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# EFFECTIVE UNSTEADY FLOW CALCULATION METHOD BY THE PREISSMANN SCHEME FOR LOOPED CHANNEL NETWORK

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## Preface

Many Asian deltas located downstream of big rivers have been developed as paddy fields area, but they appear to be suffered from floods and also serious drainage problems. In these areas, tributary rivers, branch rivers and drainage channels often form networks, and flow direction in such waterways is usually affected by tide and is not consistent. In order to design flood control facilities or to improve drainage systems, such a technique is required as can exactly estimate and calculate water levels in rivers or channels. Usual runoff analysis which is carried out by water balance equation and simplified movement equation cannot deal with such complex phenomena and the unsteady flow analysis method should be utilized for such problems.

There have been developed many numerical methods including the finite difference method and the finite element method for unsteady flow simulation. Although they have their own merits and demerits, the finite difference method is simple and effective, when we analyze the movement of one dimensional flow which can approximate river or channel flow. Especially, the implicit finite difference method, which has usually complex calculation procedure, has a high practical value, because it can take a very long time step and its computational time is much shorter than that of the explicit method. When we solve a problem by an implicit method, we must solve a simultaneous nonlinear equation system once every time step, which means we must solve a simultaneous linear equation system several times every time step. If we can not solve such simultaneous linear equation system effectively, the calculation time becomes enormous for numerous mesh number, which leads to the loss of the implicit method's merits. The Double Sweep Method (1980) is already devised for the branched channel network calculation and it can solve the simultaneous linear equation system very effectively. On the other hand, various kinds of procedures are employed to solve a looped channel network's calculation by trial and error base, although the principle is known to solve the problem.

In this study we will devise an algorithm which can effectively solve the simultaneous linear equation system for the looped channel network based on the similar procedure used for the Double Sweep Method. We also consider the calculation order of branches and the classification of channel networks as well.

## Unsteady flow simulation in a single channel

We had better confirm the calculation method by the Preissmann Scheme (1961) before we consider about that of the looped channel network. The Saint-Venant equations (1871) are usually used for the governing equations. They are shown below

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad \text{(Continuity Equation)}$$
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(uQ) + gA\frac{\partial h}{\partial x} = gA(S_0 - S_f) \quad \text{(Momentum Equation)}$$

where $A$ - cross sectional area, $t$ - time, $Q$ - discharge, $x$ - distance, $u = Q/A$ - velocity, $h$ - depth, $S_0$ - channel slope, $g$ - gravitational acceleration, and $S_f$ - friction slope. If the above governing equations are described by the conservation form of $\frac{\partial f}{\partial t} + \frac{\partial G}{\partial x} = H$, then they are discretized according to the procedure of the Preissmann Scheme as follows. See Figure 1.

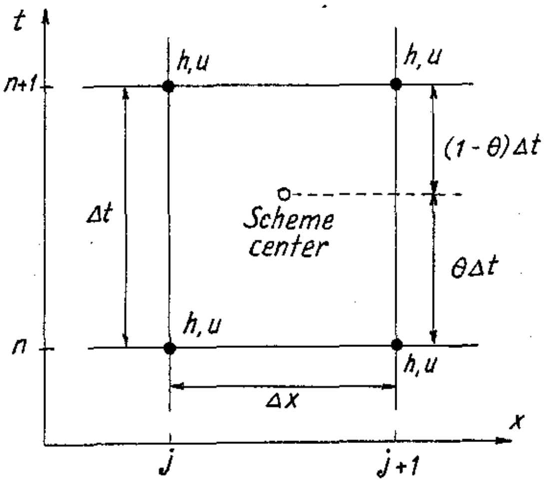


*Fig. 1.* Layout of the Preissmann Scheme

$$\frac{\partial f}{\partial t} = \frac{f_{j+1}^{n+1} + f_j^{n+1} - f_{j+1}^n - f_j^n}{2\Delta t},$$
$$\frac{\partial G}{\partial x} = \frac{\theta(G_{j+1}^{n+1} - G_j^{n+1}) + (1-\theta)(G_{j+1}^n - G_j^n)}{\Delta x},$$
$$H = \frac{1}{2}\theta(H_{j+1}^{n+1} + H_j^{n+1}) + \frac{1}{2}(1-\theta)(H_{j+1}^n + H_j^n),$$

where $n$ - time step index, $j$ - computational point index, $\Delta t$ - time interval, and $\Delta x$ - distance between two adjacent computational points. According to the above procedure, the governing equations are discretized showing relations between eight hydraulic quantities on four mesh points. Among them four hydraulic quantities on two mesh points $(j, n)$ and $(j+1, n)$ are already known, therefore we can obtain two algebraic equations showing relations between unknowns $h_{j+1}^{n+1}$, $h_j^{n+1}$, $u_{j+1}^{n+1}$ and $u_j^{n+1}$. Superscripts of $n+1$ in the four unknowns will be omitted from now on for the simplicity. Two algebraic equations shown below correspond to continuity and momentum (movement) equations

$$F_{i,j}(h_{j+1}, h_j, u_{j+1}, u_j) = 0, \quad (i = 1 : \text{continuity}, \ i = 2 : \text{movement}).$$

