STABILITY OF THE EQUILIBRIUM REGIME OF A SYSTEM OF TWO DEGREES OF FREEDOM IN AMPLITUDE-PHASE VARIABLES

NGUYEN VAN DINH
Institute of Mechanics, NCNST of Vietnam

In [1] Hans Kauderer has used the amplitude-phase variables to study the stability of the equilibrium regime which is considered as a special oscillation of amplitude $r = 0$ and of constant dephase θ^*.

In [2] this dephase has been explained as that of such motion called characteristic and this explanation permits us to propose a lightly modification in Hans Kauderer's method. The same problem will be examined below for the equilibrium regime of an oscillating system of two degrees of freedom. It will be shown that the results obtained in [2] can be applied without difficulty.

§1. SYSTEM UNDER CONSIDERATION AND ITS AVERAGED EQUATIONS

Let us consider a quasi-linear oscillating system of two degrees of freedom described by the following differential equations:

$$\ddot{x}_\mu + \omega_\mu^2 x_\mu = \varepsilon f^{(\mu)}(x_1, \dot{x}_1, x_2, \dot{x}_2, \omega_1 t, \omega_2 t) \quad (\mu = 1, 2) \tag{1.1}$$

where x_1, x_2 - oscillatory variables; $\varepsilon > 0$ - small parameter; overdot denotes time derivative; ω_1, ω_2 - exciting frequencies near the natural ones, respectively; $f^{(1)}$, $f^{(2)}$ - functions of the form:

$$f^{(\mu)} = \sum_{\nu=1}^{2} \Big\{ x_\nu \Big[A_\nu^{(\mu 0)} + \sum_{m=1}^{N} \sum_{r=1}^{2} \big(C_{\nu r}^{(\mu m)} \cos m\omega_r t + S_{\nu r}^{(\mu m)} \sin m\omega_r t \big) \Big] + \dot{x}_\nu \Big[\overline{A}_\nu^{(\mu 0)} + \\ + \sum_{m=1}^{N} \sum_{r=1}^{2} \big(\overline{C}_{\nu r}^{(\mu m)} \cos m\omega_r t + \overline{S}_{\nu r}^{(\mu m)} \sin m\omega_r t \big) \Big] \Big\} + (\ldots) \quad (\mu = 1, 2) \tag{1.2}$$

with: N - a positive integer; $A_\nu^{(\mu 0)}$, $C_{\nu r}^{(\mu m)}$, $S_{\nu r}^{(\mu m)}$, ... - constant coefficients, $(\ldots)$ represents the terms of powers equal or greater than 2 relative to x_1, $\dot{x}_1$, x_2, $\dot{x}_2$.

It is assumed that ω_1, ω_2 don't satisfy the relations of type:

$$n_1 \omega_1 + n_2 \omega_2 = 0; \qquad n_1,\ n_2 \text{ - integers} \tag{1.3}$$

In other words, the system considered is not in internal resonant situation.

Introducing slowly varying variables either of type (a, b) or (r, θ) we put, respectively:

$$x_\mu = a_\mu \cos \omega_\mu t + b_\mu \sin \omega_\mu t, \quad \dot{x}_\mu = -\omega_\mu a_\mu \sin \omega_\mu t + \omega_\mu b_\mu \cos \omega_\mu t \tag{1.4}$$

or

$$x_\mu = r_\mu \cos\psi_\mu, \quad \dot{x}_\mu = \quad r_\mu \omega_\mu \sin\psi_\mu, \quad \psi_\mu = \omega_\mu t - \theta_\mu \tag{1.5}$$

The corresponding averaged systems are

$$\dot{a}_\mu = \frac{-\varepsilon}{\omega_\mu}\langle f^{(\mu)} \sin\omega_\mu t\rangle, \quad \dot{b}_\mu = \frac{\varepsilon}{\omega_\mu}\langle f^{(\mu)} \cos\omega_\mu t\rangle \tag{1.6}$$

and

$$\dot{r}_\mu = \frac{-\varepsilon}{\omega_\mu}\langle f^{(\mu)} \sin\psi_\mu\rangle, \quad r_\mu\dot{\theta}_\mu = \frac{\varepsilon}{\omega_\mu}\langle f^{(\mu)} \cos\psi_\mu\rangle \quad (\mu = 1, 2) \tag{1.7}$$

where $\langle \quad \rangle$ is the averaging operator.

