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ON THE PROBLEM OF OPTIMAL STRUCTURE DESIGN SUBJECTED TO REGULAR LIFETIME CTITERION

NGUYEN VAN PHO
Hanoi University of Civil Engineering

SUMMARY. The objective function of problem of optimal structure design can be choosen differently. For example the objective function can be the weight, the net cost, the regular resistance, the regular reliability, an so on.

In this paper the problem of optimal structure design, subjected to regular lifetime condition for all elements is considered.

The author sets up the problem, proposes an iterative method for solving and apply the obtained results to a simple example.

§1. INTRODUCTION

Up to now the problem of optimal structure design has been paid attention by many authors [1, 2, 3,...]

According to the goal the problem is aimed at, the objective function is choosen differently.

It's well known that the different structural components or the different parts of a single contruction are generally made of different sorts of materials, having different load conditions, suffering from different erosion environment, etc.

Therfore these components or parts have their resistance capacity decreasing upon the time with different laws and different intensities. As a result, the lifetime (the untilisation time) of the structural components will not be the same. To get security in the exploitation one has recourse to the way replacing or strengthening.

It is regrettable, however, that some parts of a construction can not be replaceable or their replacing are too expensive, moreover the resulted stoppage of exploitation is unacceptable by the manager, because of heavy economic loss. Thiss gives rise to the problem of designing different parts of a construction, which have the same or likely the same lifetime. These are called constructions with regular lifetime. The problem of regular lifetime is a class of optimization problems. Here the lifetime means the time within it the exploitation of the construction is according to a given reliability.

The criterion of regular lifetime is like that of regular resistance [4]. However the regular resistance problem considers only at the initial moment (the moment of the design), while the regular lifetime problem also takes into account other factors, intervening during the process of exploitation, like the crosion, the fatigue, and others.

In this article the erosion laws of irons, the low of stiffness decreasing of brick stone are not studied. It is assumed that, owning to the results of observation and measurement data, these laws, depending upon the time, are allready established [5]. Our problem is, these laws being given, to study the optimal structure design subjected to regular lifetime criterion.

It is noteworthy that, if the construction includes parts, not expensive, replaceable easily without breaking the exploitation, then these parts need not be considered in the problem. In this case one is concerned with the problem of optimal structure design subjected to the criterion of regular lifetime for a part of the elements. The setting up and solving of both problems are the like.

In this article the author sets up the problem of optimal design subjected to regular lifetime condition, proposes the method of its solution, and gives an illustrative example.

§2. DECREASE OF RESISTANCE CAPACITY OF THE STRUCTURAL COMPONENTS

2.1. The decrease law

Suppose that the construction is divided into parts, made of different materials under different load conditions. Let the lifetime of the construction be given a number T of year, according to the design obligation. Let the characteristies of the resistance capacity of the construction (dimensions, sections, constants of the materials,...) be the vector $\vec{K}$

$$\vec{K} = \{K_i\}$$
$$K_j = K_j(h_i) \qquad j = \overline{1,n}$$
$$h_i - \text{the design parameters.}$$

Like in [5], $\vec{K}$ can here, for example, be the form

$$\vec{K} = \vec{\phi}(\vec{K}_0, \vec{\lambda}, t) \tag{2.1}$$

where $\vec{K}_0 = \{K_i^{(0)}\}$ is the characteristic vector of the initial resistance capacity (at the design moment), t - the time, $\vec{\lambda}$ - the vector of experimental constants.

Note that $\vec{K}_0$ is vector - function of the design parameters $\vec{h}$

$$\vec{h} = \{h_i\}$$
$$\vec{K}_0 = \{K_j^{(0)}(h_i^{(0)})\}$$
$$\vec{K} = \vec{K}(\vec{h}, \vec{\lambda}, t)$$

The law (2.1) are built up from the experimental data, with a statistical treatment by the method of least squares. Whenever the dates are not yet enough to define (2.1) one seeks for a reasonable way of completion or one advances a prevision law for the purpose of designing [5].

