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ON THE INSTABILITY OF A MECHANICAL SYSTEM OF 2 - DEGREE OF FREEDOM TAKING INTO ACCOUNT THE RESISTING FORCE

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High projects, soft structure are destroyed due to aerodynamic instability. The action mechanism of aerodynamic force is very complicated, which makes the mathematical problem of multi-degree freedom system more difficult to be solved. The system of mono-degree of freedom with some type of aerodynamic force and the analysis of solution for the case has been considered in [1, 2]. In [3] we have conducted a survey in the non-resisting equation system of two-degree of freedom with the aerodynamic force as in [5]. In this article, we will continue to study the above-mentioned mathematical problem with the regard to the resisting element of environment make some suggestions for the solution and survey its stability.

1. Formulation of the problem

The mechanical system is considered as a high column, the mass quantity M is concentrated on the column top end, the vibration of the system with the regard to the resisting force is described in the following equation:

$$\begin{gathered}\ddot{r} + 2\nu\dot{r} + \omega^2 r - r\dot{\varphi}^2 = (1-a)\frac{\dot{r}^2}{r}\\ \ddot{\varphi} + \alpha\dot{\varphi} = b\frac{\dot{r}}{r}\dot{\varphi}\end{gathered} \tag{1.1}$$

where: ν, α - resisting coefficient,

$$\omega^2 = \frac{c}{M}$$

c - corresponding rigidity of the system; a, b - coefficient of aerodynamic force.

The second equation of system (1.1) has an initial integral:

$$\dot{\varphi} = \sqrt{C_1}\, r^b e^{-\alpha t} \tag{1.2}$$

where: C_1 - integral constant.

Substitute (1.2) into (1.1), the first equation of (1.1) reduces to:

$$\ddot{r} + 2\nu\dot{r} + \omega^2 r - C_1 r^{2b+1} e^{-2\alpha t} = (1-a)\frac{\dot{r}^2}{r} \tag{1.3}$$

Using variable change method:

$$r = (sp)^{1/s} e^{-\lambda t} \tag{1.4}$$

s, λ are arbitrary coefficients, which will be chosen in the solution process, from (1.3) we have:

$$\ddot{p} + 2(\nu - \lambda a)\dot{p} + s(\omega^2 + a\lambda^2 - 2\nu\lambda)p = \Big(1 - \frac{a}{s}\Big)\frac{\dot{p}^2}{p} + C_1(sp)^{\frac{2b}{s}+1} e^{-2(\alpha+\lambda b)t} \tag{1.5}$$

We choose s and λ providing that:

$$\alpha + \lambda b = 0 \quad \text{and} \quad \frac{2b}{s} + 1 = 0, \tag{1.6}$$

then the equation (1.5) will be taken of the form:

$$\ddot{p} + 2\Big(\nu + \frac{a}{b}\alpha\Big)\dot{p} - 2b\Big(\alpha\omega^2 + \frac{a^2}{b^2}\alpha^2 + 2\frac{a}{b}\alpha\nu\Big)p = \Big(1 + \frac{a}{2b}\Big)\frac{\dot{p}^2}{p} + C_1. \tag{1.7}$$

Hereafter. we will consider some cases, when the equation (1.7) has analytical solutions.

2. Cases

$$1 - \frac{a}{s} = 0 \tag{2.1}$$

Equation (1.7) has form:

$$\ddot{p} + 2(\nu - 2\alpha)\dot{p} + (\alpha\omega^2 + 4\alpha^2 - 4\alpha\nu)p = C_1 \tag{2.2}$$

$$\Delta = (\nu - 2\alpha)^2 - (a\omega^2 + 4\alpha^2 - 4\alpha\nu) = \nu^2 - a\omega^2 \tag{2.3}$$

2.1. $\Delta > 0$ the equation (2.2) has a solution of following style:

$$p = C_2 e^{-(\nu - 2\alpha)t}\mathrm{ch}\Big[\big(\nu^2 - a\omega^2\big)^{1/2}t + \gamma\Big] + \frac{C_1}{4\alpha^2 - 4\alpha\nu + a\omega^2}, \tag{2.4}$$

where C_2, γ - integral constants.