If there exist $N$ mesh points in a single channel, the number of unknowns is $2N$ because there are two unknowns on each mesh point. On the contrary, since there are $N-1$ intervals between mesh points and there are two algebraic equations for each interval, we can obtain $2(N-1)$ equations. We also obtain two more equations shown below corresponding to the boundary conditions being given to both sides of the channel. As the result we can obtain $2N$ algebraic equations in all. See Fig. 2

$$F_L(h_1, u_1) = 0, \qquad F_R(h_N, u_N) = 0$$

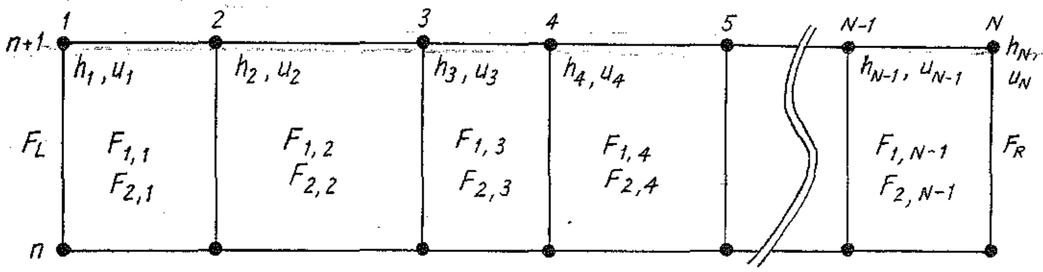


*Fig. 2.* Equation System for the Preissmann Scheme

The above algebraic equations are usually nonlinear and the Newton-Raphson method is commonly used to solve this kind of nonlinear equation system. In the Newton-Raphson method a nonlinear equation is expanded by the Taylor expansion around the temporary solutions, and the nonlinear equations is approximated by a linear equation, which has also four unknowns but they are differences between the temporary solutions and revised solutions; $\Delta h_{j+i}$, $\Delta h_j$, $\Delta u_{j+1}$, and $\Delta u_j$. They are shown below

$$\begin{aligned}
L_L &= b_L \Delta h_1 + d_L \Delta u_1 + e_L = 0 \quad \text{(Left boundary)}, \\
L_{i,j} &= a_{i,j} \Delta h_{j+1} + b_{i,j} \Delta h_j + c_{i,j} \Delta u_{j+1} + d_{i,j} \Delta u_j + e_{i,j} = 0, \\
&\qquad (i = 1, 2), \ (j = 1, N-1), \\
L_R &= a_R \Delta h_N + c_L \Delta u_N + e_R = 0 \quad \text{(Right boundary)}.
\end{aligned}$$

The Double Sweep Method is used to solve the above simultaneous linear equations system which consists of $2N$ equations. Firstly, the left side boundary condition $L_L$ is used and is solved with respect to $\Delta u_1$ as shown below

$$\Delta u_1 = W_1 \Delta h_1 + Y_1.$$

Secondly, this equation is substituted into two linear equations $L_{i,1}$ $(i = 1, 2)$, and from two equations two unknowns $\Delta u_1$ and $\Delta h_1$ are eliminated. Then we can obtain a linear equation having two unknowns $\Delta u_2$ and $\Delta h_2$, which is equivalent to the left boundary condition. We can again solve this equation with respect to $\Delta u_2$, and by iterating this procedure we can obtain following relations from $j = 2$ to $j = N$

$$\Delta u_j = W_j \Delta h_j + Y_j, \quad (j = 2, N).$$

The above process is called the Forward Sweep.

The last equation of $\Delta u_N = W_N \Delta h_N + Y_N$ has two unknowns $\Delta u_N$ and $\Delta h_N$, and the right side boundary condition has also two same unknowns. Therefore, two simultaneous equations can be solved for $\Delta u_N$ and $\Delta h_N$. By substituting $\Delta u_N$ and $\Delta h_N$ into the linear equations $L_{i,N}$ we can obtain $\Delta u_{N-1}$ and $\Delta h_{N-1}$. Same procedure can be used to solve other solutions $\Delta u_j$ and $\Delta h_j$ $(j = 1, N-2)$. This process is called the Back Sweep. The solutions of the simultaneous linear equations are used to improve the temporary solutions of the original nonlinear equations, and finally we can approach very closely to the real solutions.