2.2. Determination of the initial resistance capacity

With a give T and for a concrete sort of materials one relies on the limite state of norms to define the criterial resistance capacity $\vec{K}_{limit}$,

$$\vec{K}_{limit} = \{\vec{K}_{jlimit}\} = \vec{\phi}(\vec{K}_0, \vec{\lambda}, T)$$

$\vec{K}_{jlimit}$ characterizes the minimal permissible resistance capacity (at the end of the exploitation period of T years); it corresponds to a determined reliability, and so on it depends upon the degree of importance of the construction.

From T and $\vec{K}_{limit}$, one shall be able to determine $\vec{K}_0$. The methods to determine $\vec{K}_0$ vary with respect to the categries of structions one is concerned with. Once $\vec{K}_0$ is determined, the problem of optimal structure design subjected to regular lifetime condition is considered as solved.

§3. THE METHOD OF SOLUTION OF THE PROBLEM

3.1. The general problem

Suppose that the problem has been set up, and the equations the finite element method are [6]

$$D\vec{U} = \vec{F} \tag{3.1}$$

where

$D = D(\vec{K}_0) = D(\vec{h})$ is the stiffness matrix,

$\vec{U} = \{U_i\}$ is the nodal vector of displacement,

$\vec{F} = \{F_i\}$ is the nodal force vector.

One still has to add to (3.1) the regular lifetime condition

$$\vec{K}_{limit} = \vec{\phi}(\vec{K}_0, \vec{\lambda}, T) \tag{3.2}$$

From the system of equations (3.1), (3.2) one has to determine $\vec{U}$, $\vec{K}_0$ and the design parameters $h_i^{(0)}$. Obviously with the $h_i^{(0)}$ determined as above one still has to verify whether the so determined $\vec{U}$ will satisfy standards.

Note that

$$\begin{aligned} D &= D(h_i^{(0)}) \\ \vec{K}_0 &= \vec{K}_0(\vec{h}_i^{(0)}) \\ \vec{K}_{limit} &= \vec{K}_{limit}(\vec{h}_i^{(0)}, \vec{\lambda}, T) \end{aligned} \tag{3.3}$$

In general, equation (3.3) are non-linear or transcendental functions, D, $\vec{K}_0$, $\vec{K}_{limit}$ can be multivalued functions of $h_i^{(0)}$; the latter means that to a set of D, $\vec{K}_0$, $K_{limit} \ldots$ of values of $h_i^{(0)}$. For these reasons the finding of an exact solution of the system (3.1), (3.2) is very difficult; one could find the approximate solution in the concrete cases. In what follows author propposes an iterative method to find the approximate solution of (3.1), (3.2), and shows some particular cases the problem can be solved part in an easier manner.

3.2. The method of solution

a. The general case. The problem of structural design with regular lifetime can be solved, following the procedure as illustrated in fig. 1 (It is a block-scheme of the computation). It is required that whenever one changes the design parameters, there arises a new distribution of internal forces, which makes changing the lifetime of the elements or parts of the construction. Therefore it must be used iteration process.

