Tạp chí Cơ học Journal of Mechanics, NCNST of Vietnam T. XVIII, 1996, No 2 (7 - 12)

DYNAMIC ABSORBER FOR THE PARAMETRIC OSCILLATION OF THE RECTANGULAR THIN PLAT OF CREEP ON ELASTIC FOUNDATION

HOANG VAN DA
Technical University of Mining and Geology, Hanoi SRV

§0. INTRODUCTION

The effect of dynamic absorber for the oscillation of the mechanical systems with distributed parameters (beam and string) has been investigated (see for example [2, 3]).

However, the effect of dynamic absorber for the parametric oscillation of the rectangular thin plate of creep on the elastic foundation, to the author's knowledge, has not been hitheto examined.

This problem will be studied here by means of an asymptotic method for high-order systems [1] and boundary value problem [4].

§1. FORMULATION OF THE PROBLEM THE EQUATION OF MOTION

Now, let's consider the parametric oscillation of a rectangular thin plate of creep, having thickness h, Young's modulus E, specific mass ρ and lengths of edges b, c, which is supported on four edges and lying on the elastic foundation with one coefficient.

Its motion is loaded by longitudinal force, equally distributed $q = q(t)$. To decrease or to damps this oscillation, we use the weak dynamic absorber as shown in Fig. 1.

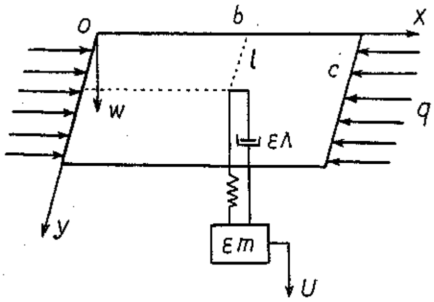

Fig. 1

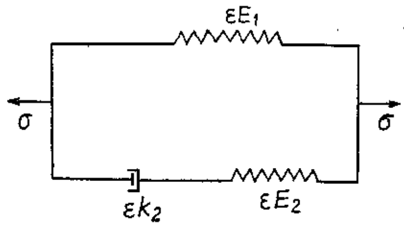

Fig. 2

The mechanical properties of material, when being strained, has been described by the model of the standard linear body [5]. The state equation in operator is written of the following form

$$\sigma = Ee, \tag{1.1}$$

$$E = \frac{E_1 + K_2\left(1 + \dfrac{E_1}{E_2}\right)\dfrac{\partial}{\partial t}}{\left(1 + \dfrac{K_2}{E_2}\right)\dfrac{\partial}{\partial t}} \,. \tag{1.2}$$

The motion equation in classical form of considered system will be

$$D\nabla^4 W + KW + \frac{\partial^2 W}{\partial t^2} M = \varepsilon\Big\{[U - W(d,\ell,t)]K_1\delta(x-d)\delta(y-\ell) + \lambda\Big[\frac{dU}{dt} - \frac{\partial}{\partial t}W(d,\ell,t)\Big]\delta(x-d)\delta(y-\ell) - hq\frac{\partial^2 W}{\partial x^2} + f\Big\}. \tag{1.3}$$

where δ - function of Dirac, ε - a small parameter, K - coefficient of the elastic foundation, D - the bending hardness

$$D = \frac{Eh^3}{12(1-\nu^2)}\ ; \qquad M = \rho h \tag{1.4}$$

Superseding the elastic modulus E by the analogous operator (1.2) into the expression D of the equation (1.3), we get the equational system of the problem

