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NEW SIZE NUMBER AND THE FRACTURE STATE OF CONCRETE STRUCTURE

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ABSTRACT. A new size number describing the fracture state of concrete structures is developed in which the structural size and the main fracture properties of concrete are dealt with. The relation between the fracture state of concrete beams loaded in bending and the newly proposed size number is presented graphically. The shape of the stress - crack curve, which is a typical property of most quasi-brittle composites like concrete, is found to be an important factor in determining the size number together with the fracture energy, the elastic modulus and the ultimate tensile strength of materials. The influence of the structural size on the fracture state is also graphically presented.

1. INTRODUCTION

As it has been empirically demonstrated that the state of the crack propagation (stable or unstable) in concrete structures strongly depends on the structural size, configuration and the fracture properties of materials. For structures made of the same material and with similar configuration, differing only in the structural size, the structures of smaller size seems to be more ductile as subjected to a load. The fracture state of concrete beams in three-point bending has been proved well by Carpinteri (1986), [1] using the energy brittleness number (S_E) as

$$S_E = \frac{G_F}{bf_t} \tag{1.1}$$

In which the influence of the beam size and some material fracture properties as the energy fracture (G_F) and the tensile strength (f_t) on the fracture state are covered. It has been demonstrated graphically (Fig. 1) that the fracture mode changes from the more ductile state (S_E is higher) to the more brittle one (S_E is lower). However the number S_E does not deal with the influence of the stress - crack opening relation ($\sigma - w$ relation) which is a typical property of concrete and most quasi- brittle composite materials. As it has been well known that this relation changes for different composition of concrete and it is proved to strongly influence the fracture parameters in concrete structures (Carpinteri et al., 1987; Duda and König, 1992; Roelfstra and Wittmann, 1986; Tran tu and Kasperkiewicz, 1994).

In the text the influence of the stress - crack opening curve on the fracture state of concrete structures is checked by theoretically analyzing the fracture state of concrete elements loaded in the uniaxial tension and in bending. The general equation describing the fracture state is found. From this equation the size numbers proposed by Carpinteri (S_E), [1] and Hillerborg (b/ℓ_{ch}), [6] are obtained. A newly proposed size number is developed and it may be called the critical size of materials. The numerical analytic results of the fracture obtained using the fictitious crack model for beams in three-point bending have been proved to be good for determination of the fracture state by the new size number.

In order to follow the matters presented in the text, a new term called the shape index of $\sigma - w$ relation (S_T) proposed by Tran Tu and Kasperkiewicz (1994), [14] is presented briefly. It is defined as follows:

$$S_T = \frac{G_F}{f_t w_c} \tag{1.2}$$

We can consider some typical properties of the shape index S_T. Using Eq. (1.2), it may be said that S_T approaches infinity in case of elastic materials (w_c reduces to zero). For the plastic fracture materials S_T equals 1 (in this case $G_F = w_c f_t$). With concrete when the $\sigma - w$ curve is taken to be mono-straight $S_T = 0.5$ ($G_F = w_c f_t/2$). From Eq. (1.2) we expect S_T to be one of the parameters determining the shape of the $\sigma - w$ curve.

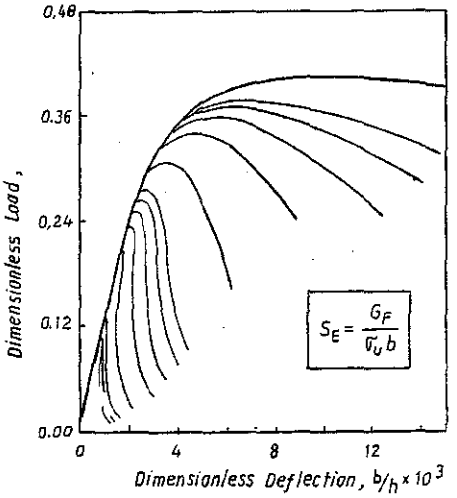

Fig. 1. The fracture state of beam in three-point bending and the energy brittle number S_E [2]