Recall that (a_μ, b_μ) and (r_μ, θ_μ) are related by:

$$a_\mu = r_\mu \cos\theta_\mu, \quad b_\mu = r_\mu \sin\theta_\mu \quad (\mu = 1, 2) \tag{1.8}$$

§2. THE EQUILIBRIUM REGIME, CHARACTERISTIC MOTION AND STABILITY

Obviously, the system considered admits the equilibrium regime which corresponds to the trivial solution $x_1 = x_2 = 0$ or, in (a, b) variables, to $a_1 = b_1 = a_2 = b_2 = 0$.

The stability study of this regime will be based on the variational system which coincides with the linear part of the averaged one (1.6):

$$\begin{aligned} \dot{a}_\mu &= \frac{\varepsilon}{2\omega_\mu}\Big\{(A_\mu - B_\mu)a_\mu + (C_\mu - D_\mu)b_\mu\Big\} \\ \dot{b}_\mu &= \frac{\varepsilon}{2\omega_\mu}\Big\{(C_\mu + D_\mu)a_\mu + (A_\mu + B_\mu)b_\mu\Big\} \quad (\mu = 1, 2) \end{aligned} \tag{2.1}$$

where

$$A_\mu = \omega_\mu \overline{A}_\mu^{(\mu 0)}, \quad B_\mu = \frac{1}{2}\big(S_{\mu\mu}^{(\mu 2)} + \omega_\mu \overline{C}_{\mu\mu}^{(\mu 2)}\big), \quad C_\mu = \frac{1}{2}\big(C_{\mu\mu}^{(\mu 2)} - \omega_\mu \overline{S}_{\mu\mu}^{(\mu 2)}\big), \quad D_\mu = A_\mu^{(\mu 0)}$$

It is noted that, in (2.1), the two couples (a_1, b_1) and (a_2, b_2) are separated; consequently, the stability study of the origin in each plane (a_μ, b_μ) can be accomplished independently.

As usual, from the characteristic equations:

$$\rho_\mu^2 + \frac{\varepsilon}{\omega_\mu}A_\mu \rho_\mu + \frac{\varepsilon^2}{4\omega_\mu^2}\Big\{A_\mu^2 + D_\mu^2 - B_\mu^2 - C_\mu^2\Big\} = 0 \quad (\mu = 1, 2) \tag{2.2}$$

we deduce these asymptotically stable conditions:

$$\operatorname{Re}\rho_\mu^\pm < 0, \quad \rho_\mu^\pm = \frac{\varepsilon}{2\omega_\mu}\Big\{ - A_\mu \pm \sqrt{B_\mu^2 + C_\mu^2 - D_\mu^2}\Big\} \quad (\mu = 1, 2) \tag{2.3}$$

The stability conditions obtained can be interpreted as follows. For each μ, the system (2.1) being a linear one of two differential equations of constants coefficients, so that [3]:

- In the plane (a_μ, b_μ) each (simple or double, real or complex) characteristic value ρ_μ corresponds, to a family (of one two parameters) of such motions called characteristic, defined as:

$$a_\mu = \xi_\mu e^{\rho_\mu t}, \quad b_\mu = \eta_\mu e^{\rho_\mu t} \tag{2.4}$$

where ξ_μ, η_μ are constants, satisfying the equations:

$$\left[\frac{\varepsilon}{2\omega_\mu}(A_\mu - B_\mu) - \rho\right]\xi_\mu + \frac{\varepsilon}{2\omega_\mu}(C_\mu - D_\mu)\eta_\mu = 0$$
$$\frac{\varepsilon}{2\omega_\mu}(C_\mu + D_\mu)\xi_\mu + \left[\frac{\varepsilon}{2\omega_\mu}(A_\mu + B_\mu) - \rho\right]\eta_\mu = 0 \tag{2.5}$$