## Unsteady Flow Simulation in a Branched Channel Network

We can apply the Double Sweep Method to a branched channel network with a minor procedure change. We will demonstrate the calculation procedure for a $Y$ shaped channel system, where two channels $A$ and $B$ meet and merge to form channel $C$. see Fig. 3

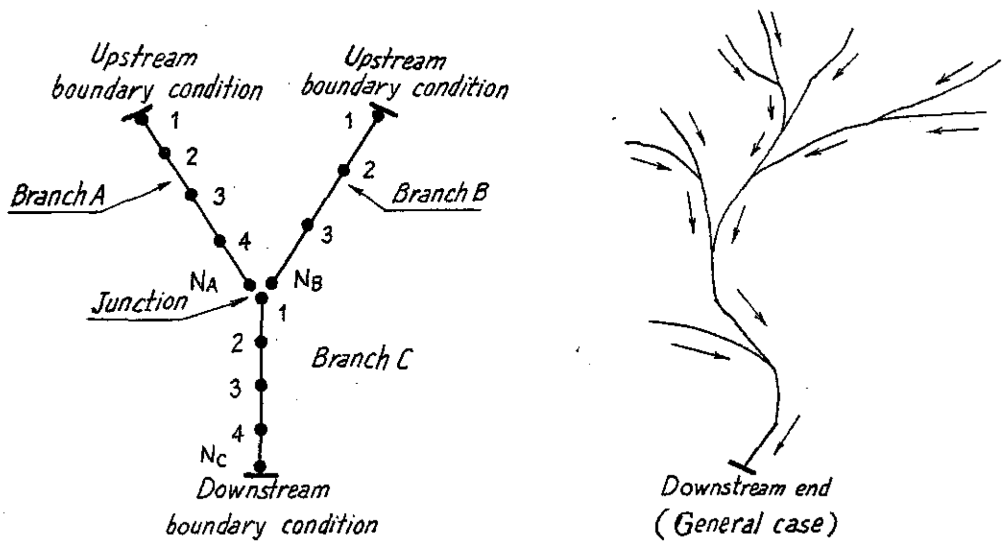


*Fig. 3.* Branched Channel Network

Each channel $A$, $B$, or $C$ is called branch, and boundary condition is given for each channel; upstream boundary conditions for branches $A$ and $B$, and a downstream boundary condition for branch $C$. we can perform the Forward Sweep for two branches $A$ and $B$ in the same way as we did for a single channel. When the numbers of mesh points in branches $A$ and $B$ are $N_A$ and $N_B$, respectively, we can obtain following relations for branches $A$ and $B$

$$\Delta u_{A,j} = W_{A,j}\Delta h_{A,j} + Y_{A,j}, \quad (j = 1, N_A) \quad \text{for Branch } A$$
$$\Delta u_{B,j} = W_{B,j}\Delta h_{B,j} + Y_{B,j}, \quad (j = 1, N_B) \quad \text{for Branch } B.$$

We want a similar relation with the above two for branch $C$, but we cannot go far beyond the junction without using other three equations at the junction. Among three equations, two are energy equations usually showing same water levels at ends of three branches, and one is continuity equation. They are shown below

$$h_{A,N_A} + Z_{A,N_A} = h_{B,N_B} + Z_{B,N_B} = h_{C,N_C} + Z_{C,N_C} \quad \text{(Energy Equations)}$$
$$Q_{A,N_A} + Q_{B,N_B} = Q_{C,N_C} \quad \text{(Continuity Equation).}$$

The above equations can be approximated by following three linear equations having six unknowns; $\Delta h_{A,N_A}$, $\Delta u_{A,N_A}$, $\Delta h_{B,N_B}$, $\Delta u_{B,N_B}$, $\Delta h_{C,1}$, and $\Delta u_{C,1}$

$$P_{i,A}\Delta h_{A,N_A} + P_{i,B}\Delta h_{B,N_B} + P_{i,C}\Delta h_{C,1} + q_{i,A}\Delta u_{A,N_A} + q_{i,B}\Delta u_{B,N_B} + q_{i,C}\Delta u_{C,1} = \Gamma_i$$

($i = 1$ and 2: Energy Equations, $i = 3$: Continuity Equation)