Basing on (1.6), (2.1) from (2.4), (1.4) we have

$$r = a^{\frac{1}{a}}\Big\{C_2 e^{-\nu t}\mathrm{ch}\big[\big(\nu^2 - a\omega^2\big)^{1/2}t + \gamma\big] + \frac{C_1 e^{-2\alpha t}}{4\alpha^2 - 4\alpha\nu + a\omega^2}\Big\}^{1/a}. \tag{2.5}$$

Solution (2.5) has a special structure: it seems to be the sum of two terms, each of which has own resisting coefficient and integral constant.

- When $a > 0$, the solution (2.5) will be stable if ν, α at the same time are positive values, unstable if at least one of ν, α has negative value. The resisting coefficient ν or α negative value when aerodynamic resistance is bigger than structure resistance.

- When $a < 0$ the solution (2.5) will be written in the form:

$$r = \frac{1}{a^{1/a}\Big\{C_2 e^{-\nu t}\mathrm{ch}\big[\big(\nu^2 - a\omega^2\big)^{1/2}t + \gamma\big] + \dfrac{C_1 e^{-2\alpha t}}{4\alpha^2 - 4\alpha\nu + a\omega^2}\Big\}^{1/a}}. \tag{2.6}$$

The toot of (2.6) is stable, not depending on the sign of ν, α.

2.2. $\Delta < 0$, the equation (2.2) will have a solution of following style:

$$p = C_2 e^{-(\nu - 2\alpha)t}\cos\Big[\big(a\omega^2 - \nu^2\big)^{1/2}t + \gamma\Big] + \frac{C_1}{4\alpha^2 - 4\alpha\nu + a\omega^2}, \tag{2.7}$$

where C_2, γ - integral constants.

Replace (2.7) into (1.4) paying attention to (1.6), (2.1) we will have:

$$r = a^{1/a}\Big\{C_2e^{-\nu t}\cos\left[\left(a\omega^2-\nu^2\right)^{1/2}t+\gamma\right]+\frac{C_1e^{-2\alpha t}}{4\alpha^2-4\alpha\nu+a\omega^2}\Big\}^{1/a} \tag{2.8}$$

The solution (2.8) will be stable when a, ν, α are positive, unstable when at least one of ν, α has negative value.

3. Cases

$$\nu+\frac{a}{b}\alpha=0. \tag{3.1}$$

Equation (1.7) will have following form:

$$\ddot{p}-\frac{2b}{a}(a\omega^2-\nu^2)p=\Big(1+\frac{a}{2b}\Big)\frac{\dot{p}^2}{p}+C_1. \tag{3.2}$$

Setting up $\dot{p}^2=z$; $z'=\dfrac{dz}{dp}$ the equation (3.2) will be taken of Bernoulli equation type:

$$z'-\Big(\frac{a}{b}+2\Big)\frac{z}{p}=2C_1+4b\Big(\omega^2-\frac{\nu^2}{a}\Big)p. \tag{3.3}$$

The equation (3.3) has a solution

$$z=-2C_1\frac{1}{\frac{a}{b}+1}p-4b\Big(\omega^2-\frac{\nu^2}{a}\Big)\frac{b}{a}p^2+C_2p^{\frac{a}{b}+2}$$

from ehich we have:

$$\dot{p}^2=-2C_1\frac{b}{a+b}p-4\frac{b^2}{a^2}(a\omega^2-\nu^2)p^2+C_2p^{\frac{a}{b}+2}, \tag{3.4}$$

where C_2 - integral constants.

According to [3], the equation (3.4) can be solved in cases, when the right member is the 2, 3, 4 degree functions in accordance with p.

3.1. When

$$\frac{a}{b}=1, \tag{3.5}$$

equation (3.4) has form:

$$\dot{p}^2=-C_1p-4(a\omega^2-\nu^2)p^2+C_2p^3 \tag{3.6}$$

a. If to satisfy the condition:

$$4\left(a\omega^2-\nu^2\right)^2+C_1C_2\geq 0,$$

and suggest that the right member of (3.6) has 3 real solutions, which are arranged as follows $e_1\geq e_2\geq e_3=0$, the equation (3.6) will have solution as following

$$p=e_2sn^2\Big[\Big(\frac{\sqrt{C_2e_1}}{2}t+\gamma\Big),k\Big], \tag{3.7}$$

where: γ - integral constant, k - module of elliptic function, $k=\dfrac{e_2}{e_1}$.