- The dephase θ_μ^* of any characteristic motion is constant and determined as:

$$\xi_\mu \sin\theta_\mu^* = \eta_\mu \cos\theta_\mu^* \tag{2.6}$$

All characteristic motions of a family of one parameter have the same dephase; so, this family is represented by any motion of the family.

For a family of two parameters, the dephases of characteristic motions are arbitrary and this family is represented by any two motions of different dephases.

The above presented properties recalled, the stability conditions (2.3) can be considered as the requirements imposed on representative characteristic motions:

In each plane (a_μ, b_μ) the equilibrium regime is asymptoticallty stable if all representative characteristic motions posseses the same property i.e. if they tend asymptotically to the origin.

Remark

- The equilibrium regime is supposed to be isolated i.e. the following nonegality is imposed:

$$\prod_{\mu=1}^{2}\left\{A_\mu^2 + D_\mu^2 - B_\mu^2 - C_\mu^2\right\} \neq 0 \quad (\mu = 1, 2) \tag{2.7}$$

- A double (real) characteristic value corresponds either to a one or a two parameters family of characteristic motions. For the second case, $B_\mu = C_\mu = D_\mu = 0$ and the averaged system (2.1) becomes:

$$\dot{a}_\mu = \frac{\varepsilon}{2\omega_\mu}A_\mu \cdot a_\mu, \quad \dot{b}_\mu = \frac{\varepsilon}{2\omega_\mu}A_\mu \cdot b_\mu \tag{2.8}$$

All characteristic motions have straight trajectories, passing through the origin.

§3. THE EQUILIBRIUM REGIME, THE VARIATIONAL SYSTEM IN (r, θ) VARIABLES

Let us pass over to the method of studying the stability of the equilibrium regime in amplitude-phase variables. Using (1.8) the variational system (2.1) is transformed into:

$$\dot{r}_\mu = \frac{\varepsilon r_\mu}{2\omega_\mu}(A_\mu - B_\mu\cos 2\theta_\mu + C_\mu \sin 2\theta_\mu)$$
$$r_\mu\dot{\theta}_\mu = \frac{\varepsilon r_\mu}{2\omega_\mu}(D_\mu + C_\mu\cos 2\theta_\mu + B_\mu \sin 2\theta_\mu) \quad (\mu = 1, 2) \tag{3.1}$$

It is not difficult to prove that (3.1) is just the linear part of the averaged system (1.7) relative to r_1, r_2.

Thus, in (r, θ) variables, the system playing the role of the variational one is obtained by neglecting in the averaged system (1.7) the terms of powers equal or greater than 2 relative to r_1, r_2.

The next step consists in examining the representative characteristic motions. As it has been shown in §2, the dephases θ_μ^* of these motions are constants, so:

$$\dot{\theta}_\mu^* = 0 \qquad (\mu = 1, 2) \tag{3.2}$$

The dephases θ_μ^* can be thus determined by vanishing the right-hand sides of the second and the fourth equations of the system (3.1):

$$D_\mu + C_\mu \cos 2\theta_\mu^* + B_\mu \sin 2\theta_\mu^* = 0 \quad (\mu = 1,2) \tag{3.3}$$

Having found θ_μ^*, we rewrite the first and the third equations of the system (3.1) in the form:

$$\dot{r}_\mu = \frac{\varepsilon r_\mu}{2\omega_\mu}\left\{A_\mu \pm \sqrt{B_\mu^2 + C_\mu^2 - D_\mu^2}\right\} \quad (\mu = 1,2) \tag{3.4}$$

and the stability conditions (2.3) can be easily obtained.

