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REDUCE IMPEDANCE FOR BRANCH COMPONENT IN STRUCTURAL DYNAMIC SYNTHESIS

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1. INSTRUCTION

The practical success of the structural modal synthesis using system depend widely on how well a small number of chosen modes can represent the response of the system [1]. In order to improve modal representation Benfield and Hruda added reduced stiffness and mass matrices to each component before computing its modes [2]. An exact component representation coos then presented by Berman [3] using reduced impedance.

This paper is intended to formulate the reduced impedance for a statically determinate branch component and to present an approach which allows an improvement of that impedance in case of modal truncation.

2. REDUCED IMPEDANCE

Consider a structural system composed of the main component "M" and a branch components "k" as illustrated in figure 1. It is convenient to rearrange and partition the elements of the two impedance matrices in the following fashion

$$Z_M = \begin{bmatrix} Z_{\ell\ell} & Z_{\ell r} \\ Z_{r\ell} & Z_{rr} \end{bmatrix}, \qquad Z_k = \begin{bmatrix} Z^k_{rr} & Z^k_{r\ell} \\ Z^k_{\ell r} & Z^k_{\ell\ell} \end{bmatrix}$$

where r refers to interface coordinates and ℓ refers to non-interface coordinates

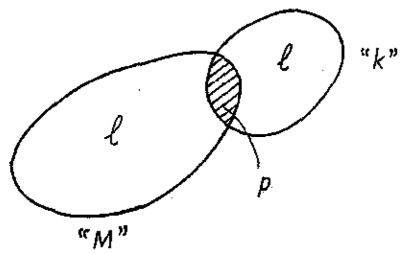

Fig. 1

The impedance of the system, then, may be exactly formed by superimposing these matrices as follows

$$Z_s = \begin{bmatrix} Z_{\ell\ell} & Z_{\ell r} & 0 \\ Z_{r\ell} & Z_{rr} + Z^k_{rr} & Z^k_{r\ell} \\ 0 & Z^k_{\ell r} & Z^k_{\ell\ell} \end{bmatrix}$$

If a valid model of component "k" could be formed using only the interface coordinates, the impedance of the system could be written as

$$Z_s = \begin{bmatrix} Z_{\ell\ell} & Z_{\ell r} \\ Z_{r\ell} & Z_{rr} + \tilde{Z}^k_{rr} \end{bmatrix}$$

where $\tilde{Z}^k_{rr}$ is called the reduced impedance of the component "k".

This concept readily extends to a structural system of several branch components ($k = 1, 2, \ldots, s$).

3. DETERMINATION OF REDUCED IMPEDANCE

Consider a branch component the interface of which is assumed isostatic. Its displacement may be expressed by the vector

$$q(t) = \begin{bmatrix} q_r \\ q_\ell \end{bmatrix} \tag{3.1}$$

The equation of motion of the component is

$$M\ddot{q} + Kq = F \tag{3.2}$$

where the stiffness, mass and force matrices can then be partitioned as

$$M = \begin{bmatrix} M_{rr} & M_{r\ell} \\ M_{\ell r} & M_{\ell\ell} \end{bmatrix}; \quad K = \begin{bmatrix} K_{rr} & K_{r\ell} \\ K_{\ell r} & K_{\ell\ell} \end{bmatrix}; \quad F = \begin{bmatrix} F_r \\ 0 \end{bmatrix}$$

The component motion is described by superimposition of the rigid-body motion excited by the main component and the flexible motion relative to the latter. This will result in

$$q = \begin{bmatrix} \phi_r & \varphi_p \end{bmatrix} \begin{bmatrix} q_r \\ y_p \end{bmatrix} \tag{3.3}$$

where ϕ_r is rigid-mode matrix defined by the interface coordinates, φ_p is elastic mode matrix the cantilevered branch component.

It is easily seen that, in this case, the rigid-mode matrix can be partitioned as

$$\phi = \begin{bmatrix} I \\ \phi_{\ell r} \end{bmatrix}$$

which must satisfy the basic condition

$$K\phi_r = 0 \tag{3.4}$$

and the elastic mode matrix may be expressed under the form:

$$\varphi_p = \begin{bmatrix} 0 \\ \varphi_{\ell p} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ x_\ell^{(1)} & x_\ell^{(2)} & \ldots & x_\ell^{(n)} \end{bmatrix}$$

where $x_\ell^{(i)}$ are eigenvectors of the equation

$$[K_{\ell\ell} - \lambda M_{\ell\ell}]x_\ell = 0$$

and the corresponding eigenvalues will be designated by $\omega_1, \omega_2, \ldots, \omega_n$.

