

OBTAINING EQUATION SET FOR TWO-PHASE BUBBLE DYNAMIC DESCRIPTION

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§1. INTRODUCTION

One process that is able to produce fast fine scale fragmentation is called thermal fragmentation or violent boiling. There are indications that this type of fragmentation is due to the explosive vapourisation (a microexplosion) of small amounts of coolant that are injected into the fuel drop by a hydrodynamic instability that develops when the bubble surrounding the fuel drop collapses at the action of a sharp pressure increase in the surrounding coolant.

Previously two types of models had been applied to the problem:

- * models that describe the thermal conditions in the vapour film but assume spherical symmetry which is quite obviously inappropriate and
- * a model that allows for ellipsoidal droplets and bubbles with an arbitrary height of the fuel within the bubble but does not take into account thermal effects and thus vapourisation at the bubble wall.

Improving existed models, taking into account thermal effects and vapourisation at the bubble and rotational symmetric shape of droplets and bubbles the aim of present work is obtaining equation set for two-phase bubble dynamic description and some important estimations. With regard to one-phase gas (with or without phase transition) bubble dynamics and dynamics of vapour of fluids with these bubbles it can be seen in publications, for instance, by Plesset M. S. Prosperetti A. (1977) [6], Nigmatulin R. I., Duong Ngoc Hai et al. (1988, 1991) [4, 5] or Van Van Diep (1993) [3] and in the cited there literature.

§2. MAIN ASSUMPTIONS

The dynamic of two-phase vapour-drop bubble is considered in spherical coordinates, assuming rotational symmetry (Fig. 1) and under the following simplified assumptions:

- (i) the drop is rigid and fixed in position;
- (ii) the collapsing liquid is incompressible and rigid;
- (iii) heat is transferred through the vapour film
 - thermal conduction (assuming the vapour at rest);
 - thermal radiation (assuming parallel flat surfaces);
- (iv) the compressibility of vapour is modeled by treating it as a polytropic gas.

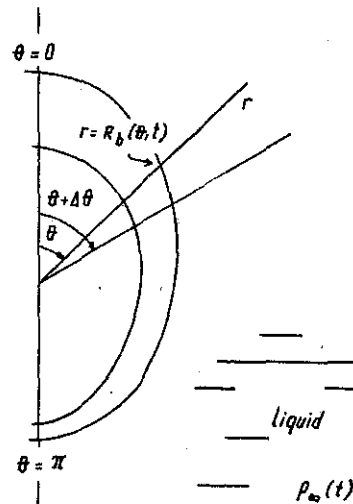


Fig. 1

§3. OUTSIDE LIQUID FLOW

For an incompressible liquid and vortex-free flow the velocity potential $\Phi(r, \theta, t)$ can be introduced:

$$\begin{aligned} \vec{u}_\ell &= \nabla\Phi, & \nabla^2\Phi &= 0, \\ \left(\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)\right), \end{aligned} \quad (3.1)$$

where \vec{u} is a velocity vector; r is a radial coordinate; θ is a polar distance. The subscript ℓ refers to the parameters of liquid.

In order to obtain a well-posed problem for partial differential equations (3.1) an initial and two boundary conditions must be presented. Initially the collapsing liquid generally proposed has some velocity field:

$$t = 0, \quad \vec{u}_\ell = \vec{u}_{\ell 0}. \quad (3.2)$$

The first boundary condition is an inquiry for liquid velocity at infinity:

$$r \rightarrow \infty, \quad \vec{u}_\ell \rightarrow \vec{u}_{\ell 0} \quad (3.3)$$

and the second one is a condition for normal stress on the bubble wall that can be derived from the Bernoulli (momentum) equation:

$$p_\ell + \rho_\ell g \cos \theta = -\rho_\ell \left(\frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla\Phi \cdot \nabla\Phi \right) + p_\infty(t) \quad (3.4)$$

and has the following form:

$$\begin{aligned} \frac{\partial \Phi}{\partial t} &= -\frac{1}{2} \nabla\Phi \cdot \nabla\Phi - g \cos \theta - \frac{p_g - p_\infty(t)}{\rho_\ell} \\ &- \frac{\sigma}{\rho_\ell R_b} \left\{ \frac{1 + 2 \left(\frac{1}{R_b} \frac{\partial R_b}{\partial \theta} \right)^2 - \frac{1}{R_b} \frac{\partial^2 R_b}{\partial \theta^2}}{\left[1 + \left(\frac{1}{R_b} \frac{\partial R_b}{\partial \theta} \right)^2 \right]^{3/2}} + \frac{1 - \frac{1}{R_b} \frac{\partial R_b}{\partial \theta} \cos \theta}{\left[1 + \left(\frac{1}{R_b} \frac{\partial R_b}{\partial \theta} \right)^2 \right]^{1/2}} \right\}, \end{aligned} \quad (3.5)$$

where p and ρ are pressure and density, respectively; g is gravitational acceleration; σ is surface tension coefficient; R_b is bubble radius; t is time. The subscripts g and ∞ refer to parameters of gas and at infinity, respectively.

It should be noted that $\Phi = \Phi(r, \theta, t)$ therefore on the phase-interface:

$$\left. \frac{d\Phi}{dt} \right|_\theta = \frac{\partial \Phi}{\partial t} + \frac{\partial \Phi}{\partial r} \frac{\partial R_b}{\partial t}. \quad (3.6)$$

§4. INSIDE GAS FLOW

4.1. Mass conservation equation

The vapour in the annular gap between a hot drop and surrounding liquid is considered in the framework of one dimensional tangential flow. Within the framework of this simplification consider the geometrical body bounded by the drop surface, bubble wall and conical segments at angles θ and $\theta + \Delta\theta$. The vapour amount contained in this volume can be calculated by the following integral:

$$\begin{aligned} dM &= \int_0^{2\pi} \int_0^{\theta + \Delta\theta} \int_{R_d}^{R_b} \rho_g(\theta, t) r^2 \sin \theta dr d\theta d\phi = \\ &= \rho_g(\theta, t) \frac{R_b^3 - R_d^3}{3} 2\pi \sin \theta d\theta + O((d\theta)^2), \end{aligned} \quad (4.1)$$

where R_d is a drop radius.

The mass flux came into the volume due to the vapour flow through two conical segments surfaces at angles θ and $\theta + \Delta\theta$ has the following form:

$$\begin{aligned} dP_m &= - \int_0^{2\pi} \int_{R_d}^{R_b} \rho_g u_g r \sin \theta dr d\phi \Big|_{\theta+\Delta\theta} + \int_0^{2\pi} \int_{R_d}^{R_b} \rho_g u_g r \sin \theta dr d\phi \Big|_{\theta} = \\ &= -d\theta \frac{\partial}{\partial \theta} \int_0^{2\pi} \int_{R_d}^{R_b} \rho_g u_g r \sin \theta dr d\phi + O((d\theta)^2) = \\ &= -d\theta \frac{\partial}{\partial \theta} \left(\rho_g u_g 2\pi \sin \theta \frac{R_b^2 - R_d^2}{2} \right) + O((d\theta)^2). \end{aligned} \quad (4.2)$$

and the mass flux due to phase transfer has the form:

$$dJ_m = \int_0^{2\pi} \int_{\theta}^{\theta+d\theta} j R_b^2 \sin \theta d\theta d\phi = j R_b^2 2\pi \sin \theta d\theta + O((d\theta)^2), \quad (4.3)$$

where j is a phase transition rate per interfacial surface unit.