Remark - The right-hand sides of (3.1) don't depend on θ_μ if $B_\mu = C_\mu = 0$ $(\mu = 1,2)$. In this case, the averaged system becomes:

$$\dot{r}_\mu = \frac{\varepsilon}{2\omega_\mu} A_\mu \cdot r_\mu, \quad r_\mu \dot{\theta}_\mu = \frac{\varepsilon}{2\omega_\mu} D_\mu \cdot r_\mu \quad (\mu = 1,2) \tag{3.5}$$

and the stability conditions are:

$$A_\mu < 0 \qquad (\mu = 1,2) \tag{3.6}$$

§4. ONE FREQUENCY OSCILLATORY REGIME. STABILITY

The method presented above can be applied successfully to study the stability of the regime where, for instance, "the part" x_1 is in oscillation while "the other part" x_2 is in equilibrium. For this case, we put $f^{(2)}$ in the form:

$$f^{(2)} = x_2 F(x_1, \dot{x}_1, \omega_1 t, \omega_2 t) + \dot{x}_2 \overline{F}(x_1, \dot{x}_1, \omega_1 t, \omega_2 t) + (\dots) \tag{4.1}$$

where F, $\overline{F}$ are functions of same structure relative to x_1, $\dot{x}_1$ as $f^{(1)}$, $f^{(2)}$ relative to x_1, $\dot{x}_1$, x_2, $\dot{x}_2$; $(\dots)$ represents the terms of powers equal or greater than 2 relative to x_2, $\dot{x}_2$. The averaged system can be written as:

$$\begin{aligned} \dot{a}_1 &= \frac{-\varepsilon}{\omega_1}\left\{K + a_2 L + b_2 M + (\dots)\right\}, \quad \dot{b}_1 = \frac{\varepsilon}{\omega_1}\left\{\overline{K} + a_2 \overline{L} + b_2 \overline{M} + (\dots)\right\} \\ \dot{a}_2 &= \frac{-\varepsilon}{\omega_2}\left\{a_2 P + b_2 Q + (\dots)\right\}, \qquad \dot{b}_2 = \frac{\varepsilon}{\omega_2}\left\{a_2 \overline{P} + b_2 \overline{Q} + (\dots)\right\} \end{aligned} \tag{4.2}$$

where K, L ... are functions of a_1, b_1, $(\dots)$ represents the terms of power equal or greater than 2 relative to a_2, b_2. Suppose that (4.2) admits the solution:

$$a_1 = a_1^0, \quad b_1 = b_1^0, \quad a_2 = b_2 = 0 \tag{4.3}$$

where a_1^0, b_1^0 are constants, satisfying the equations:

$$K(a_1^0, b_1^0) = 0, \quad \overline{K}(a_1^0, b_1^0) = 0 \tag{4.4}$$

Introducing ther perturbations $\delta a_1 = a_1 - a_1^0$, $\delta b_1 = b_1 - b_1^0$, $\delta a_2 = a_2$, $\delta b_2 = b_2$ we form the variational system:

$$\begin{aligned} \delta\dot{a}_1 &= \frac{-\varepsilon}{\omega_1}\left\{\frac{\partial K}{\partial a_1}\delta a_1 + \frac{\partial K}{\partial b_1}\delta b_1 + a_2 L(a_1^0, b_1^0) + b_2 M(a_1^0, b_1^0)\right\} \\ \delta\dot{b}_1 &= \frac{\varepsilon}{\omega_1}\left\{\frac{\partial \overline{K}}{\partial a_1}\delta a_1 + \frac{\partial \overline{K}}{\partial b_1}\delta b_1 + a_2 \overline{L}(a_1^0, b_1^0) + b_2 \overline{M}(a_1^0, b_1^0)\right\} \\ \dot{a}_2 &= \frac{-\varepsilon}{\omega_2}\left\{a_2 P(a_1^0, b_1^0) + b_2 Q(a_1^0, b_1^0)\right\} \\ \dot{b}_2 &= \frac{\varepsilon}{\omega_2}\left\{a_2 \overline{P}(a_1^0, b_1^0) + b_2 \overline{Q}(a_1^0, b_1^0)\right\} \end{aligned} \tag{4.5}$$

