm q}{2pa} \tag{2.8}$$

The corresponding algebraic - trigonometrical equations (2.1) in this subcase admit the solution:

$$\frac{3\gamma}{4}a^2 = \Delta - p, \quad a^2 \geq \frac{q^2}{4p^2}, \quad \cos\theta = \frac{\pm q}{2pa}, \quad \sin\theta = \pm\sqrt{1-\cos^2\theta} \tag{2.9}$$

Thus:

1. If $\sigma \neq 0,\ \pi/2,\ \pi,\ 3\pi/2$, the resonance curve - the entire resonance curve - is given by (2.5)
2. If $\sigma = 0,\ \pi$, the resonance curve consists of two branches : - the first branch is given by (2.5) - the second one is given by (2.9).
3. If $\sigma = \pi/2,\ 3\pi/2$, the resonance curve consists of two branches too : - the first also given by (2.5) and the second one by (2.7)

In figure 1, the heavy line represents the resonance curve corresponding to the values $\sigma = 0$, $\gamma = 0,8$, $p = 0,25$, $q = 0,2$, given by (2.5) and (2.9).

For the same values γ, p, q, the resonance curve (2.5) corresponding to $\sigma = \pi/4$ is plotted in figure 2.

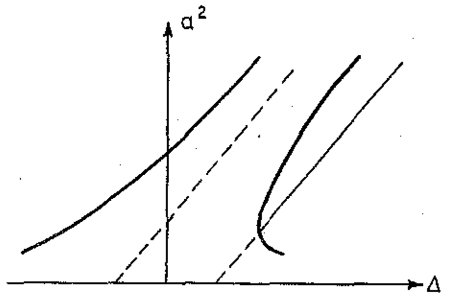

Fig. 1

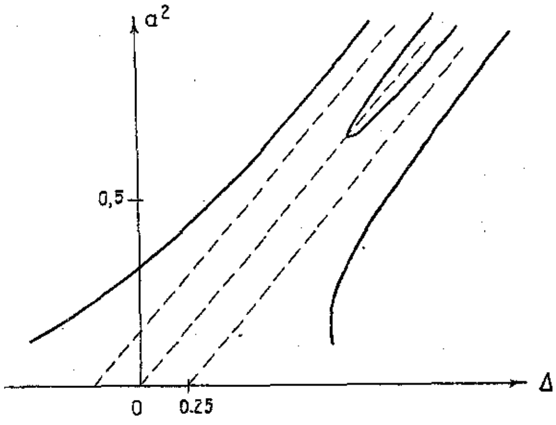

Fig. 2

Remark - (2.5) is often replaced by:

$$W = q^2\left\{\left[\left(\frac{3\gamma}{4}a^2-\Delta\right)+p\right]^2\cos^2\sigma+\left[\left(\frac{3\gamma}{4}a^2-\Delta\right)-p\right]^2\sin^2\sigma\right\}-a^2\left[\left(\frac{3\gamma}{4}a^2-\Delta\right)^2-p^2\right]^2 = 0 \tag{2.10}$$

It is necessary to note that (2.5) and (2.10) are equivalent only if $D \neq 0$. In the critical case where $D = 0$, the relationship (2.10) gives us also "parts" (2.7), (2.9) but not the inequality $a^2 \geq q^2/4p^2$.

§3. RESONANCE CURVE OF THE SYSTEM WITH DAMPING

For the system with damping, $h > 0$.

1. If $D \neq 0$, from (1.5), we deduce:

$$\begin{aligned} \sin\theta &= -\frac{q}{aD}\Big\{h\omega\cos\sigma - \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) - p\Big]\sin\sigma\Big\} \\ \cos\theta &= \frac{q}{aD}\Big\{h\omega\sin\sigma + \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]\cos\sigma\Big\} \end{aligned} \tag{3.1}$$

and the amplitude - frequency relationship is of the form:

$$W_1 = \frac{q^2}{a^2D^2}\Big\{\Big(h\omega\cos\sigma - \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) - p\Big]\sin\sigma\Big)^2 + \Big(h\omega\sin\sigma + \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]\cos\sigma\Big)^2\Big\} - 1 = 0 \tag{3.2a}$$

or

$$W_1 = \frac{q^2}{a^2D^2}\Big\{\Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\cos 2\sigma\Big]^2 + \big[h\omega + p\sin 2\sigma\big]^2\Big\} - 1 = 0 \tag{3.2b}$$

