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GYROSCOP ON THE ACCELERATED ROTATIVE BASIS

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In this paper, authors consider the balanced gyroscop on the accelerated rotative basis. Have been found interesting his nature.

§1. THE MOTION EQUATION OF GYRO

Suppose that: Gyro is balanced, he rotates at symmetric axis of rotor. There are only liquid bearing friction. The others supposition and system coordinate have been kept as [1].

Made hypothesis that: $v_1 = v_2 = 0$; $v_3 = \omega_0 t \quad 0 \le t \le t_0$. So:

$$\tilde{r}_2 = \omega_0 t \cos x; \quad \tilde{q}_2 = \omega_0 t \sin x, \quad \tilde{\psi} = d_3 \omega_0 \sin x;$$

$$\tilde{\varphi} = \sin 2y \sin x(-d_0 - C_1)\frac{\omega_0}{2} + d_3\omega_0 \sin x$$

System quasi (2.2) in [1] will be written

$$\frac{dx}{dt} = \omega_1; \qquad \frac{dy}{dt} = \omega_2$$

$$\begin{aligned}
\mu A(y)\frac{d\omega_1}{dt} &+ \mu[d_2\omega_1\omega_2 \sin 2y + \omega_0 t\omega_2 \cos x(d_2 \cos 2y - d_3) \\
&+ \frac{C_1}{2}(\omega_0 t)^2 \sin 2x \cos^2 y + (\omega_0 t)^2 \frac{\sin 2x}{2}(d_0 \sin^2 y - d_3 + d_4) \\
&+ \omega_0 t \frac{\sin 2y}{2} \sin x(C_1 - d_0)] + \omega_2 \cos y + \omega_0 t \sin x \cos y - -n\chi\omega_1 \qquad (1.1)\\
\mu d_3 \frac{d\omega_2}{dt} &+ \mu\Big\{\omega_1[d_0\omega_0 t \cos x - d_2(\omega_1 \sin y + \omega_0 t\frac{\sin 2y}{2}) \cos y] \\
&- d_3\omega_0 t \sin x\Big\} - \omega_1 \cos y + \omega_0 t \cos x \sin y = -n\chi\omega_2
\end{aligned}$$

System (1.1) will be studied by separation method of motion [2]

§2. BUILD ASYMPTOTIC SOLUTION

Consider (x, y) - small

a) The solution in outside boundary layer

At "0" order of smallness parameter μ, we have

$$\begin{aligned}
&\frac{dx^{(0)}}{dt} = \omega_1^{(0)}; \qquad \frac{dy^{(0)}}{dt} = \omega_2^{(0)} \\
&-\omega_2^{(0)} \cos y^{(0)} - \omega_0 t \sin x^{(0)} \cos y^{(0)} - n\chi\omega_1^{(0)} = 0 \qquad (2.1)\\
&\omega_1^{(0)} \cos y^{(0)} - \omega_0 t \sin y^{(0)} \cos x^{(0)} - n\chi\omega_2^{(0)} = 0.
\end{aligned}$$

Initial condition for (2.1):

$$x^{(0)}(t)\big|_{t=0} = x_0; \quad y^{(0)}(t)\big|_{t=0} = 0$$

Linearizing (2.1), receive

$$\frac{dy^{(0)}}{dt} + \omega_0 t x^{(0)} + n\chi\frac{dx^{(0)}}{dt} = 0$$
$$-n\chi\frac{dy^{(0)}}{dt} - \omega_0 t y^{(0)} + \frac{dx^{(0)}}{dt} = 0$$

Multiplying the second quasi on i-supposed and adding one with the first; result is:

$$(n\chi + i)\frac{d}{dt}(x^{(0)} - iy^{(0)}) + \omega_0 t(x^{(0)} - iy^{(0)}) = 0$$

Solution have been represented in form:

$$\begin{aligned} x^{(0)} &= x_0 e^{-\tilde{\omega}_0 n\chi t^2}\cos\tilde{\omega}_0 t^2 \\ y^{(0)} &= x_0 e^{-\tilde{\omega}_0 n\chi t^2}\sin\tilde{\omega}_0 t^2; \quad \tilde{\omega}_0 = \frac{\omega_0}{1+n^2\chi^2}. \end{aligned} \tag{2.2}$$