However, in this case in order to have 3 solutions of the right member of equation (3.6) arranged in the above order we must have $C_2 < 0$, therefore in this case the solution (3.7) does not exist.

If satisfying the condition: $-C_1/C_2 < 0$, from (1.2) $C_1 > 0$ we find out that $C_2 > 0$, then the right member of (3.6) will have 3 solutions, which are arranged as follows: $e_1 > e_2 = 0 > e_3$, then the equation (3.6) will have the solution of following

$$p = e_3 cn^2\Big[\big(\sqrt{C_2(e_1 - e_3)}t + \gamma\big), k\Big] \tag{3.8}$$

where $k = \dfrac{-e_3}{e_1 - e_3}$.

Replace (3.8) into (1.4) we will have:

$$r = \Big\{ -2ae_3 cn^2\Big[\big(\sqrt{C_2(e_1 - e_3)}t + \gamma\big), k\Big]\Big\}^{\frac{1}{-2a}} e^{-\frac{\nu}{a}t}. \tag{3.9}$$

The solution of (3.9) is stable, periodical and moderating when both ν, a are negative and the score $1/2a$ must be even. It is unstable when both ν, a are positive or have opposite signs.

b. If satisfying the condition:

$$4\big(a\omega^2 - \nu^2\big)^2 + C_1C_2 < 0$$

the right member of equation (3.6) will have one real solution and two combined complex solutions:

$$e_1 = 0, \quad e_2 \pm ie_3, \quad e_3 > 0.$$

In order to solve equation (3.6) we set up:

$$p = -\frac{e_3}{\cos\theta_1}\,\frac{1 - \sqrt{1 - q^2}}{1 + \sqrt{1 - q^2}}, \tag{3.10}$$

here, $\mathrm{tg}\theta_1 = \dfrac{-e_2}{e_1}$, θ_1 is an obtuse angle.

Then the equation (3.6) will be changed to the form:

$$\dot{q}^2 = C_2\mu_0^2(1 - q^2)(1 - k^2q^2) \tag{3.11}$$

in which:

$$k^2 = \sin^2\left(\frac{\theta_1}{2} + \frac{\pi}{4}\right) = \frac{1}{2}\left(1 + \frac{e_2}{\sqrt{e_1^2 + e_3^2}}\right),$$
$$\mu_0^2 = \frac{-e_3}{\cos\theta_1} = \sqrt{e_1^2 + e_3^2}.$$

Solving the equation (3.11) we find the solution:

$$q = sn\Big\{\big[\sqrt{C_2}(e_1^2 + e_3^2)^{1/4}t + \gamma\big], k\Big\}, \tag{3.12}$$

where: γ - integral constant, k - module of elliptic function.

Replace (3.12) into (3.10) we will have solution of equation (3.6)

$$p = \sqrt{e_1^2 + e_3^2}\,\frac{1 - cn\Big\{\big[\sqrt{C_2}(e_1^2+e_3^2)^{1/4}t + \gamma\big], k\Big\}}{1 + cn\Big\{\big[\sqrt{C_2}(e_1^2+e_3^2)^{1/4}t + \gamma\big], k\Big\}}. \tag{3.13}$$

Replace (3.13) into (1.4) we will have:

$$r = \left\{-2a\sqrt{e_1^2 + e_3^2}\,\frac{1 - cn\Big\{\big[\sqrt{C_2}(e_1^2+e_3^2)^{1/4}t + \gamma\big], k\Big\}}{1 + cn\Big\{\big[\sqrt{C_2}(e_1^2+e_3^2)^{1/4}t + \gamma\big], k\Big\}}\right\}^{\frac{1}{-2a}} e^{-\frac{k}{a}t} \tag{3.14}$$

in which: $k^2 = \frac{1}{2}\Big(1 + \frac{e_2}{\sqrt{e_1^2 + e_3^2}}\Big)$.

The solution (3.14) is not stable, because $1 + cn\Big\{\big[\sqrt{C_2}(e_1^2 + e_3^2)^{1/4}t + \alpha\big], k\Big\}$ is periodical according to t, therefore in (3.14), at the zero value of denominator becomes infinite.