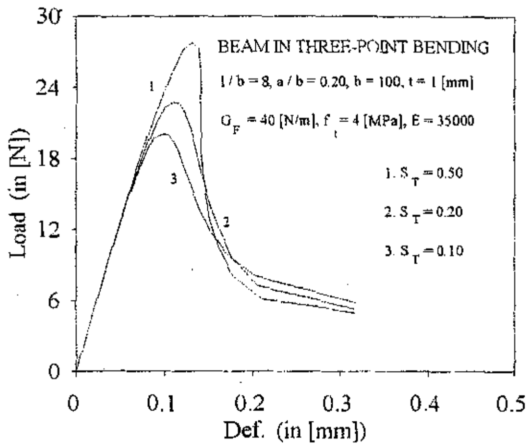

Fig. 2. The fracture state of beam in three-point bending depends on the shape of the $\sigma - w$ curve

2. FRACTURE STATE OF CONCRETE STRUCTURES

According to Carpinteri (1990), [2] stable or unstable fracture state is defined by the slope of the post-failure branch of load-deflection diagram referring to the vertical direction. The following example shows that the size number S_E is insufficient to estimate the fracture state. Fig. 2 gives three load-deflection diagrams taken from numerical fracture analysis for notched beam in three-point bending with $G_F = 40$ [N/m], $f_t = 4$ [MPa], $E = 35000$ [MPa] with S_T ranging between 0.10 and 0.5 (as shown in Figure). In this case the value S_E calculated from Carpinteri's proposal is constant: $S_E = 10^{-4}$. As it has been seen in Figure, fracture state depends on the shape of the $\sigma - w$ curve.

At first, let us study an uniaxial tensile specimen, (Fig. 3) in which a part B is assumed to be very short so that its boundary displacement is to the crack opening displacement. The total displacement of a system composing of the testing machine and the specimen d is determined as

$$\delta = C_m P + \frac{L}{EF} P + w \tag{2.1}$$

where L and F are the length and cross-sectional area of the part A of specimen, E - elastic modulus, C_m - compliance of the test machine, P - tensile load and w - boundary displacement of the part B evaluated by the relation between the tensile stress and the crack opening displacement

: $g(\sigma/f_t)w_c$, that is the intrinsic fracture property of the material. In order to study the fracture .e of the specimen, let us differentiate d with respect to P and denoting $D_w = -\partial g/\partial(\sigma/f_t)$ as

$$\frac{\partial \delta}{\partial P} = C_m + \frac{L}{EF} - \frac{w_c}{f_t F_w} D_w \tag{2.2}$$

ıre F_w is the cross-sectional area at notch.

The fracture state of specimen is determined by the value $\partial\delta/\partial P$. The stable state occurs ın $\partial\delta/\partial P < 0$ and the unstable one when $\partial\delta/\partial P \geq 0$. Let us study the limiting state from the ɔwing

$$\frac{w_c}{f_t F_w} D_w = C_m + \frac{L}{EF} \tag{2.3}$$

The influence of the stiffness of a testing machine on the fracture state of a specimen is nulated in Eq. (2.2). With the machine controlled in loading well as in displacement, the first n in the right side of Eq. (2.2) can be neglected, we arrive at the following expression:

$$\frac{w_c}{f_t} D_w = \frac{LF_w}{EF} \tag{2.4}$$

From Eq. (2.4) we can derive the size numbers proposed by Hillerborg and Carpinteri assuming t the $\sigma - w$ curve is a mono-straight, this means that $D_w = -1$, substituting $G_F = w_c f_t/2$ and ring E to the left side, Eq. (2.4) becomes:

$$\frac{G_F E}{F f_t^2} = \frac{LF_w}{2F^2} \tag{2.5}$$

The term in the right side is the size number proposed by Hillerborg. Similarly, moving f_t n the left side of Eq. (2.4) to the right side, we get

$$\frac{G_F}{F f_t} = \frac{L f_t F_w}{2EF^2} = \frac{LW_e F_w}{2 f_t F^2} \tag{2.6}$$