The conservation of vapour mass in the considered volume means:

$$\frac{\partial}{\partial t} (dM) = dP_m + dJ_m. \quad (4.4)$$

Substituting eqs. (4.1)-(4.3) into eq. (4.4) and simplifying the following equation is obtained:

$$\frac{\partial}{\partial t} \left(\frac{R_b^3 - R_d^3}{3} \rho_g \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\frac{R_b^2 - R_d^2}{2} \rho_g u_g \sin \theta \right) = j R_b^2. \quad (4.5)$$

4.2. Relation between vapour velocity and pressure gradient

Let suppose that the variation of vapour velocity with time is small compared to the variation with respect to spatial coordinates. In addition let also that the inertial terms are small compare to the viscous terms, i.e. low vapour Reynolds number. The equations of mass and momentum (Navier-Stokes equation) conservation have the following forms, respectively:

$$\begin{aligned} \frac{\partial \rho_g}{\partial t} + \nabla \cdot \rho_g \vec{u}_g &\equiv \frac{d\rho_g}{dt} + \rho_g \nabla \cdot \vec{u}_g = 0; \\ \rho_g \frac{d\vec{u}_g}{dt} &= \rho_g \vec{F} + \nabla \cdot \mathbf{P}_g, \end{aligned} \quad (4.6)$$

where \vec{F} is an external force vector, \mathbf{P}_g is a tension tensor:

$$\begin{aligned} \mathbf{P}_g^{ij} &= -p_g \delta^{ij} + 2\mu_g \left(e^{ij} - \frac{1}{3} \Delta \delta^{ij} \right), \\ e^{ij} &= 0.5 \left(\frac{\partial u_g^i}{\partial x^j} + \frac{\partial u_g^j}{\partial x^i} \right); \\ \Delta &= e^{kk} = \nabla \cdot \vec{u}_g; \quad \delta^{ij} = 0, \quad \text{when } i \neq j \quad \text{and } \delta^{ij} = 1, \quad \text{when } i = j. \end{aligned}$$

When gas dynamical viscosity does not depend upon coordinates the second equation of (4.6) can be rewritten in the following form:

$$\rho_g \frac{d\vec{u}_g}{dt} = \rho_g \vec{F} - \nabla \cdot p_g + \mu_g (\nabla^2 \vec{u}_g + \frac{1}{3} \nabla (\nabla \cdot \vec{u}_g)). \quad (4.7)$$

Or due to rotational symmetry in two dimensional r and θ coordinates these equations have the following forms:

$$\begin{aligned}
& \frac{\partial \rho_g}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho_g u_{gr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho_g u_{g\theta} \sin \theta) \equiv \\
& \equiv \frac{d\rho_g}{dt} + \rho_g \left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_{gr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (u_{g\theta} \sin \theta) \right] = 0; \\
& \rho_g \left(\frac{\partial u_{gr}}{\partial t} + \vec{u}_g \cdot \nabla u_{gr} - \frac{u_{g\theta}^2}{r} \right) = -\rho_g g \cos \theta - \frac{\partial p_g}{\partial r} + \\
& + \mu_g \left[\nabla^2 u_{gr} - \frac{2u_{gr}}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (u_{g\theta} \sin \theta) + \frac{1}{3} \frac{\partial}{\partial r} (\nabla \cdot \vec{u}_g) \right]; \\
& \rho_g \left(\frac{\partial u_{g\theta}}{\partial t} + \vec{u}_g \cdot \nabla u_{g\theta} + \frac{u_{gr} u_{g\theta}}{r} \right) = \rho_g g \sin \theta - \frac{1}{r} \frac{\partial p_g}{\partial \theta} + \\
& + \mu_g \left[\nabla^2 u_{g\theta} + \frac{2}{r^2} \frac{\partial u_{gr}}{\partial \theta} - \frac{u_{g\theta}}{r^2 \sin \theta} + \frac{1}{3r} \frac{\partial}{\partial \theta} (\nabla \cdot \vec{u}_g) \right],
\end{aligned} \tag{4.8}$$

where:

$$\nabla \cdot \vec{u}_g = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_{gr}) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_{g\theta} \sin \theta)$$

and the subscripts r and θ refer to radial and azimuthal directions, respectively.