where $\frac{\partial K}{\partial a_1}, \frac{\partial K}{\partial b_1}, \frac{\partial \overline{K}}{\partial a_1}, \frac{\partial \overline{K}}{\partial b_1}$ as all coefficients of a_2, b_2 are taken at a_1^0, b_1^0.

The structure of (4.6) shows that:

- The stability of the couple $(a_2 = 0, b_2 = 0)$ can be studied directly and independently of that of the one (a_1^0, b_1^0),

- If the couple $(a_2 = 0, b_2 = 0)$ is asymptotically stable i.e. if $\lim_{t\to\infty} a_2 = 0$, $\lim_{t\to\infty} b_2 = 0$, the stability of the couple (a_1^0, b_1^0) can be based on the system:

$$\begin{aligned} \delta\dot{a}_1 &= \frac{-\varepsilon}{\omega_1}\left\{\frac{\partial K}{\partial a_1}\delta\dot{a}_1 + \frac{\partial K}{\partial b_1}\delta b_1\right\} \\ \delta\dot{b}_1 &= \frac{\varepsilon}{\omega_1}\left\{\frac{\partial \overline{K}}{\partial a_1}\delta a_1 + \frac{\partial \overline{K}}{\partial b_1}\delta b_1\right\} \end{aligned} \tag{4.6}$$

In other words, the stability study of the couple (a_1^0, b_1^0) is reduced to that of the stationary regime $x_1^0 = a_1^0 \cos\omega_1 t + b_1^0 \sin\omega_1 t$ of the subsystem:

$$\ddot{x}_1 + \omega_1^2 x = \varepsilon f^{(1)}(x_1, \dot{x}_1, 0, 0, \omega_1 t, \omega_2 t) \tag{4.7}$$

It is easy to translate all these remarks into (r, θ) language. The averaged system in (r, θ) variables is of the form:

$$\begin{aligned} \dot{r}_1 &= \frac{-\varepsilon}{\omega_1}\left\{U(r_1, \theta_1) + r_2 V(r_1, \theta_1, \theta_2) + (\ldots)\right\} \\ r_1\dot{\theta}_1 &= \frac{\varepsilon}{\omega_1}\left\{\overline{U}(r_1, \theta_1) + r_2 \overline{V}(r_1, \theta_1, \theta_2) + (\ldots)\right\} \\ \dot{r}_2 &= \frac{-\varepsilon}{\omega_2}\left\{r_2 W(r_1, \theta_1, \theta_2) + (\ldots)\right\} \\ r_2\dot{\theta}_2 &= \frac{\varepsilon}{\omega_2}\left\{r_2 \overline{W}(r_1, \theta_1, \theta_2) + (\ldots)\right\} \end{aligned} \tag{4.8}$$

where U, $\overline{U}$ (V, $\overline{V}$, W, $\overline{W}$) are functions of r_1, θ_1 (r_1, θ_1, θ_2); $(\ldots)$ represents the terms of powers equal or greater than 2 relative to r_2. The stationary regime (4.3) corresponds to the solution

$$r_1 = r_1^0, \quad \theta_1 = \theta_1^0, \quad r_2 = 0 \tag{4.9}$$

where r_1^0, θ_1^0 are constants satisfying the equations:

$$U(r_1^0, \theta_1^0) = 0, \quad \overline{U}(r_1^0, \theta_1^0) = 0 \tag{4.10}$$