The right side of Eq. (2.6) represents the elastic strain energy stored in body divided by maximum tensile load and the energy brittleness number proposed by Carpinteri derived. The ve has proved that use of the size numbers proposed by Hillerborg and Carpinteri for simulation he effect of the size on the fracture parameters and fracture state is only an approximation. s was shown clearly in Fig. 2. Previous investigations (Hillerborg 1987,[7]; Carpinteri 1990, [2]; ınd Liang 1992, [9]) proved that the maximum load reached for the crack opening displacement bout 1/3 to 1/2 of w_c. Therefore in order to study the fracture state, the value D_w is taken r this range. As it has been seen in Fig. 3b, D_w increases with decreasing the shape index S_T, approximate replacement of $D_w = 1/S_T$ in Eq. (2.4) we arrive at the following equation

$$\frac{EG_F}{f_t^2 S_T^2} = \frac{LF_w}{F} \tag{2.7}$$

which, the ratio L/F represents the slender and F_w/F represents the notched length of a cimen. The term in the left side of Eq. (2.7) is a function of the fracture properties, on the er hand the term on the right side is a function of the characteristic size of the specimen. Taking left side of Eq. (2.7) to be a term having the dimension $[L]$ and denoting L_s as

$$L_s = \frac{G_F E}{(f_t S_T)^2} \tag{2.8}$$

The above results are obtained from the uniaxial tensile specimen, we need to know how they ıave in elements loaded in bending. Let us assume that a notched beam subjected to bending load P, notched length is a, span and depth of the beam is ℓ and b respectively, length of the fracture process region is ℓ_c, cohesive forces is P_c. The deflection δ at the mid-span of the beam is determined from the following equation:

$$\delta = C_m P + C_2 P + C_3 P \tag{2.9}$$

where C_m is the compliance of the test machine, C_2 and C_3 - compliancies of the specimen caused by the action of P and P_c respectively. The value of P_c is determined from Fig. 4 by putting $\ell_F = \ell_c/b$ and $z = x/b$:

$$P_c = \int_c^{L_F} f(w/w_c) f_t b dz \tag{2.10}$$

in which x-axis coincides with the crack growing direction and a function $\sigma/f_t = f(w/w_c)$.

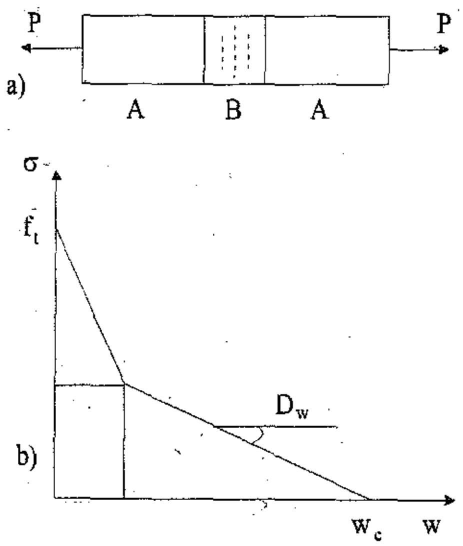

Fig. 3. A specimen loaded in tension and a bilinear diagram of the $\sigma - w$ curve

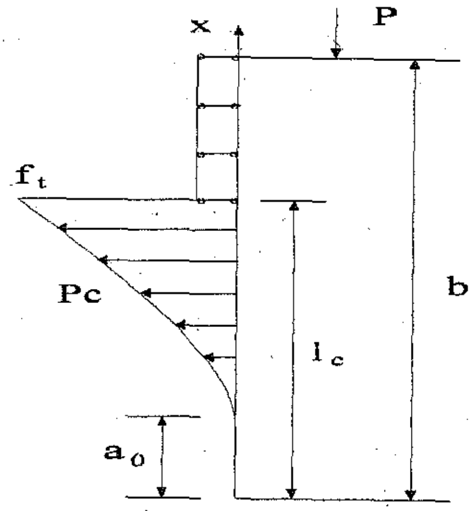

Fig. 4. A speciment loaded in bending and the distribution of cohesive forces

The value w is determined by the superposition of P and P_c:

$$w = C_4 P + C_5 P_c \tag{2.11}$$

where, C_4 and C_5 are compliancies of the specimen in different cases of the studied point. Derivative δ with respect to P and substituting (2.10), (2.11) into (2.9):

$$\frac{\partial \delta}{\partial P} = C_m + C_2 + C_3 \frac{\int_0^{L_F} C_4 dz}{\frac{w_c}{f_t b f(w/w_c)} - \int_0^{L_f} C_3 dz} \tag{2.12}$$

putting:

$$A = \int_0^{L_F} C_4 dz = A\left(\frac{a}{b}, \frac{L_c}{b}\right), \quad D = \int_0^{L_F} C_5 dz = D\left(\frac{a}{b}, \frac{L_c}{b}\right)$$

we derive the following formula:

$$\frac{1}{b}\frac{w_c}{f_t}D_w = \frac{C_3}{C_2}A - D \tag{2.13}$$

The left side of Eq. (2.13) similar to Eq. (2.4) is L_S, whereas the length of the fracture process zone has a functional relation with L_S, formally we can derive an expression as

$$\frac{EG_F}{(f_t S_T)^2} = \beta\left(\frac{a}{b}, \lambda, \lambda_p\right)b \tag{2.14}$$

where λ is the slender of beam and λ_p is a coefficient depending on the loading position.

Again we can see that the term in the left side of Eq. (2.14) is L_S, Whereas the right side only describes the dependence of the beam size on the loading condition. The term L_S is described by Eq. (2.8) and depends only on the intrinsic fracture properties of materials. Remembering that Eqs (2.4) and (2.14) express the limiting fracture state, L_S may thus be called the critical size of materials. As it has been seen in Fig. 5 the fracture state of a beam depends on the critical size of materials (L_S/b changes from 1.75 to 26.25).

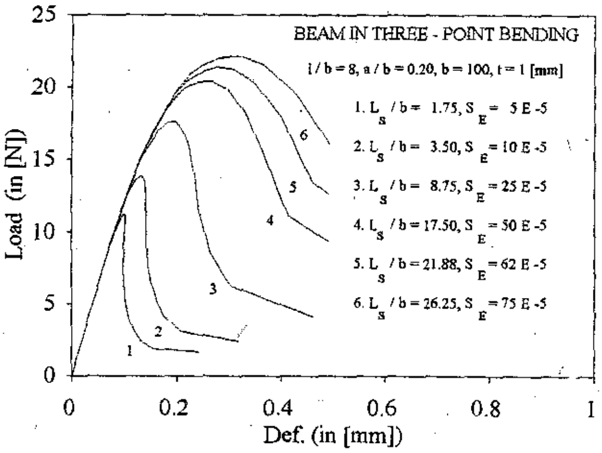

Fig. 5. The dependence of the fracture state of the critical size of concrete

3. PRESENTATION OF THE FRACTURE STATE IN NOTCHED BEAMS

3.1. Numerical experiment

Important in the fictitious model for creating the cohesive forces and controlling them in the crack propagating process. The cohesive forces are calculated by steps based on the stress-crack opening relation. This relation were formulated by many researches such as Reinhardt (1984), [11]; Gopalaratnam and Shah (1985), [5]; Cornelissen et al. (1987), [3] and Tran Tu and Kasperkiewicz (1994), [14]. As it is replaced by multi-linear diagrams as by Hillerborg et al. (1976), [6]; Peterson and Gustarsson (1981), [10]; Roelfstra and Wittmann (1986), [12]; Carpinteri (1987), [1]; etc. As it has been proved by Tran Tu and Kasperkiewicz (1994), [14] that the load-deflection diagrams for beams in bending obtained by applying the equations of Cornelissen et al., Gopalaratnam and Shah, Tran Tu and Kasperkiewicz and the consistent bilinear diagrams, are rather similar. In this text Equation describing the stress-crack opening curve proposed by Tran Tu and Kasperkiewicz (1994) is used:

$$\frac{\sigma}{f_t} = (1 - A)(1 - x^k) + A(1 - x)^{1/k} \tag{3.1}$$

$$x = \frac{w}{w_c}, \quad k = \frac{S_T}{1 - S_T}$$

where the coefficient A is chosen to be 0.5. The change of A may be offset by changing the shape index S_T which does not have much influence on the obtained results. The use of Eq. (3.1) is convenient for covering the influence of the stress-crack opening curve on the fracture parameters in concrete structures. In which a change is only in the shape index S_T that is considered to be the intrinsic fracture property of concrete.

Numerical approach can be presented in brief as follows:

1) Calculating the load where the micro crack starts developing. The length of the real crack is the notched length. The fracture criterion of the critical stress intensity factor is employed.