As it has been in §2, under condition $D = 0$, (3.2) can be replaced by

$$W = q^2\Big\{\Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\cos 2\sigma\Big]^2 + \big[h\omega + p\sin 2\sigma\big]^2\Big\} - a^2D^2 = 0 \tag{3.3}$$

2. If

$$D = \Big(\frac{3\gamma}{4}a^2 - \Delta\Big)^2 + h^2\omega^2 - p^2 = 0 \tag{3.4}$$

the two linear algebraic equations (1.5) is in "critical" situation respectively by $[\overline{D}]$ and $[\tilde{D}]$, we denote the coefficient matrix and the extended one:

$$[\overline{D}] = \begin{vmatrix} ha\omega & -\Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) - p\Big]a \\ \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]a & ha\omega \end{vmatrix} \tag{3.5}$$

$$[\tilde{D}] = \begin{vmatrix} ha\omega & -\Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) - p\Big]a & -q\cos\sigma \\ \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]a & ha\omega & q\sin\sigma \end{vmatrix} \tag{3.6}$$

Since $h > 0$, $\omega \approx 1$, if $D = 0$ we have:

$$\text{rang } [\overline{D}] = 1 \tag{3.7}$$

Hence, the algebraic equations (1.5) can be resolved only if:

$$\text{rang } [\tilde{D}] = 1 \tag{3.8}$$

This requirement leads to two equations:

$$\begin{vmatrix} h\omega a & -q\cos\sigma \\ \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]a & q\sin\sigma \end{vmatrix} = aq\Big\{h\omega\sin\sigma + \Big[\Big(\frac{3\gamma}{4}a^2 - \Delta\Big) + p\Big]\cos\sigma\Big\} = 0 \tag{3.9a}$$

$$\begin{vmatrix} -\left[\left(\frac{3\gamma}{4}a^2-\Delta\right)+p\right]a & -q\cos\sigma \\ h\omega a & q\sin\sigma \end{vmatrix} = aq\left\{h\omega\cos\sigma - \left[\left(\frac{3\gamma}{4}a^2-\Delta\right)-p\right]\sin\sigma\right\} = 0 \qquad (3.9b)$$

or, in equivalent form:

$$h\omega + p\sin 2\sigma = 0 \qquad (3.10a)$$

$$\left(\frac{3\gamma}{4}a^2-\Delta\right) + p\cos 2\sigma = 0 \qquad (3.10b)$$

It is easy to prove that the equations (3.10) admit an unique solution (Δ_*, a_*^2) satisfying (3.4). It means that, in the curve $D = 0$ (the resonance curve of the pure parametrically excited system). there always exists a point $C_*(\Delta_*, a_*^2)$ at which the algebraic equations (1.5) can be resolved and can be reduced - for instance - to (1.5a):

$$h\omega_* a_* u - \left[\left(\frac{3\gamma}{4}a_*^2-\Delta_*\right)-p\right]av = -q\cos\sigma \qquad (3.11)$$

However, C_* is acceptable only if $a_*^2 > 0$ and if Δ_* is in the neighbourhood of zero (in the resonance region), if not, C_* must be rejected. Moreover, even in the case where C_* is acceptable, it may be that the trigonometrical equation corresponding to (3.11) i.e. the following one

$$h\omega_* a_* \sin\theta - \left[\left(\frac{3\gamma}{4}a_*^2-\Delta_*\right)-p\right]a_*\cos\theta = -q\cos\sigma \qquad (3.12)$$

does not admit any solution. Trigonometrical solutions exist only if:

$$a_*^2\left\{h^2\omega_*^2 + \left[\left(\frac{3\gamma}{4}a_*^2-\Delta_*\right)-p\right]^2\right\} \geq q^2\cos^2\sigma. \qquad (3.13)$$

or, by using (3.13):

$$a_*^2 \geq \frac{q^2}{4p^2} \qquad (3.14)$$

Thus, if (3.14) is satisfied, the point C_* corresponds to determinated stationary oscillations so that C_* is the second "branch" of the resonance curve.