The variational system (4.5) is replaced by:

$$\begin{aligned} \delta\dot{r}_1 &= \frac{-\varepsilon}{\omega_1}\left\{\frac{\partial U}{\partial r_1}\delta r_1 + \frac{\partial U}{\partial \theta_1}\delta\theta_1 + r_2 V(r_1^0, \theta_1^0, \theta_2)\right\} \\ r_1^0 \cdot \delta\dot{\theta}_1 &= \frac{\varepsilon}{\omega_1}\left\{\frac{\partial \overline{U}}{\partial r_1}\delta r_1 + \frac{\partial \overline{U}}{\partial \theta_1}\delta\theta_1 + r_2 \overline{V}(r_1^0, \theta_1^0, \theta_2)\right\} \\ \dot{r}_2 &= \frac{-\varepsilon}{\omega_2} r_2 W(r_1^0, \theta_1^0, \theta_2) \\ r_2 \cdot \dot{\theta}_2 &= \frac{\varepsilon}{\omega_2} r_2 \overline{W}(r_1^0, \theta_1^0, \theta_2) \end{aligned} \tag{4.11}$$

where $\delta r_1 = r_1 - r_1^0$, $\delta\theta_1 = \theta_1 - \theta_1^0$ - the perturbations; $\frac{\partial U}{\partial a_1}, \frac{\partial \overline{U}}{\partial r_1}, \frac{\partial U}{\partial \theta_1}, \frac{\partial \overline{U}}{\partial \theta_1}$ as all coefficients of r_2 are taken at r_1^0, θ_1^0. In the plane (r_2, θ_2) the constant dephases θ_2^* of characteristic motions satisfy the equations:

$$\overline{W}(r_1^0, \theta_1^0, \theta_2^*) = 0 \tag{4.12}$$

and the stability conditions of the equilibrium in the plane (r_2, θ_2) are:

$$\mathrm{Re}\, W(r_1^0, \theta_1^0, \theta_2^*) > 0 \tag{4.13}$$

For the couple (r_1^0, θ_1^0) we form the characteristic equation:

$$\begin{vmatrix} \frac{-\varepsilon}{\omega_1}\frac{\partial U}{\partial r_1} - \rho & \frac{-\varepsilon}{\omega_1}\frac{\partial U}{\partial \theta_1} \\ \frac{\varepsilon}{\omega_1}\frac{\partial \overline{U}}{\partial r_1} & \frac{\varepsilon}{\omega_1}\frac{\partial \overline{U}}{\partial \theta_1} - \rho \end{vmatrix} = 0 \tag{4.14}$$

and the stability conditions are $\mathrm{Re}\,\rho < 0$.

Remark - W and $\overline{W}$, relative to θ_2, are of same structure as the right-hand sides of (3.1), relative to θ_μ. Therefore, the dephase θ_2 can be absent in W and $\overline{W}$ only simultaneously. In this case, the stability condition of the couple $(a_2 = b_2 = 0)$ is $W(r_1^0, \theta_1^0) > 0$.

Example

Let us consider an oscillating system described by the differential equations:

$$\ddot{x} + \omega^2 x = \varepsilon\{-h_1\dot{x} - \beta x^3 + cxy^2\}$$
$$\ddot{y} + \nu^2 y = q\sin\gamma t + \varepsilon\{-h_2\dot{y} - \beta y^3 + cx^2 y\}$$

where h_1, h_2 are positive constants; $\omega^2 = \gamma^2 - \varepsilon\Delta$, $m\omega + n\nu \neq 0$, (m, n - interger); other symbols retain the same significations as in [4] (pp 288-294).

Using the amplitude-phase variables, we put:

$$x = r\cos\psi, \quad \dot{x} = -r\gamma\sin\psi, \quad \psi = \gamma t - \theta,$$
$$y = \rho\cos\psi + q_*\sin\gamma t, \quad \dot{y} = -\rho\nu\sin\varphi + \gamma q_*\cos\gamma t$$
$$\varphi = \nu t - \sigma, \quad q_* = \frac{q}{\nu^2 - \gamma^2}$$

where r, ρ; θ, σ are slowly varying amplitudes and dephase angles, respectively.