$$\begin{aligned}
&\frac{\partial^3 W}{\partial t^3} + \xi\frac{\partial^2 W}{\partial t^2} + \frac{D_1}{M}\frac{\partial}{\partial t}(\nabla^4 W + KW) + \xi(D_1\nabla^4 W + KW) = \\
&= \varepsilon\frac{\xi}{M}\Big\{K_1[U - W(d,\ell,t)]\delta(x-d)\delta(y-\ell) + \lambda\Big[\frac{dU}{dt} - \frac{\partial}{\partial t}W(d,\ell,t)\Big]\delta(x-d)\delta(y-\ell) \\
&- hq\frac{\partial^2 W}{\partial x^2} + f\Big\} + \frac{\varepsilon}{M}\frac{\partial}{\partial t}\Big\{K_1[U - W(d,\ell,t)]\delta(x-d)\delta(y-\ell) \\
&+ \lambda\Big[\frac{dU}{dt} - \frac{\partial}{\partial t}W(d,\ell,t)\Big]\delta(x-d)\delta(y-\ell) - hq\frac{\partial^2 W}{\partial x^2} + f + D_1\frac{E_2}{E_1}\nabla^4 W\Big\},
\end{aligned} \tag{1.5}$$

$$m\frac{d^2 U}{dt^2} + K_1[U - W(d,\ell,t)] = -\lambda\Big[\frac{dU}{dt} - \frac{\partial}{\partial t}W(d,\ell,t)\Big], \tag{1.6}$$

here

$$\begin{aligned}
&W = W(x,y,t) \text{ - deflection of the plate, } \nu \text{ - Poisson's ratio,} \\
&\nabla^4 = \Big(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}\Big) \text{ - Laplace's operator,} \\
&f = f\Big(W, \frac{\partial W}{\partial x}, \ldots\Big) \text{ - nonlinear function,} \\
&D_1 = \frac{E_1 h^3}{12(1-\nu^2)}, \qquad \xi = \frac{E_2}{K_2}.
\end{aligned} \tag{1.7}$$

The relevant homogeneous boundary conditions are as follow

$$\begin{aligned}
&W\Big|_{x=0,\ b} = 0, \qquad \frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2}\Big|_{x=0,\ b} = 0, \\
&W\Big|_{y=0,\ c} = 0, \qquad \frac{\partial^2 W}{\partial y^2} + \nu\frac{\partial^2 W}{\partial x^2}\Big|_{y=0,\ c} = 0,
\end{aligned} \tag{1.8}$$

For simplicity, it is supposed that $M = 1$ and we put $\Omega_1^2 = \dfrac{D_1}{M}$, $\omega^2 = \dfrac{K_1}{m}$, then the equational system (1.5), (1.6) is possibly written in the following form

$$\frac{\partial^3 W}{\partial t^3} + \xi\frac{\partial^2 W}{\partial t^2} + \frac{\partial}{\partial t}(\Omega^2\nabla^4 W + KW) + \xi(\Omega^2\nabla^4 W + KW) = \varepsilon F\Big(U, \frac{dU}{dt}, W, \frac{\partial W}{\partial t}, \ldots\Big), \tag{1.9}$$

$$\frac{d^2 U}{dt^2} + \omega^2[U - W(d,\ell,t)] = -\frac{\lambda}{m}\Big[\frac{dU}{dt} - \frac{\partial}{\partial t}W(d,\ell,t)\Big], \tag{1.10}$$

with the boundary conditions

$$
\begin{aligned}
W\Big|_{x=0,\ b} &= 0, \qquad \frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\Big|_{x=0,\ b} = 0, \\
W\Big|_{y=0,\ c} &= 0, \qquad \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\Big|_{y=0,\ c} = 0.
\end{aligned} \tag{1.11}
$$

§2. CONSTRUCTION OF THE ASYMPTOTIC SOLUTION

The solution of the equation (1.9) with the boundary conditions (1.11) can be found in the form

$$
W(x,y,t) = \sum_{n,m=1}^{\infty} S_{nm}(t) Z_{nm}(x,y), \quad Z_{nm}(x,y) = \sin\frac{n\pi x}{b}\sin\frac{m\pi y}{c}. \tag{2.1}
$$