As we now have five equations for six unknowns, four unknowns can be eliminated and we get a following relation similar to the boundary condition

$$\Delta u_{C,1} = W_{C,1}\Delta h_{C,1} + Y_{C,1}.$$

Then we can apply the Forward Sweep to branch $C$, and obtain following relation on each mesh point in branch $C$.

$$\Delta u_{C,j} = W_{C,j}\Delta h_{C,j} + Y_{C,j}, \quad (j = 2, N_C) \quad \text{for Branch } C.$$

When we go to the final mesh point of branch $C$, that is, at $j = N_C$, we can use another boundary condition, and $\Delta h_{C,N_C}$ and $\Delta u_{C,N_C}$ can be solved. The procedure after this is almost the same with that used for a single channel. In case of a branched channel network we can use the Double Sweep Method, and calculation can be performed effectively. Such a junction as connects two already-wept branches to one not-yet-swept branch will be called junction of Type 1.

## Classification of Channel Networks

Although we have already tacitly classified channel networks as a single channel, a branched channel network, and a looped channel network, we had better divide the looped network further into "simple looped network" and "complex looped network", because calculation procedures for two types of looped networks are different from each other.

### Simple Looped Network

We consider a channel network system consists of six branches $A$, $B$, $C$, $D$, $E$, and $F$ as shown in Fig. 4. Three branches $D$, $E$, and $F$ compose a triangle, and other three branches $A$, $B$, and $C$ are connected to three vertexes of the triangle. We can perform the Forward Sweep for three branches $A$, $B$, and $C$, but we cannot go further anymore. Here we notice that we need not to distinguish $a$ downstream end from upstream ends and there exist only ends, because the former calculation procedure that proceeds from upstream branches to downstream ones is no more effective.

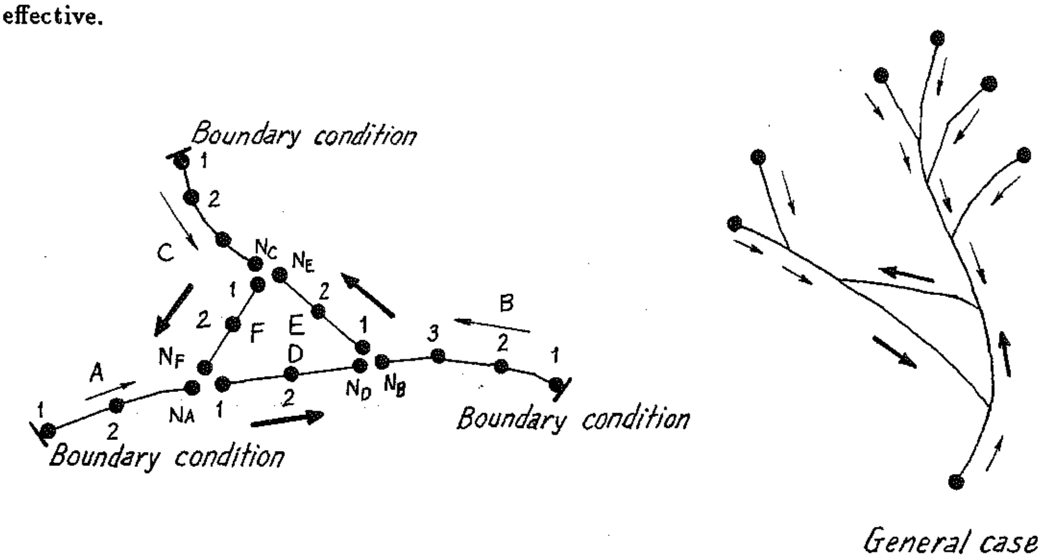


*Fig. 4.* Simple Looped Network

If we perform the Forward Sweep at all events, three branches $D$, $E$, and $F$ composing a triangle remain untreated. In this stage we can see the feature of the single looped network. When we see all junctions located at three vertexes of the triangle, we can find that one already-swept branch and two not-yet-swept branches join at each junction. This kind of junction will be called junction of Type II here. This is a typical feature of the single looped network. In case of the complex looped network, on the contrary, there exist some junctions where three not-yet-swept branches join. Such a junction will be called junction of Type III.