From the first equation of (4.8) we must get:

$$u_{g\theta} \sin \theta = Q(r). \tag{4.9}$$

And the second and third equations have the following forms:

$$\begin{aligned}
& g \cos \theta + \frac{1}{\rho_g} \frac{\partial p_g}{\partial r} - \frac{u_{g\theta}^2}{r} = 0; \\
& g \sin \theta - \frac{1}{\rho_g r} \frac{\partial p_g}{\partial \theta} + \frac{\mu_g}{\rho_g r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_{g\theta}}{\partial r} \right) = 0.
\end{aligned} \tag{4.10}$$

From the first equation of (4.10) the following dimensionless equation can be obtained:

$$\frac{\partial \bar{p}_g}{\partial \bar{r}} = n M^2 L \frac{\bar{\rho}_g \bar{u}_g^2}{\bar{r}} - \bar{g} \cos \theta, \tag{4.11}$$

where: $\bar{p}_g = p_g/p_0$; $\bar{u}_g = u_g/u_0$; $\bar{r} = r/R_0$;

$$M = \frac{u_0}{a_0} \text{ (Mach number); } L = \frac{(R_{b0} - R_{d0})}{R_0}; \quad \bar{g} = \frac{g(R_{b0} - R_{d0})}{p_0}.$$

(a_0 is a sound speed).

Usually $\bar{g} \ll 1$, therefore for wide number of cases, when whether flow is under-sonic ($M^2 \ll 1$) or the vapour film is thin ($L \ll 1$) the approach $p_g = p_g(\theta)$, i.e. gas pressure is uniform over r , is well satisfied. Neglecting the influence of gravity from the second equation of (4.10) the following relation can be derived:

$$\sin \theta \frac{\partial p_g}{\partial \theta} = \frac{\mu_g}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial Q(r)}{\partial r} \right). \tag{4.12}$$

Because r and θ are two independent coordinates, therefore two parts of eq. (4.12) must equal constant:

$$\frac{\mu_g}{r} \frac{\partial}{\partial r} \left(r^2 \frac{\partial Q(r)}{\partial r} \right) = \Delta p_g, \tag{4.13}$$

where:

$$\Delta p_g = p_g(\theta = \pi) - p_g(\theta = 0) = \text{const.}$$

The solution of eq. (4.13) can be obtained:

$$Q(r) = \frac{\Delta p_g}{2\mu_g} \left[r + \frac{R_b R_d}{r} - (R_b + R_d) \right] \quad (4.14)$$

and from eqs. (4.9) and (4.14) we have:

$$u_{g\theta}(r, \theta) = \frac{Q(r)}{\sin \theta} = \frac{\Delta p_g}{2\mu_g \sin \theta} \left[r + \frac{R_b R_d}{r} - (R_b + R_d) \right]. \quad (4.15)$$

Keeping the constant of mass flow, i.e.:

$$\int_0^{2\pi} \int_{R_d}^{R_b} \rho_g \bar{u}_{g\theta} r \sin \theta dr d\phi = \int_0^{2\pi} \int_{R_d}^{R_b} \rho_g u_{g\theta} r \sin \theta dr d\phi. \quad (4.16)$$

and integrating we must have:

$$\bar{u}_{g\theta} = \frac{2}{R_b^2 - R_d^2} \int_{R_d}^{R_b} u_{g\theta} r dr. \quad (4.17)$$

Substituting eq. (4.15) into eq. (4.17) and integrating the following formula is obtained:

$$\bar{u}_{g\theta}(\theta) = -\frac{\Delta p_g (R_b - R_d)^2}{k\mu_g (R_b + R_d)}; \quad k = 6. \quad (4.18)$$

Taking into account eqs. (4.12) and (4.13) from eq. (4.18) the following relation is obtained:

$$\bar{u}_g = -\frac{(R_b - R_d)^2}{k\mu_g (R_b + R_d)} \frac{\partial p_g}{\partial \theta}. \quad (4.19)$$

Also from eq. (4.18) we have:

$$\frac{\Delta p_g}{\bar{R} \Delta \theta} = -\frac{12\mu_g \bar{u}_{g\theta}}{h^2}; \quad h = R_b - R_d; \quad \bar{R} = 0.5(R_b + R_d) \quad (4.20)$$

and at $\bar{R} \rightarrow \infty$, $\bar{R} \Delta \theta \simeq \Delta x$ the relation (4.20) leads to the form for Poiseuille flow between two infinite parallel flat surfaces.