Substituting (2.1) into the equation (1.9) and then applying Galerkin-Bubnov's method, we obtain the following equations for unknown functions $S_{nm}(t)$, $U(t)$

$$
\dddot{S}_{nm}(t) + \xi \ddot{S}_{nm}(t) + \Omega_{nm}^2 \dot{S}_{nm}(t) + \xi \Omega_{nm}^2 S_{nm}(t) = \varepsilon F(t), \tag{2.2}
$$

$$
\frac{d^2 U}{dt^2} + \omega^2 \Big[U - \sum_{n,m=1}^{\infty} S_{nm}(t) Z_{nm}(d,\ell)\Big] = -\frac{\lambda}{m}\Big[\frac{dU}{dt} - \sum \dot{S}_{nm}(t) Z_{nm}(d,\ell)\Big], \tag{2.3}
$$

where

$$
\Omega_{nm}^2 = \Omega^2\Big[\big(\frac{n\pi}{b}\big)^2 + \big(\frac{m\pi}{c}\big)^2\Big]^2 + K. \tag{2.4}
$$

It is supposed that there exists a form of particular oscillation $Z_{nm}(x,y) = \sin\frac{2\pi x}{b}\sin\frac{m\pi y}{c}$ and it is the most important when $n = m = 1$. Then equational system (2.2), (2.3) has the form

$$
\dddot{S}_{nm}(t) + \xi \ddot{S}_{nm}(t) + \Omega_{nm}^2 \dot{S}_{nm}(t) + \xi \Omega_{nm}^2 S_{nm}(t) = \varepsilon F(t), \tag{2.5}
$$

$$
\frac{d^2 U}{dt^2} + \omega^2 \Big[U - S_{nm}(t) X_{nm}(d,\ell)\Big] = -\frac{\lambda}{m}\Big[\frac{dU}{dt} - \dot{S}_{nm}(t) Z_{nm}(d,\ell)\Big], \tag{2.6}
$$

$$
\begin{aligned}
F(t) = {} & \varepsilon\frac{4\xi}{bc}\Big\{K_1[U - S_{nm}(t)Z_{nm}(d,\ell)]Z_{nm}(d,\ell) + \lambda\Big[\frac{dU}{dt} - \dot{S}_{nm}(t)Z_{nm}(d,\ell)\Big]Z_{nm}(d,\ell) \\
& + \frac{hbc}{4}\big(\frac{n\pi}{b}\big)^2 S_{nm}(t)q(t) + \int_0^b\int_0^c f Z_{nm}(x,y)\,dx\,dy\Big\} \\
& + \varepsilon\frac{4}{bc}\frac{\partial}{\partial t}\Big\{K_1[U - S_{nm}(t)Z_{nm}(d,\ell)]Z_{nm}(d,\ell) + \lambda\Big[\frac{dU}{dt} - \dot{S}_{nm}(t)Z_{nm}(d,\ell)\Big]Z_{nm}(d,\ell) \\
& + \frac{hbc}{4}\big(\frac{n\pi}{b}\big)^2 S_{nm}(t)q(t) + \int_0^b\int_0^c f Z_{nm}(x,y)\,dx\,dy\Big\} - \varepsilon H_{nm}\dot{S}_{nm}(t).
\end{aligned} \tag{2.7}
$$

here

$$
H_{nm} = \frac{E_2}{E_1}\Omega^2\Big[\big(\frac{n\pi}{b}\big)^2 + \big(\frac{m\pi}{c}\big)^2\Big]^2.
$$

Now, we shall study the case, when

$$
q = q_0 \sin\gamma t, \quad q_0 = \text{const}, \quad f = -\beta W^3, \quad \beta > 0. \tag{2.8}
$$