Although all branches in a simple looped network are even and we can start calculation anywhere, we will start from branch $A$ counterclockwise. Let $N_D$ mesh points exists in the branch $D$ and let all mesh points be numbered also counterclockwise. We suppose here that $\Delta u_{D,2}$ and $\Delta h_{D,2}$ can be expressed by using $\Delta u_{D,1}$ and $\Delta h_{D,1}$ as follows

$$\Delta u_{D,2} = W1_{D,2}\Delta u_{D,1} + W2_{D,2}\Delta h_{D,1} + W3_{D,2}$$

$$\Delta h_{D,2} = Y1_{D,2}\Delta u_{D,1} + Y2_{D,2}\Delta h_{D,1} + Y3_{D,2}$$

where $\Delta u_{D,1}$, $\Delta h_{D,1}$, $\Delta u_{D,2}$ and $\Delta h_{D,2}$ are differences between temporary and revised solutions on mesh points 1 and 2, respectively.

The above expressions are always possible, because there are two linear equations approximating continuity and movement equations against four unknowns; $\Delta u_{D,1}$, $\Delta h_{D,1}$, $\Delta u_{D,2}$ and $\Delta h_{D,2}$ on two mesh points 1 and 2, and $\Delta u_{D,2}$ or $\Delta h_{D,2}$ can be eliminated from two equations each other. The same treatment is also possible for four unknowns; $\Delta u_{D,2}$, $\Delta h_{D,2}$, $\Delta u_{D,3}$ and $\Delta h_{D,3}$ on mesh points 2 and 3, and $\Delta u_{D,3}$ and $\Delta h_{D,3}$ are expressed by $\Delta u_{D,2}$ and $\Delta h_{D,2}$. as two unknowns $\Delta u_{D,2}$ and $\Delta h_{D,2}$ are already expressed by $\Delta u_{D,1}$ and $\Delta h_{D,1}$, finally $\Delta u_{D,3}$ and $\Delta h_{D,3}$ can be expressed by $\Delta u_{D,1}$ and $\Delta h_{D,1}$. By iterating this procedure, we can obtain following relation on each mesh point in the branch $D$

$$\Delta u_{D,j} = W1_{D,j}\Delta u_{D,1} + W2_{D,j}\Delta h_{D,1} + W3_{D,j} \quad (j = 1, N_D)$$
$$\Delta h_{D,j} = Y1_{D,j}\Delta u_{D,1} + Y2_{D,j}\Delta h_{D,1} + Y3_{D,j} \quad (j = 1, N_D)$$

Through the above process resembling to the Forward Sweep, two unknowns $\Delta u_{D,N_D}$ and $\Delta h_{D,N_D}$ at the end of the branch can be expressed by two unknowns $\Delta u_{D,1}$ and $\Delta h_{D,1}$ at another end of the branch. At the junction where branches $B$, $D$, and $E$ join, we have already known the information about $\Delta u_{B,N_B}$ and $\Delta h_{B,N_B}$ as shown below

$$\Delta u_{B,N_B} = W_{B,N_B}\Delta h_{B,N_B} + Y_{B,N_B} \quad \text{for Branch } B.$$

Moreover, we have other three equations showing continuity and energy relations among three branches $B$, $D$, and $E$. Three equations are the same with those shown for a junction of Type I, and are approximated by three linear equations shown before. Now as we have five equations against eight unknowns, five unknowns can be eliminated. Then we can express $\Delta u_{E,1}$ and $\Delta h_{E,1}$, and further $\Delta u_{E,1}$ and $\Delta h_{E,j}$ $(j = 2, N_E)$ in the branch $E$ by two unknowns $\Delta u_{D,1}$ and $\Delta h_{D,1}$ in the same way. For unknowns on the branch $F$, we can also express them by $\Delta u_{D,1}$ and $\Delta h_{D,1}$, and finally two unknowns $\Delta u_{D,1}$, and $\Delta h_{D,1}$, on the branch $D$ can be expressed by themselves as shown below.

$$\Delta u_{D,1} = W1_{DEF}\Delta u_{D,1} + W2_{DEF}\Delta h_{D,1} + W3_{DEF}$$
$$\Delta h_{D,1} = Y1_{DEF}\Delta u_{D,1} + Y2_{DEF}\Delta h_{D,1} + Y3_{DEF}.$$

As above two equations have only two unknowns $\Delta u_{D,1}$ and $\Delta h_{D,1}$, they can be solved. Once we get the values of $\Delta u_{D,1}$ and $\Delta h_{D,1}$, all other unknowns are solved because they are already expressed by $\Delta u_{D,1}$ and $\Delta h_{D,1}$.

In case of the simple looped network, we can utilize similar procedure to the Forward Sweep and we need not deal with a matrix, which saves computational time considerably. After solving the unknowns in branches $D$, $E$, and $F$ which compose a loop, unknowns in other branches $A$, $B$, and $C$ can be solved by the ordinary Back Sweep.