2) Calculating the crack extension, the maximum main stress criterion is used. The fictitious crack length is assumed to increase by steps until the total length of the real crack and the fictitious crack reaches the crack ligament. The cohesive forces are calculated according to the $\sigma - w$ relation by iteration. They are controlled by the critical deviations of two neighboring steps, not larger than the specific value (it is specified here about 0.01 [N]). It has been proved (Tran Tu and Kasperkiewicz, 1994) that the load-deflection diagrams will be discrepant if the shape of the $\sigma - w$ cure is missing.

3.2. Fracture state of concrete notched beams in bending

In this chapter the illustrations on the fracture state in the concrete notched beams in three-point bending is presented. It includes the influence of the material fracture properties and the beam depth through the critical size to beam depth ratio (L_S/b).

1) Arrangements are made, that include the change of the fracture energy G_F from 20 [N/m] to 180 [N/m] and the shape index S_T from 0.1 to 0.9. the unchanged factors are the tensile strength $f_t = 4$ [MPa], the elastic modulus $E = 35000$ [MPa], the beam sizes: depth $b = 100$ [mm], span $\ell = 800$ [mm], the thickness of unit is chosen. Fig. 6 is plotted for $G_F = 20 - 180$ [N/m] and $S_T = 0.3$. The ratio L_S/b and the energy brittleness number S_E are calculated and presented in the figure. In this case we can see that the fracture state becomes more ductile with the increase of the L_S and S_E similar to the results obtained by Carpinteri (1990).

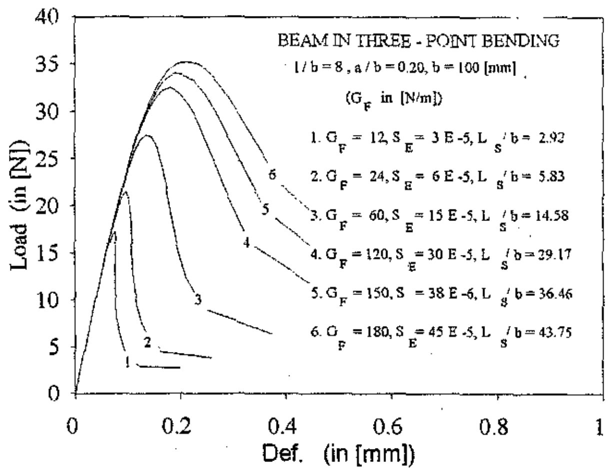

Fig. 6. **The dependence of the fracture state on the critical size with the changing fracture energy**

The change of the shape index S_T is presented in Fig. 7. The values L_S/b and S_E are also calculated and presented in graphs. We can see that the fracture state becomes more brittle as the value S_E increases contrarily to the conclusion of Carpinteri. In this case the value L_S decreases in accordance with the theoretical prediction.

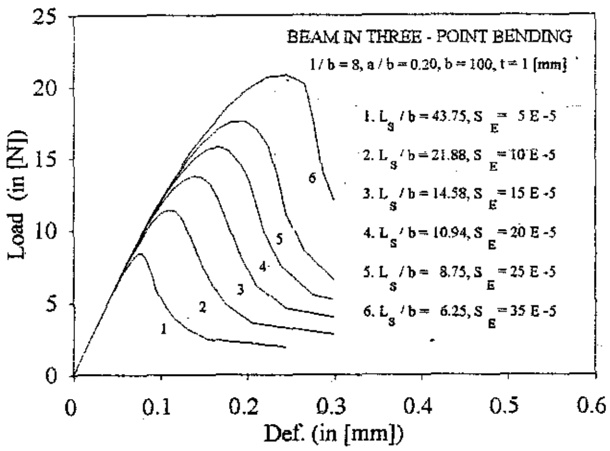

Fig. 7. The dependence of the fracture state on the critical size with the changing shape of the $\sigma - w$ curve

2) In the second case the dependence of the fracture state on the L_S with the change of elastic modulus of concrete is studied. Three load-deflection diagrams are plotted with the same $G_F = 80$ [N/m], $f_t = 4$ [MPa] and $w_c = 0.05$ [mm]. The values of the elastic modulus are shown in Fig. 8. We can see clearly in the figure that the fracture state becomes more brittle with the increase of elastic modulus.