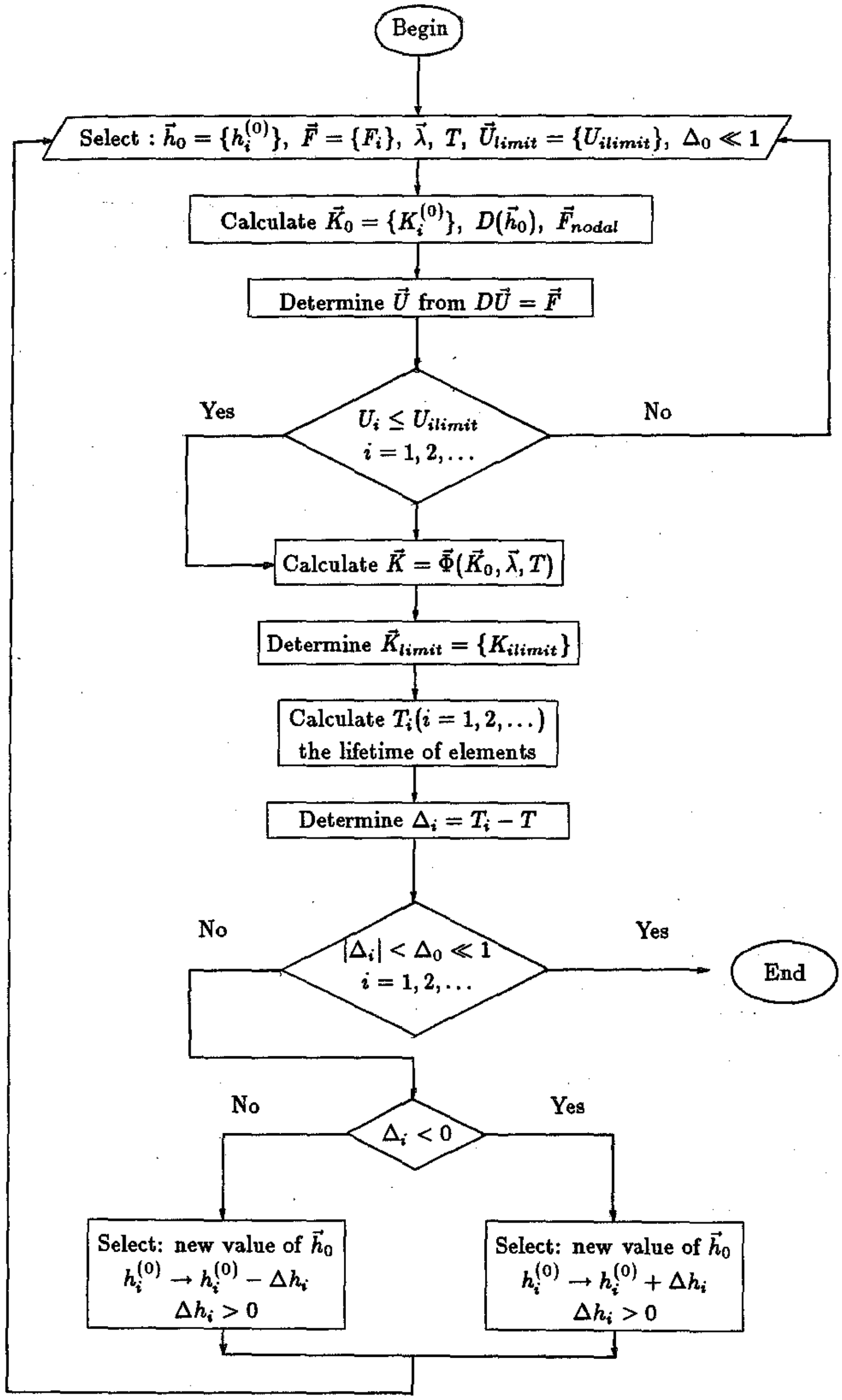
The essence of this iteration process is to strengthen the parts whose the lifetimes remain lower than they had been fixed, while to relax the parts whose the lifetimes remain higher than they had been fixed. This idea was used in [3] by Y. M. Xie and G. P. Steven. This method is like that used in the approximate solution of the problem with regular resistance criterion, and in this way the solution of the problem (3.1), (3.2) is realizable.

b. The particular cases. In the case where K_{limit} can be choosen beforhand, the solution of the problem turns back the procedure given in fig. 2

In the case where the structure can be divided into parts and the problem can be solved successively from a part to the next one, then the results obtained in a part will serve as entrance date for the computation in the next part (fig. 2).

§4. EXAMPLE

To have a comparison with the problem of regular resistance we conside a brick column made of different materials of the parts, subject to an axial pressure. without loss of generality let the column consist of three parts, having different cross sections and made of different materials (fig. 3).