Caldarola (1972) [8] used the following expressions for vapour velocity:

$$u_{g\theta}(r, \theta) = -\frac{3\dot{m}_g}{\pi\rho_g(R_b - R_d)^3} \left[r + \frac{R_b R_d}{r} - (R_b + R_d) \right] \frac{1 - \cos \theta}{\sin \theta}; \quad (4.21)$$

$$\frac{dp_g}{d\theta} = -\frac{3\mu_g(1 - \cos \theta)\dot{m}_g}{\pi\rho_g \sin \theta (R_b - R_d)^3},$$

where $\dot{m}_g = dm_g/dt$ is a total vapour production rate.

Substituting the first formula of eqs. (4.21) into eq. (4.16) and integrating we can received the following form:

$$\bar{u}_{g\theta}(\theta) = \frac{\dot{m}_g(1 - \cos \theta)}{\pi\rho_g(R_b^2 - R_d^2) \sin \theta}. \quad (4.22)$$

Excluding \dot{m} from eq. (4.22) using the second formula of (4.21) the similar to (4.19) relation is obtained with $k = 3$.

4.3. Closed relations. Initial and boundary conditions for gas flow

To close the system of dynamical equations for gas the polytropic law is used:

$$p_g = p_{g0}(\rho_g/\rho_{g0})^n \quad \text{or} \quad \rho_g = \rho_{g0}(p_g/p_{g0})^{1/n}, \quad (4.23)$$

where n is polytropic coefficient.

The rate j of mass transfer per interfacial surface unit ($j > 0$ for vapourisation and $j < 0$ for condensation) is found from heat balance condition on the bubble wall:

$$j = (q_g + q_\ell + q_r)/\ell_{eq}, \quad (4.24)$$

here q_g , q_ℓ and q_r are heat fluxes to interfacial surface from vapour, liquid and due to radiation, respectively:

$$\begin{aligned} q_g &= \lambda_g(T_d - T_s)/(R_b - R_d); & q_\ell &= -\lambda_\ell(T_s - T_\infty)/\delta; \\ q_r &= B\varepsilon(T_d^4 - T_s^4). \end{aligned} \quad (4.25)$$

where T and λ are temperature and thermal conductivity, respectively; ℓ is latent heat; δ is thermal boundary layer thickness in liquid; B is Stefan-Boltzman coefficient; ε is emission coefficient. The subscript s refers to parameters at saturation.

The relation between saturated temperature and vapour pressure is determined by using Clausius-Clapeyron equation:

$$\frac{dT_s}{dp_g} = \frac{T_s}{\ell\rho_g} \left(1 - \frac{\rho_g}{\rho_\ell}\right) \quad (4.26)$$

and the value of effective latent heat is determined by:

$$\ell_{eq} = \ell + 0.68c_g(T_d - T_s) \quad (4.27)$$

following Bromley 1952 [1], Rohsenow 1956 [7].

The set of eqs. (4.5), (4.23)-(4.27) has the following initial and by virtue of symmetry two boundary conditions:

$$\begin{aligned} t = 0, & \quad p_g(0, \theta) = p_{g0}(\theta); \quad \rho_g = \rho_{g0}(\theta); \quad u_g = u_{g0}(\theta); \\ \theta = 0, & \quad \frac{\partial p_g}{\partial \theta} = 0; \quad \theta = \pi, \quad \frac{\partial p_g}{\partial \theta} = 0. \end{aligned} \quad (4.28)$$

It should be noted that the initial values $p_{g0}(\theta)$, $\rho_{g0}(\theta)$ and $u_{g0}(\theta)$ are determined prior to time dependent calculation by requiring force balance between the pressure distribution in vapour film and the drop weight. In this calculation the pressure at the top is kept constant and equal p_{g0} that could be explained by releasing at this area vapour bubbles.