Replacing q by (3.3) into (3.2) then multiplying by $[\phi_r \quad \phi_p]^T$ the equation (3.2) taking into account (3.4) becomes

$$\begin{bmatrix} m_{rr} & L_{pr}^T \\ L_{pr} & m_p^* \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{y}_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & k_p^* \end{bmatrix} \begin{bmatrix} q_r \\ y_p \end{bmatrix} = \begin{bmatrix} F_r \\ 0 \end{bmatrix} \tag{3.5}$$

where

$m_{rr} = \phi_r^T M \phi_r$ rigid-body mass matrix
$m_p^* = \varphi_p^T M \varphi_p$ generalized mass matrix
$k_p^* = \varphi_p^T K \varphi_p$ generalized stiffness matrix
$L_{pr} = \varphi_p^T M \phi_r$

The system of equations (3.5) may be written under the alternative form

$$m_p \ddot{y}_p + k_p^* y_p = -L_{pr} q_r \tag{3.6}$$

$$F_r = m_{rr} \ddot{q}_r + L_{pr}^T \ddot{y}_p \tag{3.7}$$

For a harmonic excitation

$$F_r = \overline{F}_r e^{j\omega t} \tag{3.8}$$

a steady-state solution is assumed of the form

$$q_r = \overline{q}_r e^{j\omega t} \quad \text{and} \quad y_p = \overline{y}_p e^{j\omega t} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} e^{j\omega t} \tag{3.9}$$

Substitution of (3.9) into (3.7) yields

$$\begin{aligned} \overline{F}_r &= -\omega^2 m_{rr} \overline{q}_r - \omega^2 L_{pr}^T \overline{y}_p \\ &= -\omega^2 m_{rr} \overline{q}_r - \omega^2 \sum_{i=1}^{n} L_{ir}^T \overline{y}_i \end{aligned} \tag{3.10}$$

Substitution of (3.8), (3.9) into (3.6) yields

$$y_i = \frac{\omega^2}{\omega_i^2 - \omega^2} \cdot \frac{L_{ir}}{m_i} \overline{q}_r \tag{3.11}$$

According to (3.10) and (3.11) the following relation is obtained

$$\overline{F}_r = -\omega^2 \left[m_{rr} - \sum_{i=1}^{n} \frac{\omega^2}{\omega^2 - \omega_i^2} M_{rr,i} \right] \overline{q}_r \tag{3.12}$$

where

$$M_{rr,i} = \frac{1}{m_i} L_{ir}^T L_{ir} = \frac{1}{m_i} \phi_r^T M^T x^{(i)} x^{(i)T} M \phi_r$$

which may be called effective mass of the natural mode "i" and generally satisfy the property

$$\sum_{i=1}^{n} M_{rr,i} = m_{rr} \tag{3.13}$$

According to the definition of reduced impedance $\overline{F}_r = \tilde{Z}_{rr}\overline{q}_r$ the equation (3.12) taking account (3.13) results in

$$\tilde{Z}_{rr} = -\omega^2 \sum_{i=1}^{n} \frac{\omega_i^2}{\omega_i^2 - \omega^2} M_{rr,i} \tag{3.14}$$

the quantity $\tilde{Z}_{rr}$ is called the reduced impedance which now is directly added to the main component at the interface coordinates q_r.

4. REDUCED IMPEDANCE WITH MODAL TRUNCATION

The impedance (3.14) is an exact representation of the full model at the interface coordinates. However, in practice an incomplete truncated modal set is generally used ($m < n$) and (3.14) becomes

$$Z_{rr}^m = -\omega^2 \sum_{i=1}^{m} \frac{\omega_i^2}{\omega_i^2 - \omega^2} M_{rr,i} \tag{4.1}$$

The truncation would produce a poor accuracy because of removing the contribution of a part of modes. In order to improve the accuracy a technique similar to the one presented in [4] will be used. The impedance (3.14) is written under the form

$$\tilde{Z}_{rr} = -\omega^2 \Big(\sum_{i=1}^{m} \frac{\omega_i^2}{\omega_i^2 - \omega^2} M_{rr,i} + \sum_{i=m+1}^{n} \frac{\omega_i^2}{\omega_i^2 - \omega^2} M_{rr,i} \Big)$$