The point C_* plays a special role:

$$W\big|_{C_*} = 0, \quad \frac{\partial W}{\partial\Delta}\bigg|_{C_*} = 0, \quad \frac{\partial W}{\partial a_*}\bigg|_{C_*} = 0 \quad \text{i.e. } C_* \text{ is a critical point} \qquad (3.15)$$

of the curve C determined by the relationship $W = 0$

$$\begin{aligned} \frac{\partial^2 W}{\partial\Delta^2}\bigg|_{C_*} &= 2\left\{q^2 + \frac{h^2}{4\omega_*^2} - h^4a_*^2 - 4h^2pa_*^2\cos 2\sigma - 4p^2a_*^2\cos^2 2\sigma\right\} \\ \frac{\partial^2 W}{\partial\Delta\partial a^2}\bigg|_{C_*} &= 2\left(\frac{3\gamma}{4}\right)\{-q^2 + 2h^2pa_*^2\cos 2\sigma + 4p^2a_*^2\cos^2 2\sigma\} \\ \frac{\partial^2 W}{\partial(a^2)^2}\bigg|_{C_*} &= 2\left(\frac{3\gamma}{4}\right)^2\{q^2 - 4p^2a_*^2\cos^2 2\sigma\} \end{aligned} \qquad (3.16)$$

so that, in the neighbourhood of C_*, by neglecting the terms of powers greater than 2 relative to $x = \Delta - \Delta_*$, $y = a^2 - a_*$, the curve C is given by:

$$\begin{aligned} W = \left(\frac{3\gamma}{4}\right)^2\{q^2 - 4p^2a_*^2\cos^2 2\sigma\}y^2 + 2\left(\frac{3\gamma}{4}\right)\{4p^2a_*^2\cos^2 2\sigma + 2ph^2a_*^2\cos 2\sigma - q^2\}yx + \\ + \left\{q^2 + \frac{h^2q^2}{4\omega_*^2} - h^4a_*^2 - 4ph^2a_*^2\cos 2\sigma - 4p^2a_*^2\cos^2 2\sigma\right\}^2 = 0 \end{aligned} \qquad (3.17)$$

The quadratic form of the left hand side of (3.17) has as discriminant:

$$D = \left(\frac{3\gamma}{4}\right)^2 \{4pa_*^2 \cos^2 2\sigma + 2ph^2 a_*^2 \cos 2\sigma - q^2\}^2 -$$
$$- \left(\frac{3\gamma^2}{4}\right)^2 \{q^2 - 4p^2 a_*^2 \cos^2 2\sigma\} \left\{q^2 + \frac{h^2 q^2}{4\omega_*^2} - h^4 a_*^2 - 4ph^2 a_*^2 \cos 2\sigma - 4p^2 a_*^2 \cos^2 2\sigma\right\} =$$
$$= \left(\frac{3\gamma}{4}\right)^2 \frac{h^2 q^2}{4\omega_*^2} (4p^2 a_*^2 - q^2) \tag{3.18}$$

- If $a_*^2 < q^2/4p^2$, then $D < 0$, therefore C_* is an isolated point of the curve C determined by $W = 0$ but does not correspond to any stationary oscillation. In this case, the resonance curve is obtained from C after excluding C_*.

- If $a_*^2 = q^2/4p^2$, then $D = 0$, therefore in the neighbourhood of C_*, there exists two branches of C, connecting themselves at C_*, having at C_* the common tangent and form so a sharp cap (this remark can be deduced by retaining the terms of powers 3 in the expression of W in the neighbourhood of C_*).

- If $a_*^2 > q^2/4p^2$, then $D > 0$, therefore in the neighbourhood of C_*, there exists two branches of C, intersecting themselves at C_*. In two last cases. C_* corresponds to determinated stationary oscillations and the resonance curve is given by C ($W = 0$) including C_*.