At the first order of small parameter μ receive following system

$$\begin{aligned} \frac{dy^{(1)}}{dt} + n\chi\frac{dx^{(1)}}{dt} + \omega_0 t x^{(1)} &= -(a_2 + a_1 + a_0)\frac{d^2x^{(0)}}{dt^2} - (d_2 - d_3)\omega_0 t \\ &\quad - (\omega_0 t)^2[x^{(0)}(d_4 - d_3) + C_1] \\ -\frac{dx^{(1)}}{dt} + n\chi\frac{dy^{(1)}}{dt} + \omega_0 t y^{(1)} &= -(a_0 + b_1)\frac{d^2y^{(0)}}{dt^2} - d_0\omega_0 t\frac{dx^{(1)}}{dt} - d_3\omega_0 t x^{(0)} \end{aligned} \tag{2.3}$$

Initial condition

$$x^{(1)}(t)\big|_{t=0} = y^{(1)}(t)\big|_{t=0} = 0$$

Solution of (2.3) easy describe by form:

$$\begin{aligned} x^{(1)}(t) &= e^{-\tilde{\omega}_0 n\chi t^2}\left[\frac{d_0 x_0}{12}t^3\cos\tilde{\omega}_0 t^2 + M_1\cos\tilde{\omega}_0 t^2 + M_2\sin\tilde{\omega}_0 t^2 + M_0\right] \\ y^{(1)}(t) &= e^{-\tilde{\omega}_0 n\chi t^2}\left[\frac{d_0 x_0\tilde{\omega}_0 t}{8}\sin\tilde{\omega}_0 t^2 + N_1\cos\tilde{\omega}_0 t^2 + N_2\sin\tilde{\omega}_0 t^2 + N_0\right] \end{aligned} \tag{2.4}$$

where M_i, N_i - constants only dependent on a_i, d_i, x_0, $n\chi$, $\tilde{\omega}_0$.

b) The asymptotic solution at inside boundary layer

At the "zero" order of μ we have:

$$\frac{dx^{(0)}}{d\tau} = 0; \quad \frac{dy^{(0)}}{d\tau} = 0; \quad \tau = \frac{t}{\mu}.$$

Initial condition

$$x^{(0)}(\tau)\big|_{\tau=0} = y^{(0)}(\tau)\big|_{\tau=0} = 0$$

So: $x^{(0)}(\tau) = y^{(0)}(\tau) \equiv 0$

The "first" order of μ, receive system of quasi:

$$\frac{dx^{(1)}}{d\tau} = \omega_1^{(0)}; \quad \frac{dy^{(1)}}{d\tau} = \omega_2^{(0)}; \quad x^{(1)}(\tau)\big|_{\tau=0} = y^{(1)}(\tau)\big|_{\tau=0} = 0$$
$$(a_2 + a_1 + a_0)\frac{d\omega_1^{(0)}}{d\tau} + n\chi\omega_1^{(0)} + \omega_2^{(0)} = 0 \tag{2.5}$$
$$-\omega_1^{(0)} + (a_0 + b_1)\frac{d\omega_2^{(0)}}{d\tau} + n\chi\omega_2^{(0)} = 0$$

System (2.5) has quasi of feature:

$$\begin{vmatrix} (a_2 + a_1 + a_0)\lambda + n\chi & 1 \\ 1 & (a_0 + b_1)\lambda + n\chi \end{vmatrix} = 0 \tag{2.6}$$

Solution of (2.6) have been represented by form:

$$\lambda_{1/2} = -K_0 + iK_1$$

where

$$K_0 = \frac{n\chi(a_2 + a_1 + 2a_0 + b_1)}{2(a_2 + a_1 + a_0)(a_0 + b_1)}; \quad K_1 = \frac{\sqrt{n\chi^2 a_0^2 - 4(a_0 + b_1)(a_2 + a_1 + a_0)}}{2(a_2 + a_1 + a_0)(a_0 + b_1)}$$