Begin
Select : $\vec{h}_0 = \{h_i^{(0)}\}$, $\vec{F} = \{F_i\}$, $\vec{\lambda}$, T, $\vec{U}_{limit} = \{U_{ilimit}\}$, $\Delta_0 \ll 1$
Calculate $\vec{K}_0 = \{K_i^{(0)}\}$, $D(\vec{h}_0)$, $\vec{F}_{nodal}$
Determine $\vec{U}$ from $D\vec{U} = \vec{F}$
Yes
$U_i \leq U_{ilimit}$
$i = 1, 2, \ldots$
No
Calculate $\vec{K} = \vec{\Phi}(\vec{K}_0, \vec{\lambda}, T)$
Determine $\vec{K}_{limit} = \{K_{ilimit}\}$
Calculate $T_i (i = 1, 2, \ldots)$
the lifetime of elements
Determine $\Delta_i = T_i - T$
No
$|\Delta_i| < \Delta_0 \ll 1$
$i = 1, 2, \ldots$
Yes
End
No
$\Delta_i < 0$
Yes
Select: new value of $\vec{h}_0$
$h_i^{(0)} \to h_i^{(0)} - \Delta h_i$
$\Delta h_i > 0$
Select: new value of $\vec{h}_0$
$h_i^{(0)} \to h_i^{(0)} + \Delta h_i$
$\Delta h_i > 0$

Fig. 1

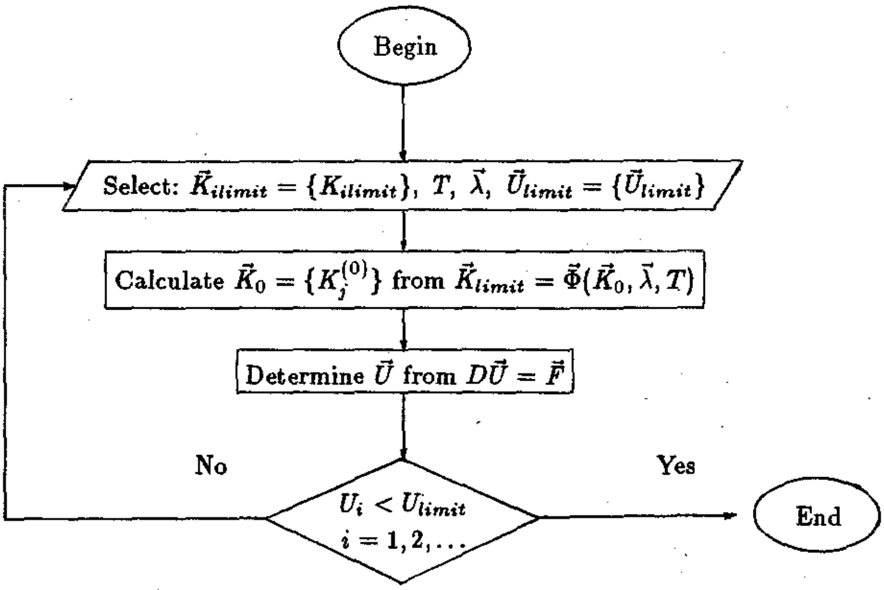


Fig. 2

The design is so that the section 1-1, 2-2, 3-3 have the same lifetime.

G_i $(i = 1, 2, 3)$ - the weight of the parts,

F_i $(i = 1, 2, 3)$ - the areas of the sections,

ℓ_i $(i = 1, 2, 3)$ - the hights of the parts.

According to the standard of structure of brick - stones [7, 8]

$$N < [N]$$
$$[N] = \varphi R F$$

where

$[N]$ - the permisible equivalent longitudinal force

φ - the coefficient of longitudinal bending

F - the area of the sections.

According to the observation and measurement data, by the statistical streatment, we find for the laws of decreasing of the resistance capacify of the materials [5].

$$R_t^{(i)} = R_i^{(0)} e^{-\lambda_i t} \qquad i = 1, 2, 3 \tag{4.1}$$

where

$\lambda_1 = 0.0037$; $\lambda_2 = 0.0042$; $\lambda_3 = 0.0046$

If the given lifetime of the column T is equal 100 years, the

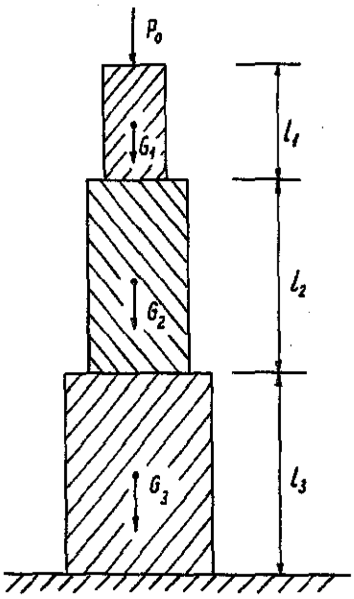


Fig. 3

$$R_i(100) = R_i^{(0)} e^{-\lambda_i . 100}, \quad i = 1, 2, 3$$

we choose $\quad R_i(100) = R_{i_{limit}} = R_i^{(0)} e^{-\lambda_i 100}, \quad i = 1, 2, 3$