Function F (2.7) is of the form

$$F(t) = \varepsilon\frac{4\xi}{bc}\Big\{K_1[U - S_{nm}(t)Z_{nm}(d,\ell)]Z_{nm}(d,\ell) + \lambda\Big[\frac{dU}{dt} - \dot{S}_{nm}(t)Z_{nm}(d,\ell)\Big]Z_{nm}(d,\ell)$$
$$+ \frac{hbc}{4}\Big(\frac{n\pi}{b}\Big)^2 S_{nm}(t)q_0\sin\gamma t - \frac{9bc}{64}\beta S^3_{nm}(t)\Big\}$$
$$+ \varepsilon\frac{4}{bc}\frac{\partial}{\partial t}\Big\{K_1[U - S_{nm}(t)Z_{nm}(d,\ell)]Z_{nm}(d,\ell) + \lambda\Big[\frac{dU}{dt} - \dot{S}_{nm}(t)Z_{nm}(d,\ell)\Big]Z_{nm}(d,\ell)$$
$$+ \frac{hbc}{4}\Big(\frac{n\pi}{b}\Big)^2 S_{nm}(t)q_0\sin\gamma t - \frac{9bc}{64}\beta S^3_{nm}(t)\Big\} - \varepsilon H_{nm}\dot{S}_{nm}(t). \tag{2.9}$$

It is supposed that there is a resonance relation

$$\Omega_{nm} = \frac{1}{2}\gamma + \varepsilon\Delta, \tag{2.10}$$

Δ is the detuning coefficient.

With these assumptions, we are going to find the partial solution of the equational system (2.5), (2.6) in the following form [2]

$$S_{nm}(t) = a\cos\varphi, \quad \varphi = \Big(\frac{1}{2}\gamma t + \psi\Big), \tag{2.11}$$
$$U = a(L\cos\varphi + N\sin\varphi), \tag{2.12}$$

where L, N - constant and the quantities a, ψ are determined from the equations

$$\begin{aligned}\frac{da}{dt} &= \varepsilon A_1(a,\psi) + \varepsilon^2 A_2(a,\psi) + \dots,\\ \frac{d\psi}{dt} &= \Big(\Omega_{nm} - \frac{\gamma}{2}\Big) + \varepsilon B_1(a,\psi) + \varepsilon^2 B_2(a,\psi) + \dots\end{aligned} \tag{2.13}$$

By substituting (2.11), (2.12) into equation (2.6) we obtain after simple manipulation

$$L = \frac{\Big[\omega^2(\omega^2 - \Omega^2_{nm}) + \dfrac{\lambda^2}{m^2}\Omega^2_{nm}\Big]}{\Big[(\omega^2 - \Omega^2_{nm})^2 + \dfrac{\lambda^2}{m^2}\Omega^2_{nm}\Big]}Z_{nm}(d,\ell), \quad N = \frac{\dfrac{\lambda}{m}\Omega^3_{nm}Z_{nm}(d,\ell)}{\Big[(\omega^2 - \Omega^2_{nm})^2 + \dfrac{\lambda^2}{m^2}\Omega^2_{nm}\Big]}. \tag{2.14}$$

The quantities A_1, B_1 are determined from the following expressions [3]

$$A_1 = -\frac{(\Omega_{nm}G + \xi H)}{2\Omega_{nm}(\Omega^2_{nm} + \xi^2)}, \quad B_1 = -\frac{(\xi G - \Omega_{nm}H)}{2a\Omega_{nm}(\Omega^2_{nm} + \xi^2)}, \tag{2.15}$$

here

$$G = \frac{1}{\pi}\int_0^{2\pi} F\cos\varphi d\varphi, \quad H = \frac{1}{\pi}\int_0^{2\pi} F\sin\varphi d\varphi \tag{2.16}$$