### Complex Looped Network

As an example, we consider a looped network system which consists of seven branches as shown in Fig. 5. Two branches $A$ and $B$ have their own ends to which boundary conditions are given, and we can conduct the Forward Sweep for these branches

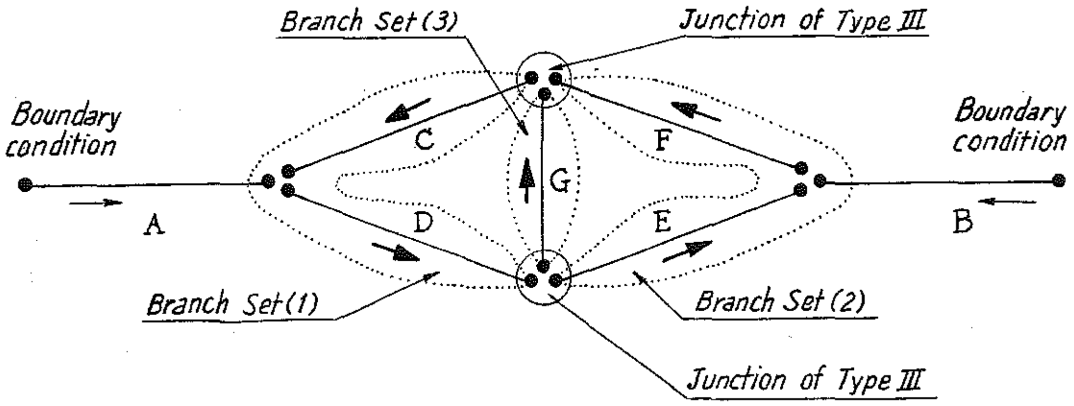


*Fig. 5.* Complex looped network

We cannot use the quite same procedure used for the simple looped network, because there are two junctions of Type III here. Two junctions of branches $C$, $F$, and $G$ and of branches $D$, $E$, and $G$ are Type III. Other two junctions of branches $A$, $C$, and $D$ and of branches $B$, $E$, and $F$ are Type II.

We need to consider some branch set here. A branch set consists of a single or a series of branches connecting two junctions of Type III. There exist three branch sets; set (1) composed of branches $C$ and $D$, set (2) composed of branches $E$ and $F$, and set (3) composed of branch $G$. All unknowns $\Delta u_{X,J}$ and $\Delta h_{X,j}$ in branch $X$ in a certain branch set (i) can be expressed by two unknowns $\Delta u_{S,1}$ and $\Delta h_{S,1}$ in top branch $S$ in the same branch set (i). This situation is similar to that of the simple looped network except that first two unknowns cannot be expressed by themselves. Through this procedure, last two unknowns $\Delta u_{T,N_T}$ and $\Delta h_{T,N_T}$ can be expressed by first two unknowns $\Delta u_{S,1}$ and $\Delta h_{S,1}$ as shown below

$$\Delta u_{T,N_T} = W1_{(i)}\Delta u_{S,1} + W2_{(i)}\Delta h_{S,1} + W3_{(i)}$$
$$\Delta h_{T,N_T} = Y1_{(i)}\Delta u_{S,1} + Y2_{(i)}\Delta h_{S,1} + Y3_{(i)}$$

where the branch $S$ is located at the top end, and the branch $T$ at the tail end in the branch set (i).

What is important here is that all unknowns in a branch set can be expressed only by first two unknowns. In other words, there are only two independent unknowns in a branch set. As there are three branch sets, in this example, the total number of unknowns is six. The total number of equations is also six, because there are two junctions of Type III, where six equations, two energy and one continuity equations on each junction, can be obtained. As the ratio of Type III's junction number to branch set number is always 2 : 3 in a complex looped network, we can always obtain same number of equations with that of unknowns.

Six unknowns, in this example, cannot be solved effectively without using matrix calculation like the Gauss' Elimination Method. In this method, a matrix whose components are coefficients of simultaneous linear equations is transformed to a triangular matrix (forward steps), and unknowns are solved successively from bottom to top (backward steps). Although this method is very effective, its calculation time is approximately proportional to the third power of unknowns' number. As this part may be the most time-consuming in numerical calculation process, we should not adopt such a method as uses four unknowns in a branch set. In the above example, $\Delta u_{T,N_T}$ and $\Delta h_{T,N_T}$ should not be used as unknowns, but they should be used by the forms expressed in advance by $\Delta u_{S,1}$ and $\Delta h_{S,1}$. Once six unknowns are solved by the Gauss' Elimination Method, we can solve easily all other unknowns in branch sets just like we did in the case of the simple looped network. Other branches $A$ and $B$ are solved by the Backward Sweep as well.