$$R_i^{(0)} = R_i(100) e^{100\lambda_i}, \quad i = 1, 2, 3$$

At the sections 1-1

$$G_1 + P_0 = \varphi_1(100).F_1$$
$$G_1 = F_1 \ell_1 \rho_1, \quad \rho_1 - \text{the density of material}$$

$$F_1 = \frac{P_0}{\varphi_1 R_1(100) - \rho_1 \ell_1}$$

or

$$F_1 = \frac{P_0}{\varphi_1 R_1^{(0)} \cdot e^{-0.37} - \sigma_1 \ell_1}$$

At the section 2-2

$$F_2 = \frac{P_0 + G_1}{\varphi_2 R_2^{(0)} \cdot e^{-0.42} - \rho_2 \ell_2}$$

At the section 3-3

$$F_3 = \frac{P_0 + G_1 + G_2}{\varphi_3 R_3^{(0)} \cdot e^{-0,46} - \sigma_3 \ell_3}$$

When $\lambda_1 = \lambda_2 = \lambda_3$, we have the problem of regular resistance.

§5. CONCLUSION

The problem of optimal structure design and iterative method of solution presented in this article can be applied to a wide class of problems. The procedure in fig. 1 presents only the blocks of computation, in the concrete problem, each block of computation will become a concrete subprogram. To illustrate for procedure in fig. 1, some problems are solved on computer in [5].

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REFERENCES

1. Lev O. E. Structural Optimization. Recent Development and Application. ASCE Publication, New York 1981.
2. New Direction in Optimuson Structural Design. Edited by E. Atrek, R. H. Gallagher, K. M. Ragsdell, O. C. Zienkiewicz. John wiley and Sons, New York and Chickester, 1984.
3. Xie Y. M. and Steven G. P. A simple evolutionary procedure for structural optimization. Computers and structures Vol. 49, pp 885-896, 1994.
4. Cherepanov G. P., Ersop I. V. Fracture Mechanic. Manufacturing - Machine Publisching house. Moscow 1977 (in Russian).
5. Pho. N. V. On the method of determining the lifetime of constructions. Proceedings of the fifth national conference on mechanics Vol. 3, 124-128 Hanoi Vietnam (in Vietnamese).
6. Zienkiewicz O. C. and Taylor R. I. The finite elements method. Fourth Edition - Vol. 1, McGraw-Hill book company.
7. Stone-brick and reinforced stone-brick construction. Design standard TCVN 5573-1991. Hanoi Vietnam (in Vietnamese).
8. Nguyen Hien. Hanbook for stone-brick Science-Techn. Publishing House. Hanoi 1974, (in Vietnamese).

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BÀI TOÁN THIẾT KẾ TỐI ƯU KẾT CẤU VỚI TIÊU CHUẨN TUỔI THỌ ĐỀU

Hàm mục tiêu của bài toán thiết kế tối ưu kết cấu có thể chọn khác nhau, tùy thuộc vào mục đích tối ưu. Chẳng hạn, hàm mục tiêu có thể chọn là trọng lượng, giá thành, độ bền đều, độ tin cậy đều v.v...

Trong bài này tác giả chọn mục tiêu tối ưu là tuổi thọ của các yếu tố kết cấu là bằng nhau. Bài toán tối ưu như vậy được áp dụng cho trường hợp kết cấu sửa chữa thay thế khó khăn, tốn kém, phải ngừng sử dụng ảnh hưởng đến quá trình khai thác.

Nội dung bài báo gồm các phần: thành lập bài toán, đề nghị một phương pháp lặp để giải bài toán, giải một thí dụ đơn giản để minh họa.