After series of simple calculations we get

$$\begin{aligned}\frac{\gamma}{2}\frac{da}{dt} &= -\Big[K_1R + \lambda\Omega_{nm}S + \xi\Omega_{nm}P + Q_0\cos\varphi\Big]a,\\ a\frac{\gamma}{2}\frac{d\psi}{dt} &= \Big[\Big(\Omega^2_{nm} - \frac{\gamma^2}{4}\Big) + K_1S - \lambda\Omega_{nm}R + \Omega^2_{nm}P + Q_0\sin 2\psi + Ta^2\Big]a.\end{aligned} \tag{2.17}$$

where

$$R = \frac{2N}{bc} Z_{nm}(d,\ell)\varepsilon, \qquad S = \frac{2}{bc}\left[Z_{nm}(d,\ell) - L\right] Z_{nm}(d,\ell)\varepsilon,$$
$$P = \frac{E_2\Omega^2\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{c}\right)^2\right]^2}{2E_1(\Omega_{nm}^2 + \xi^2)}, \quad Q_0 = \frac{q_0 n^2\pi^2}{4b^2}, \quad T = \frac{\beta 27}{128}. \tag{2.18}$$

Vanishing the right part of the equational system (2.17) we obtain the stationary solution a_0, ψ_0 related to the frequency γ and amplitude q_0 of the force q

$$\frac{\eta^2}{4} = (1+P) + K_1 S^* - \lambda\Omega_{nm}R^* + A_0^2 \pm \sqrt{Q^{*2} - \left[K_1 R^*(\lambda S^* + \xi P^*)\frac{\eta}{2}\right]^2}. \tag{2.19}$$

here

$$\eta^2 = \frac{\gamma^2}{\Omega_{nm}^2}, \quad S^* = \frac{S}{\Omega_{nm}^2}, \quad R^* = \frac{R}{\Omega_{nm}^2}, \quad Q^* = \frac{Q_0}{\Omega_{nm}^2}, \quad P^* = \frac{P}{\Omega_{nm}^2}, \quad A_0^2 = \frac{Ta_0^2}{\Omega_{nm}^2}.$$

The relation (2.19) is plotted in Fig. 3 for the case

$\Omega_{nm}^2 = 1$; $K_1 = 0.1$; $R^* = 0.1$; $S^* = 0.1$; $Q^{*2} = 0.04$; $P^* = 0.2$; $\lambda = 0.1$; $\xi = 0.65$ and $\xi = 0.7$

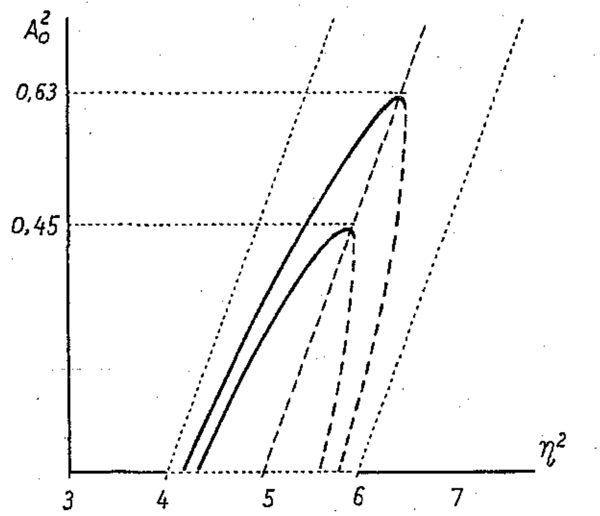

Fig. 3

It is easy to see that when $\xi = 1$, $\lambda = 0.2$ the parametric oscillation is damped and when the parameters of the examined system are chosen so that

$$\frac{Q^* - K_1R^*}{\lambda S^* + \xi P^*} < \left[(1+P) + K_1 S^* - \lambda\Omega_{nm}R^*\right], \quad \text{it is self disappeared.}$$

§3. THE STABILITY OF STATIONARY OSCILLATION

To study the stability of stationary oscillations, we have to set into the equational system (2.17) for $a = a_0 + \delta a$, $\psi = \psi_0 + \delta\psi$, where δa, $\delta\psi$ are small perturbations. Neglecting the small quantities of higher order, we receive the following variational equations for $a_0 \neq 0$

$$\begin{aligned} \frac{\gamma}{2}\frac{d\delta a}{dt} &= 2Q_0 a_0 \sin 2\psi_0 \delta\psi, \\ \frac{a\gamma}{2}\frac{d\delta\psi}{dt} &= 2Ta_0^2\delta a + 2a_0Q_0\cos 2\psi_0\delta\psi. \end{aligned} \tag{3.1}$$