Let us fix $\gamma = 0.8$, $p = 0.25$, $h = 0.22$. For $\sigma = 0$, from (3.10) we deduce $\Delta_* = -1$, $a_*^2 = -3/3\gamma(1+p)$ so that C_* is not acceptable. The resonance curves corresponding to $q = 0.007$ and $q = 0.05$ are presented in the figure 3 by the curves a, b respectively. If q is small enough, the resonance curve (a) consists of two branches, separated by the curve $D = 0$ (the resonance curve of the pure - parametrically excited system). Increasing q, the branch lying under $D = 0$ is restricted then disappears and the resonance curve consists of an unique branch lying upon $D = 0$.

For $\sigma = 3\pi/4$, from (3.11) we deduce $\frac{3\gamma}{4} a_*^2 = \Delta_* = \frac{p^2 - h^2}{h^2} \approx 0.29$ and the critical point C_* is acceptable. In figure 4 the curves a, b represent the resonance curves corresponding to the values $q = 0.1$, $q = 0.2$, respectively: the resonance curve consists of two branches, connecting at C_*, lying respectively under and upon the curve $D = 0$; increasing q, the branched lying under $D = 0$ is restricted then disappears

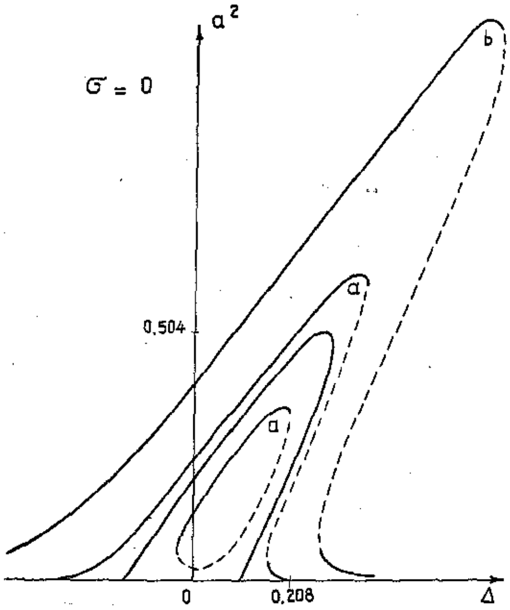

Fig. 3

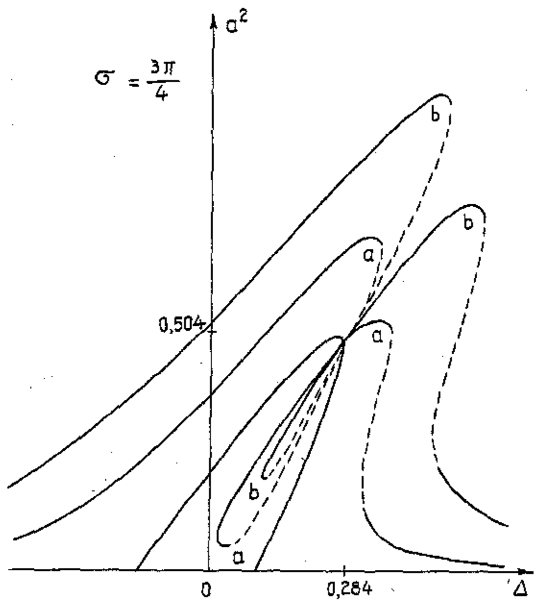

Fig. 4

§4. STABILITY OF STATIONARY OSCILLATIONS

The stability study of the stationary oscillations (a_0, θ_0) will be based on the following variational equations:

$$\begin{aligned}\delta\dot{a} &= -\frac{\varepsilon}{2\omega}\{h\omega + p\sin 2\theta_0\}\delta a - \frac{\varepsilon}{2\omega}\{2pa_0\cos 2\theta_0 + q\cos(\theta_0 - \sigma)\}\delta\theta \\ \delta\dot{\theta} &= -\frac{\varepsilon}{2\omega a_0}\Big\{p\cos 2\theta_0 + \Big(\Delta - \frac{9\gamma}{4}a_0^2\Big) + p\cos 2\theta_0\Big\}\delta a + \frac{\varepsilon}{2\omega a_0}\{2pa_0\sin 2\theta_0 + q\sin(\theta_0 - \sigma)\}\delta\theta\end{aligned} \tag{4.1}$$

where $\delta a = a - a_0$, $\delta\theta = \theta - \theta_0$ are the variations of the amplitude and the dephase, respectively.