## Practical application of computation method

Under developed technique, a practical application of computation and its calibration have been carried out in South Ninh Binh irrigation system. The system is sketched as shown in the Fig. 6

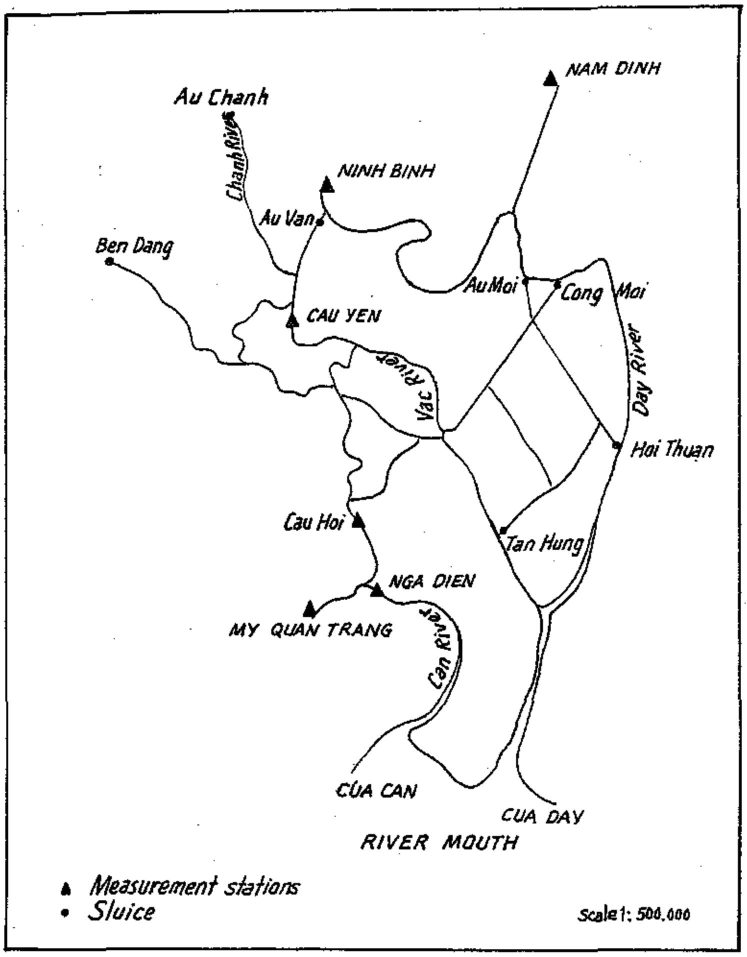


*Fig. 6.* Measurement stations in the South Ninh Binh river system

For computational purpose the system then was schematized that comprises of 7 looped cycles as shown in the Fig. 7