3.2. When $a = 2b$, the equation (3.4) will have form:

$$\dot{p}^2 = -\frac{2}{3}C_1 p - (a\omega^2 - \nu^2)p^2 + C_2 p^4. \tag{3.16}$$

Setting $a\omega^2 - \nu^2 = \theta^2$, we have:

$$\dot{p}^2 = -\frac{2}{3}C_1 p - \theta^2 p^2 + C_2 p^4. \tag{3.17}$$

If satisfying the condition:

$$\frac{\theta^2}{C_2} > 0, \qquad \Big(-\frac{\theta^2}{3C_2}\Big)^3 + \Big(\frac{C_1}{3C_2}\Big)^2 < 0,$$

the right member of equation (3.17) will have 4 real solutions $e_1 > e_2 > e_3 = 0 > e_4$, then the equation (3.17) has a solution as follows:

$$p = \frac{-e_2 e_4 sn^2\Big[\Big(\frac{1}{2}\sqrt{C_2 e_1 e_2}t + \gamma\Big), k\Big]}{e_2 - e_4 - e_2 sn^2\Big[\Big(\frac{1}{2}\sqrt{C_2 e_1 e_2}t + \gamma\Big), k\Big]}. \tag{3.18}$$

From (3.18) we can see p varies within $0 \le p \le e_2$.

Substituting (3.18) into (1.4), we have:

$$r = \left\{-a\,\frac{-e_2 e_4 sn^2\Big[\Big(\frac{1}{2}\sqrt{C_2 e_1 e_2}t + \gamma\Big), k\Big]}{e_2 - e_4 - e_2 sn^2\Big[\Big(\frac{1}{2}\sqrt{C_2 e_1 e_2}t + \gamma\Big), k\Big]}\right\}^{\frac{-1}{a}} e^{\frac{-2\alpha}{a}t}. \tag{3.19}$$

When $a < 0$: the solution is stable, if $\alpha < 0$ and unstable if $\alpha > 0$.

4. Cases

$$\frac{a}{b} = 1 \tag{4.1}$$

$$a\omega^2 - \nu^2 + \frac{1}{9}(\nu + \alpha)^2 = 0 \tag{4.2}$$

According to [2], in this case the equation (1.7) has the solution:

$$p = -\frac{C_1}{2\left(a\omega^2 + \alpha^2 + 2\nu\alpha\right)}\left[C_2 e^{\frac{2}{3}(\nu+\alpha)t} + 1\right]^2. \tag{4.3}$$

Replacing (4.3) into (1.4), we have:

$$r = \left(\frac{C_1 a}{a\omega^2 + \alpha^2 + 2\nu\alpha}\right)^{\frac{-1}{2\alpha}}\left[C_2 e^{\frac{-1}{3}(\alpha-2\nu)t} + e^{-\alpha t}\right]^{\frac{-1}{\alpha}}. \tag{4.4}$$

- If $a < 0$, the solution (4.4) will be stable when

$$\alpha > 0, \qquad \alpha - 2\nu > 0 \tag{4.5}$$

and not be stable when

$$\alpha > 0, \quad \alpha - 2\nu < 0, \quad \text{or } \alpha < 0, \quad \alpha - 2\nu > 0, \quad \text{or } \alpha < 0, \quad \alpha - 2\nu < 0. \tag{4.6}$$

- If $a > 0$, the solution (4.4) will be stable when condition (4.6) is satisfied, not be stable when (4.5) is satisfied.

5. Conclusion

In the surveyed system figure five parameters, it has analytical solution only when there exist definite relations of these parameters.

The variety, stability as well as instability of solution depend strictly on the relations and characteristics of parameters, especially a, b, α, ν.

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MẤT ỔN ĐỊNH KHÍ ĐỘNG CỦA CƠ HỆ HAI BẬC TỰ DO CÓ KỂ TỚI LỰC CẢN

Trong bài này chúng tôi xét bài toán mất ổn định khí động của một hệ cơ học có kể tới lực cản và dao động theo hai phương. Đưa ra được mối quan hệ giữa các tham số để hệ có nghiệm giải tích và phân tích ổn định của nghiệm.