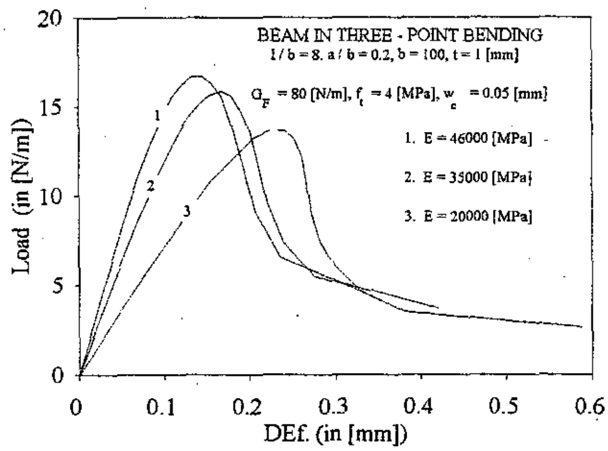

Fig. 8. The dependence of the fracture state on the elastic modulus of concrete

3) The last, the effect of the change in the beam depth is studied and otherwise unchange. This is a case that was noted by many researches. The change of beam depth between 100 [mm] and 400 [mm], the fracture energy of 80 [N/m] and S_E of 0.4 are chosen. We can see clearly that the fracture state becomes more brittle with increasing beam depth (Fig. 9).

4. CONCLUSION

The critical size L_S may be considered as the intrinsic fracture property of concrete and similar materials. It characterizes the fracture state (stable or unstable) of concrete structures.

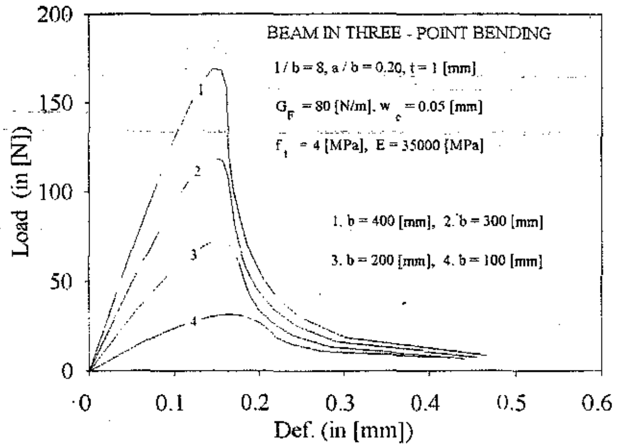

Fig. 9. The fracture state changes in accordance with the changes of beam depth

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HỆ SỐ ẢNH HƯỞNG KÍCH THƯỚC MỚI VÀ TRẠNG THÁI PHÁ HỦY CỦA KẾT CẤU BÊ TÔNG

Một hệ số ảnh hưởng kích thước của kết cấu bê tông được phát hiện trong đó kích thước của kết cấu và tính chất phá hủy của vật liệu được thể hiện trong công thức. Bằng đồ thị, tác giả chứng minh được sự phụ thuộc lớn của trạng thái phá hủy của dầm bê tông chịu uốn ba điểm vào số kích thước mới này. Hình dạng của đường cong ứng suất - độ mở vết nứt ($\sigma - w$), đây là một tính chất điển hình của vật liệu bê tông và các vật liệu composite tương tự, đã được chỉ ra như là một yếu tố đặc biệt quan trọng ảnh hưởng đến trạng thái phá hủy. Ngoài ra vai trò quan trọng của các tính chất phá hủy khác của bê tông như mô đun đàn hồi E, năng lượng phá hủy G_F cũng được thể hiện từ kết quả phân tích phá hủy.

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NGHIÊN CỨU ẢNH HƯỞNG CỦA QŨY ĐẠO BIẾN DẠNG ĐẾN ĐƯỜNG CONG GIỚI HẠN HÌNH THÀNH

Mục đích nghiên cứu ở đây là dự đoán ảnh hưởng của qũy đạo biến dạng đến đường cong giới hạn hình thành, từ biểu đồ ứng suất giới hạn. Chúng ta sẽ nghiên cứu qũy đạo cấu tạo từ hai đoạn thẳng tương ứng với các giai đoạn: kéo đúng tâm - dãn đều hoặc ngược lại.