The averaged system is of the form:

$$\dot{r} = -\frac{\varepsilon r}{2\gamma}\Big\{h_1\gamma + \frac{1}{4}cq_*^2\sin 2\theta\Big\}$$
$$r\dot{\theta} = -\frac{\varepsilon r}{2\gamma}\Big\{-\big(\Delta + \frac{c}{4}q_*^2\big) - \frac{c}{4}\rho^2 + \frac{3}{4}\beta r^2 + \frac{c}{4}q_*^2 cos2\theta\Big\}$$
$$\dot{\rho} = -\frac{\varepsilon\rho}{2\nu}\Big\{h_2\nu\Big\}$$
$$\rho\dot{\sigma} = -\frac{\varepsilon\rho}{2\nu}\Big\{\frac{3}{2}\beta q_*^2 + \frac{3}{4}\beta\rho^2 - \frac{c}{2}r^2\Big\}$$

Obviously, in the first approximation:

- The trivial solution $r_1 = \rho_1 = 0$ corresponds to the pure-forced oscillation $x = 0$, $y = q_*\sin\gamma t$
- The quasi-trivial solutions

$$\rho_2 = 0, \quad \frac{3}{4}\beta r_2^2 = \big(\Delta + \frac{c}{4}q_*^2\big) \pm \sqrt{\frac{c^2}{16}q_*^4 - h_1^2\gamma^2}$$

correspond to the combined oscillations

$$x_2 = r_2 \cos\psi, \quad y_2 = q_* \sin\gamma t$$

Following the above presented analyses, we can conclude that, in the first approximation:

- The trivial solution is asymptotically stable if

$$\mathrm{Re}\Big\{h_1\gamma + \frac{c}{4}q_*^2 \sin 2\theta\Big\} > 0$$

where

$$\sin 2\theta = \pm\sqrt{1-\cos^2 2\theta} = \pm\sqrt{1-\big(\Delta + \frac{c}{4}q_*^2\big)^2\Big(\frac{4}{cq_*^2}\Big)^2}$$

- In the combined regime, x is parametrically excited and its motion is governed by the differential equation:

$$\begin{aligned}\ddot{x} + \gamma^2 x &= \varepsilon\Big\{-h_1\dot{x} + \Delta x - \beta x^3 + cx(q_*\sin\gamma t)^2\Big\}\\ &= \varepsilon\Big\{-h_1\dot{x} + \big(\Delta + \frac{c}{2}q_*^2\big)x - \beta x^3 - \frac{c}{2}q_*^2 x\cos 2\gamma t\Big\}\end{aligned}$$

Since $h_2 > 0$ the amplitude r_2 exponentially tends to zero. Hence, the parametric oscillation with large amplitude r_2 (sign + before radical) is asymptotically stable.

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REFERENCES

1. Kauderer H. Nichtlineare mechanik, Berlin, 1958.
2. Nguyen Van Dinh. On the dephase angle in a variational system of the equilibrium regime. Journal of Mechanics, No 1, 1994.
3. Понтрягин Л. С. Обыкновенные дифференциальные уравнения, Москва, 1961.
4. Mitropolskii Yu. A., Nguyen Van Dao. Applied asymptotic methods in nonlinear oscillations. Hanoi, 1994.

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ỔN ĐỊNH CỦA CHẾ ĐỘ CÂN BẰNG Ở HỆ HAI BẬC TỰ DO TRONG BIẾN BIÊN ĐỘ - PHA

Bài toán mở rộng việc áp dụng những kết quả đạt được trong [2] về phương pháp sử dụng các biến biên độ - pha để khảo sát ổn định của chế độ cân bằng ở hệ dao động á tuyến hai bậc tự do.