For the spectrum of frequencies in the range of interest $\omega < \omega_m$ the contribution of truncated modes is not negligible, but for $\omega_i^2 \gg \omega^2$ we have $\frac{\omega_i^2}{\omega_i^2 - \omega^2} \sim 1$. This implies that the contribution of truncated modes can be approximately represented by their static contribution so that

$$\tilde{Z}_{rr}^m = -\omega^2 \Big(\sum_{i=1}^{m} \frac{\omega_i^2}{\omega_i^2 - \omega^2} M_{rr,i} + \sum_{i=m+1}^{n} M_{rr,i} \Big)$$

If the residual impedance is defined as

$$\begin{aligned} Z_{RES}^m &= -\omega^2 \sum_{i=m+1}^{n} M_{rr,i} \\ &= -\omega^2 \Big(\sum_{i=1}^{n} M_{rr,i} - \sum_{i=1}^{m} M_{rr,i} \Big) \end{aligned}$$

or

$$Z_{RES}^m = -\omega^2 \Big(m_{rr} - \sum_{i=1}^{m} M_{rr,i} \Big) \tag{4.3}$$

which represent an approximate contribution of the truncated modes so the reduced impedance (4.1), the improved reduced impedance (4.2) may be expressed as

$$\tilde{Z}_{rr}^m = Z_{rr}^m + Z_{RES}^m$$

5. EXAMPLES

A simple rod is analysed as a branch component which is assumed to interface a main component at its left end and to be submitted to a longitudinal motion $u(x)$ (fig. 2)

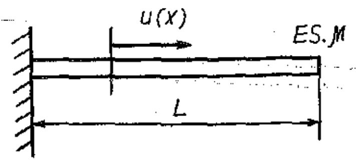

Fig. 2

The equation of motion of the cantilevered rod is

$$ESu'' - \mu u = 0$$

From which we have the modal frequencies

$$\omega_i = \frac{(2i-1)\pi}{2L}\sqrt{\frac{ES}{\mu}}$$

the corresponding mode shapes

$$\varphi_i(x) = C \sin\frac{(2i-1)\pi}{2L}x$$

the generalized masses

$$m_i = \int_0^L \mu\varphi_i^2 dx = \frac{1}{2}C^2\mu L$$

The rigid-body mode is given as

$$\phi_r(x) = 1$$

from which the rigid-body mass

$$m_{rr} = \int_0^L \mu\phi_r\phi_r dx = \mu L$$

the matrices L_{ir}

$$L_{ir} = \int_0^L \mu\varphi_i\phi_r dx = C\frac{2\mu L}{(2i-1)\pi}$$

the effective masses

$$M_{rr,i} = \frac{L_{ir}^2}{m_i} = \frac{8}{(2i-1)^2\pi^2}\mu L$$

It is easily seen that

$$\sum_{i=1}^{\infty} M_{rr,i} = \frac{8}{\pi^2}\mu L\Big(1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\Big) = \mu L = m_{rr}$$

If we retain the three first elastic modes, the residual impedance is

$$Z_{RES}^3 = -\omega^2\Big[1 - \frac{8}{\pi^2}\Big(1 + \frac{1}{3^2} + \frac{1}{5^2}\Big)\Big]\mu L$$

and the reduced impedance is

$$\tilde{Z}_{rr}^3 = Z_{RES}^3 - \omega^2\Big(\frac{\omega_1^2}{\omega_1^2-\omega^2} + \frac{\omega_2^2}{\omega_2^2-\omega^2}\cdot\frac{1}{9} + \frac{\omega_3^2}{\omega_3^2-\omega^2}\cdot\frac{1}{25}\Big)\frac{8}{\pi^2}\mu L$$

CONCLUSIONS

A technique which allows an exact representation of a branch component within the model of the main component with the help of the reduced impedance at the interface coordinates has been described. It is directly applicable in dynamic synthesis of structures which contain several branch components attached isostatically to the main component. When a modal truncation is used, a better accuracy of the reduced impedance will be expected by adding to it a residual impedance.

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TRỞ KHÁNG THU GỌN NHÁNH TRONG TỔNG HỢP ĐỘNG LỰC HỆ KẾT CẤU

Thông qua trở kháng thu gọn, đặc trưng động lực của nhánh được nối ghép trực tiếp vào nhánh chính tại các tọa độ nối. Việc lập trở kháng thu gọn cho một nhánh gắn tĩnh định vào nhánh chính đã được trình bày. Trở kháng thu gọn gần đúng do bỏ qua ảnh hưởng của một số dạng riêng được nâng cao độ chính xác nhờ việc đưa thêm vào trở kháng dư.