§5. DEFINITION OF MOVING INTERFACE

The condition of mass continuity over moving interface described by $R_b = R_b(\theta, t)$ has the following form:

$$j = \rho_\ell(\dot{X}_j n_j - (u_j n_j)_\ell) = -\rho_g(\dot{X}_j n_j - (u_j n_j)_g). \quad (5.1)$$

Under conditions far from critical usually $j/\rho_\ell \ll 1$. In this case the presented above equation leads to the condition that the normal liquid velocity at bubble surface equals the bubble wall velocity and can be used to determine the time evolution of interface:

$$\frac{dR_b}{dt} = \left(\frac{\partial \Phi}{\partial r} + \frac{1}{R_b^2} \frac{\partial \Phi}{\partial \theta} \frac{\partial R_b}{\partial \theta} \right). \quad (5.2)$$

Eq. (5.2) has a simple initial condition:

$$R_b(\theta, 0) = R_0(\theta). \quad (5.3)$$

The system of eqs. (3.1)-(3.3), (3.4)-(3.5), (4.5), (4.19), (4.23)-(4.28), (5.2)-(5.3) is closed and can be used to describe the presented above problem of two-phase vapour drop bubble dynamic.

§6. SOME ESTIMATIONS

6.1. Role of subcooling

Consider the following ratio:

$$\bar{q}_\ell = \frac{\lambda_\ell(T_s - T_\infty)}{\delta} \frac{(R_b - R_d)}{\lambda_g(T_d - T_s)}. \quad (6.1)$$

where $\delta \simeq \sqrt{D_\ell t}$ and D is thermal diffusivity.

For example, for water we have:

$$\bar{q}_\ell \sim 10^5 \frac{(T_s - T_\infty)(R_b - R_d)}{(T_d - T_s)\sqrt{10t}}. \quad (6.2)$$

For the case of $(T_s - T_\infty) \sim 10K$; $(T_d - T_s) \sim 10^3K$ and $(R_b - R_d) \sim 10^{-1} \text{ mm}$ (6.2) leads to the following relation:

$$\bar{q}_\ell \sim \frac{10^{-1}}{\sqrt{10t}} \quad (6.3)$$

which means that \bar{q}_ℓ will be smaller than 1, when $t \geq 1 \text{ ms}$, i.e. heat loss from the subcooling due to increasing the surrounding pressure will be negligible in comparison with heat transferred by thermal conductivity when the thermal boundary layer in liquid is thick enough. It should be noted that in reality the thermal boundary layer thickness in liquid could be strongly changed dependently on the bubble surface dynamic: this layer becomes thinner in the expansion phase and larger in the compress phase.

6.2. Role of radial convective heat transfer

Consider the ratio between two items presented respectively convective heat transfer and heat transfer due to thermal conductivity in equation of the vapour energy conservation in spherical symmetric approach:

$$\bar{q}_c = \frac{u_{gr} \frac{\partial T}{\partial r}}{\frac{D_g}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right)}. \quad (6.4)$$

For water we have:

$$\bar{q}_c \sim 10^4 (R_b - R_d) u_{gr}. \quad (6.5)$$

In the case of $(R_b - R_d) \sim 10^{-1} \text{ mm}$ in according to (6.5) \bar{q}_c will be smaller than 1 when $u_r < 1 \text{ m/s}$, i.e. the radial convective heat transfer will be negligible in comparison with heat transfer due to thermal conductivity when the gas motion is low ($u < 1 \text{ m/s}$).

6.3. Role of radiation heat transfer

Consider the ratio between two items presented respectively radiation heat transfer and heat transfer due to thermal conductivity:

$$\bar{q}_r = B\varepsilon(T_d^4 - T_s^4) \frac{R_b - R_d}{\lambda_g(T_d - T_s)}. \quad (6.6)$$

For the case of an alumina drop in water [2]: $\lambda_g \sim 0.05$ W/mK; $\varepsilon \sim 0.3$ therefore we have:

$$\bar{q}_r \sim 4 \times 10^{-7} T_d^3 (R_b - R_d). \quad (6.7)$$

In the case of $T_d \sim 1500 \div 2000$ K \bar{q}_r will be ~ 1 , when $(R_b - R_d) \sim 1$ mm, i.e. for the vapour film thickness with order of 1 mm heat transfer due to radiation and due to thermal conductivity will be approximately the same. If the vapour film thickness is more than 1 mm the heat transfer due to radiation will be dominated.