Accordingly, solution $\omega_i^{(0)}(\tau)$ of system (2.5) is being expressed by formula:

$$\omega_1^{(0)} = (C_1 \cos K_1\tau + C_2 \sin K_1\tau)e^{-K_0\tau} \tag{2.7}$$
$$\omega_2^{(0)} = \left[C_1(s_0 \cos K_1\tau + s_1 \sin K_1\tau) + C_2(s_2 \cos K_1\tau + s_3 \sin K_1\tau)\right]e^{-K_0\tau}$$

s_i - const:

$$\omega_1^{(0)}(\tau)\big|_{\tau=0} = \omega_2^{(0)}(tau)\big|_{\tau=0} = 0$$

From this condition, receive:

$$\omega_1^{(0)}(\tau) = \omega_2^{(0)}(\tau) = 0$$

But follow the first quasi of system (2.5); we have

$$x^{(1)} = \int \omega_1^{(0)} d\tau = 0; \quad y^{(1)} = \int \omega_2^{(0)} d\tau = 0.$$

Then, in boundary layer, receive:

$$x^{(0)}(\tau) = x^{(1)}(\tau) = 0$$
$$y^{(0)}(\tau) = y^{(1)}(\tau) = 0 \tag{2.8}$$

In the end, from (2.2), (2.4); (2.8), we have

$$x(t,\mu) = x_0 e^{-\tilde{\omega}_0 n\chi t^2} \cos\tilde{\omega}_0 t^2 + \mu x^{(1)}(t)$$
$$y(t,\mu) = x_0 e^{\tilde{\omega}_0 n\chi t^2} \sin\tilde{\omega}_0 t^2 + \mu y^{(1)}(t) \tag{2.9}$$

$$\tilde{\omega}_0 = \frac{\omega_0}{1 + n^2\chi^2}; \quad 0 \le t \le t_0$$

CONCLUSION

+ The result (2.9) shows that: Gyro has cross-bedding. This cross-bedding includes in element order μ.

+ Cross-bedding dependent on direction of rotation of basis.

+ The formula (2.8) represent that the motion happens very quickly so that, the motion in boundary layer beforche had time to expanse.

+ In general case

$$x^2(t,\mu) + y^2(t,\mu) = x_0^2 e^{-2\tilde{\omega}_0 n\chi t^2} + \mu \dots$$

So if $\tilde{\omega}_0 < 0$ then

$$x^2 + y^2 \approx x_0^2(1 - 2\tilde{\omega}_0 n\chi t^2)$$

Because t - small $\Rightarrow t^2$ - very smallness.

If $\tilde{\omega}_0 > 0$ t - big then:

$$x^2 + y^2 \to 0$$

If $\tilde{\omega}_0 < 0$, t - big then:

$$x^2 + y^2 = x_0^2 e^{2|\tilde{\omega}_0| n\chi t^2} + \mu$$

It mean must be attentively to the direction of rotative of basis.

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GYROSCOP TRÊN ĐẾ QUAY CÓ GIA TỐC

Tác giả sử dụng phương trình chuyển động của gyroscop cân bằng trong giá các đăng, đế quay có gia tốc. Nghiệm của hệ phương trình vi phân chuyển động được tìm bằng phương pháp tách chuyển động. Kết quả cho thấy ở gần đúng bậc "1" theo μ xuất hiện một tiến động nhỏ. Nói chung, chuyển động của gyroscop phụ thuộc mạnh vào chiều quay của đế.

VỀ PHƯƠNG PHÁP NGHIÊN CỨU KHẢ NĂNG ...

(tiếp theo trang 7)

SUMMARY

METHODS FOR STABILITY AND INSTABILITY OF THE SOLUTION OF ONE NON-LINEAR DIFFERENTIAL EQUATION

When research on instability states of tall and flexible structures is carried out, the solutions of non-linear differential equations have to be investigated. Althrough the fully analytical solutions of the moving rule of the structures cannot be found, but based on the conditions applied to the parameters of the moving differential equations the authors have studied the characteristics of the solutions when $t \to \infty$. Then the instability of the structures may be investigated, and the stability conditions can be concluded. This is the content which this paper would like to present.