For $a_0 = 0$

$$\begin{aligned} \frac{\gamma}{2}\frac{d\delta a}{dt} &= -[K_1 R + \lambda\Omega_{nm} S + \xi\Omega_{nm} P + Q_0\cos 2\psi_0]\delta a, \\ a\frac{\gamma}{2}\frac{d\delta\psi}{dt} &= \Big[\Big(\Omega_{nm}^2 - \frac{\gamma^2}{4}\Big) + K_1 S - \lambda\Omega_{nm} R + \Omega_{nm}^2 P + Q_0\sin 2\psi_0\Big] = 0. \end{aligned} \tag{3.2}$$

Using the Routh-HurWitz's criteria, we get the stability condition of stationary solution $a_0 \neq 0$ in the form

$$A_0^2 - \lambda\Omega_{nm} R^* + K_1 S^* + (1+P) > \frac{\eta^2}{4}\cdot \tag{3.3}$$

In Fig. 3, the solid curves correspond to states of oscillation, where the stability condition (3.3) is being valid. In the case $a_0 = 0$, from (3.2) we are

$$\frac{d\delta a}{\delta a} = \lambda dt \tag{3.4}$$

where

$$\lambda = -\Big\{CR + (\lambda S + \xi P)\Omega_{nm} \pm \sqrt{Q_0^2 - \Big[-\frac{\gamma}{4} + (1+P)\Omega_{nm}^2 - CS - \lambda R\Omega_{nm}\Big]^2}\Big\} \tag{3.5}$$

The stability condition of a stationary solution for $a_0 = 0$ will be thus given $\lambda < 0$.

It is easy to show that, if $Q^* = \dfrac{Q_0}{\Omega_{nm}^2} = [CR^* + (\lambda S^* + \xi P^*)\Omega_{nm}]$. The stationary solution ($a_0 = 0$) are always stable, doing not count the limitary value of η^*

$$\frac{\eta^{*2}}{4} = [(1+P) + CS^* - \lambda R^*\Omega_{nm}]. \tag{3.6}$$

If $Q^* > \dfrac{Q_0}{\Omega_{nm}}$, they are always stable, too with all value of η^*.

When $Q^* < \dfrac{Q_0}{\Omega_{nm}}$, it is meaned that the acting forces are large, the stationary solutions ($a_0 = 0$) will be stable with the values

$$\frac{\eta^2}{4} < \frac{\eta_1^2}{4} \quad \text{or} \quad \frac{\eta^2}{4} > \frac{\eta_2^2}{4}$$

where

$$\frac{\eta_{1,2}^2}{4} = (1+P) + CS^* - \lambda R^*\Omega_{nm} \mp \sqrt{Q^{*2} - [CR^* + (\lambda S^* + \xi P)\Omega_{nm}]^2}\,.$$

§4. CONCLUSION

1. The equation of motion a rectangular thin plate of creep on an elastic foundation, combined a dynamic absorber was set up. Its solution has been found by means of an asymptotic method, further the stability conditions of the stationary solution have been investigated.

2. The effect of a weak dynamic absorber has been considered. It is possible to choose the parameters of the examined system for the stationary oscillation to be damped.

3. It is easy to see that the effect of the absorber will be high, if $Z_{nm}(d,\ell) = \ell$, according to $d = \dfrac{b}{2n}$, $\ell = \dfrac{c}{2m}$ in particular case $n = 1$, $m = 1$ we have $d = \dfrac{b}{2}$, $\ell = \dfrac{c}{2}$

This publication is completed with financial support from the National Basic Research Program in Natural science

(xem tiếp trang 48)