Using (1.4), we write (4.1) in the form:

$$\begin{aligned}\delta\dot{a} &= -\frac{\varepsilon}{2\omega}\{h\omega + p\sin 2\theta_0\}\delta a - \frac{\varepsilon}{2\omega}\Big\{pa_0\cos 2\theta_0 + \Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big)a_0\Big\}\delta\theta \\ \delta\dot{\theta} &= -\frac{\varepsilon}{2\omega a_0}\Big\{p\cos 2\theta_0 - \Big(\frac{9\gamma}{4}a_0^2 - \Delta\Big)\Big\}\delta a - \frac{\varepsilon}{2\omega}\{h\omega - p\sin 2\theta_0\}\delta\theta\end{aligned} \tag{4.2}$$

and the characteristic equation can be established:

$$\rho^2 + \varepsilon h\rho + \frac{\varepsilon^2}{4\omega^2}\Big\{D + 2\Big(\frac{3\gamma}{4}\Big)a_0^2\Big[\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) + p\cos 2\theta_0\Big]\Big\} = 0 \tag{4.3}$$

The first stability condition $h > 0$ is satisfied for the system with damping, the second one is given by the inequality:

$$D + 2\Big(\frac{3\gamma}{4}\Big)a_0^2\Big[\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) + p\cos 2\theta_0\Big] > 0 \tag{4.4}$$

Using again (1.4) and (3.3) we find

$$p\cos 2\theta_0 = \Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) - \frac{q^2}{a_0^2 D}\Big[\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) + p\cos 2\theta_0\Big] \tag{4.5}$$

So, (4.4) can be written as:

$$\begin{aligned}&D + 2\Big(\frac{3\gamma}{4}a_0^2\Big)\Big\{2\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) - \frac{q^2}{a^2D^2}\Big[\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) + p\cos 2\sigma\Big]\Big\} = \\ &= D + 4\Big(\frac{3\gamma}{4}a_0^2\Big)\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) - 2\Big(\frac{3\gamma}{4}\Big)\frac{q^2}{D}\Big[\Big(\frac{3\gamma}{4}a_0^2 - \Delta\Big) + p\cos 2\sigma\Big] = -\frac{1}{D}\frac{\partial W}{\partial a_0^2} > 0\end{aligned} \tag{4.6}$$

Analyzing the signs of D and $\partial W/\partial a^2$, from (4.6) we can easily determine in the resonance curve, the "parts" corresponding to stable stationary oscillations and those corresponding to unstable stationary oscillations.

Since $\left.\dfrac{\partial W}{\partial a^2}\right|_a = 0$, the stability of the stationary oscillations corresponding to the critical point C_* cannot be deduced from the variational equations (4.1).

CONCLUSION

Using the asymptotic method, we have examined the interaction between parametric and forced oscillations in fundamental resonance. We have concentrated ourattention on the critical situation and the critical point in the resonance curve has been analyzed in detail. Depending on this critical point, diverse shapes of the resonance curve have been obtained.

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TƯƠNG TÁC GIỮA DAO ĐỘNG THÔNG SỐ VÀ CƯỠNG BỨC Ở CỘNG HƯỞNG CƠ BẢN

Bài báo xét tương tác giữa dao động thông số và cưỡng bức ở hệ á tuyến khi cả hai đều ở cộng hưởng cơ bản (tương ứng tần số kích động lân cận gấp đôi và bằng tần số riêng). Đã phân biệt trường hợp thường và trường hợp tới hạn khi giải phương trình đại số - lượng giác để xác định biên độ và pha của dao động dừng. Đã phân tích điểm lạ trên đường cộng hưởng và thấy mối liên quan giữa tính chất điểm lạ với các dạng đường cộng hưởng.

CONVECTION IN BINARY MIXTURE ...

(tiếp trang 4)

The publication is completed with financial support from the National Basic Research Program in Natural Sciences.

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CHUYỂN ĐỘNG ĐỐI LƯU TRONG HỖN HỢP HAI THÀNH PHẦN CÓ MẶT THOÁNG

Trong bài báo chứng minh định lý tồn tại và duy nhất nghiệm suy rộng của bài toán về chuyển động đối lưu nhiệt trong hỗn hợp hai thành phần có mặt thoáng