6.4. Quantity of vapourisation liquid

Let consider a spherical approach. The total quantity of over-heats in relation to surrounding medium contained in the drop can be calculated:

$$Q_d = \frac{4}{3} \pi R_d^3 \rho_d c_d (T_d - T_s). \quad (6.8)$$

The quantity of heat transferred from hot drop to surrounding medium can be estimated in the following way:

$$Q_g = 4\pi R_b^2 q_g t. \quad (6.9)$$

Consider the following ratio:

$$\bar{Q}_g = \frac{Q_g}{Q_d} \sim \frac{3\lambda_g t}{\rho_d c_d R_d (R_b - R_d)}. \quad (6.10)$$

Let as above consider the case of water. The relation (6.10) leads to the following estimation:

$$\bar{Q}_g \sim \frac{10^{-1} t}{10^6 R_d (R_b - R_d)}. \quad (6.11)$$

And for the case of $R_d \sim 1$ mm and $(R_b - R_d) \sim 0.1$ mm \bar{Q}_g will be less than 1 when $t < 1$ s. This means for the short processes with $t \ll 1$ s the drop temperature can be proposed to be constant.

Let estimate the quantity of vapourized liquid during process. The quantity of initially existed vapour in the gap can be calculated in the following way:

$$M_{g0} = \frac{4}{3} \pi (R_b^3 - R_d^3) \rho_{g0}. \quad (6.12)$$

The quantity of vapour produced during process can be estimated:

$$\Delta M_g \simeq \frac{4\pi R_b^2 q_g t}{\ell}. \quad (6.13)$$

And the ratio of new produced vapour quantity to the initially existed vapour quantity can be obtained:

$$\Delta \bar{M}_g = \frac{\Delta M_g}{M_0} \sim \frac{\lambda_r}{\rho_{gd} \ell} \frac{(T_d - T_s) t}{(R_b - R_d)^2}. \quad (6.14)$$

For the case of water we have:

$$\Delta \bar{M}_g \sim 10^{-7} \frac{(T_d - T_s) t}{(R_b - R_d)^2}. \quad (6.15)$$

And if $(T_d - T_s) \sim 10^3$ K and $(R_b - R_d) \sim 0.1$ mm, $\Delta \bar{M}_g$ will be less than 1 for processes with $t \leq 0.1$ ms, i.e. vapour production due to heating will be important for the processes with $t > 0.1$ ms. The estimation for pressure increasing due to the local vapourisation can be obtained

using (4.23). It should be noted that the bubble volume can be changed due to a motion of bubble surface.

§7. CONCLUSION

Basing on the general thermo-, gas- and hydrodynamic laws the equation set for two-phase bubble dynamic description and some theoretical estimations are obtained. The model taking into account thermal effects and thus vapourisation at the bubble wall is extended for the case of droplets and bubbles with rotational symmetric shape. It is the next step-forward to understanding the process that seems to be simple enough at the first consideration, but indeed, is very complicated in terms of coupling thermo- and hydro-dynamic, multi-parameter and spatial (multi-dimensional) problem with moving boundary and possible physical instability and etc.

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THU NHẬN HỆ PHƯƠNG TRÌNH MÔ TẢ ĐỘNG LỰC HỌC BỌT HAI PHA

Trên cơ sở những quy luật chung của nhiệt thủy khí động lực học trình bày kết quả thu nhận hệ phương trình đóng kín có thể dùng để mô tả động lực học bọt hai pha và một số đánh giá lý thuyết. Mô hình mở rộng cho trường hợp hạt và bọt tròn xoay có xét đến các hiệu ứng nhiệt và sự bay hơi trên biên pha. Đây là bước tiếp tiến tới tìm hiểu một hiện tượng tưởng như đơn giản nhưng thực ra rất phức tạp do sự liên hệ chặt chẽ giữa các quá trình nhiệt và thủy động lực học, do tính chất đa tham số và tính không gian của bài toán, ngoài ra còn có biên di động và có thể có khả năng mất ổn định vật lý và v. v...