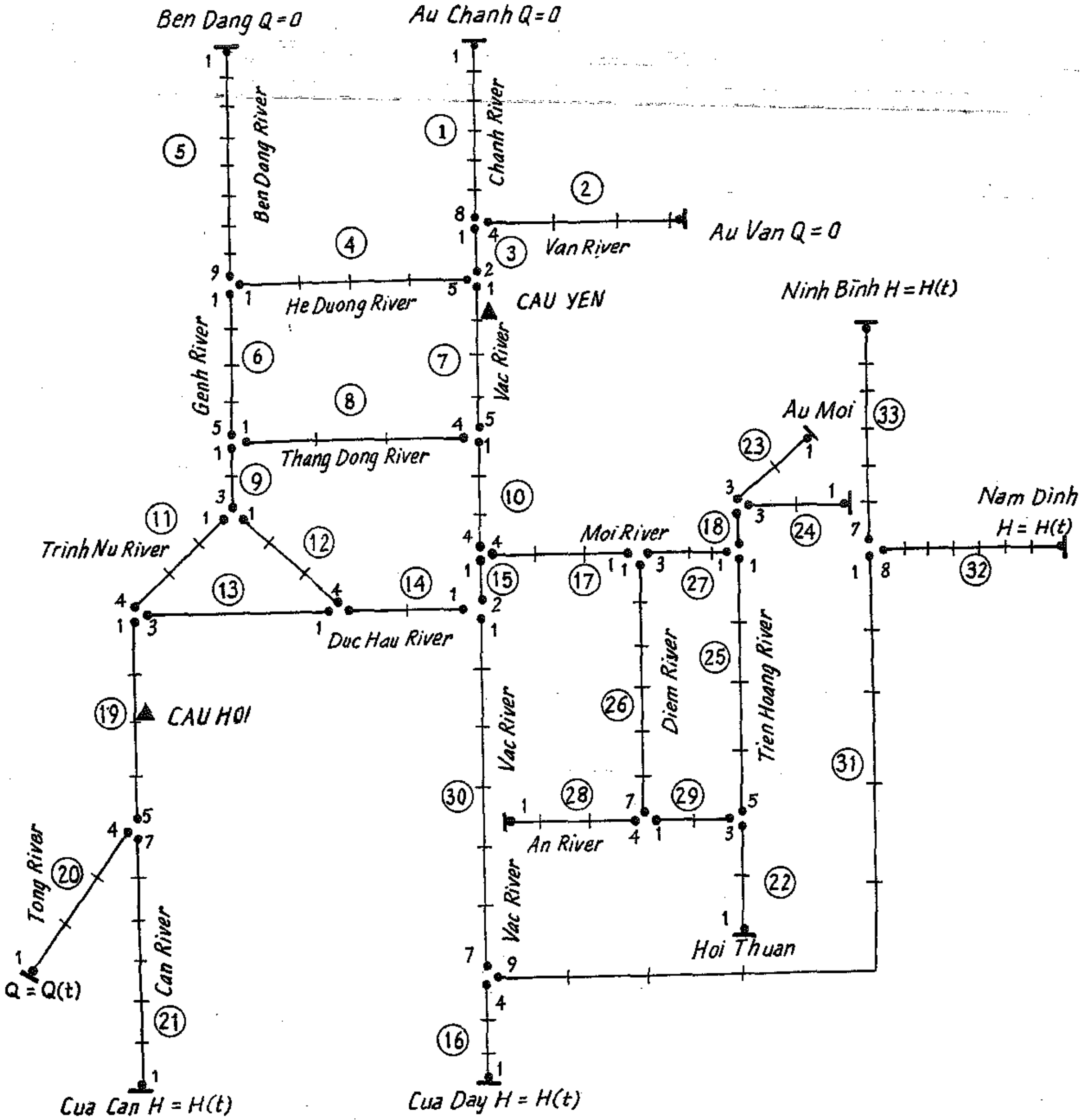


*Fig. 7.* The schematization of the river network

The computation was carried out at the Water Engineering and Management Program, School of Civil Engineering, Asian Institute of Technology, Bankok Thailand. By comparison between observed and computed results, the agreement was considered satisfactory . See figures 8 below that shows the comparison between observed and computed result at some important locations.

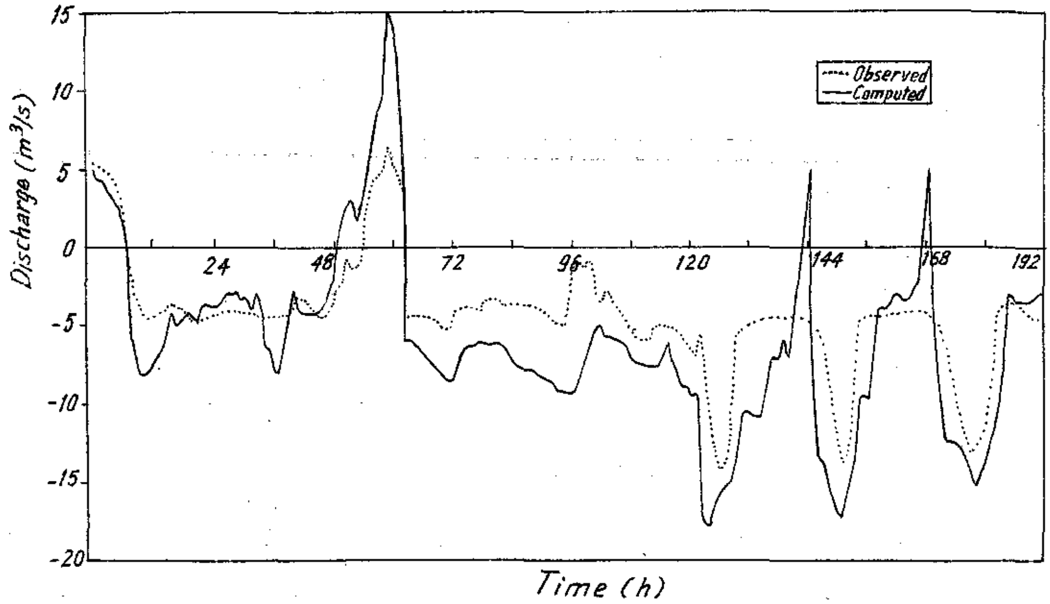


*Fig. 8a.* The Calibration of Water Level at Cau Hoi station from 28/8 to 04/9/1995

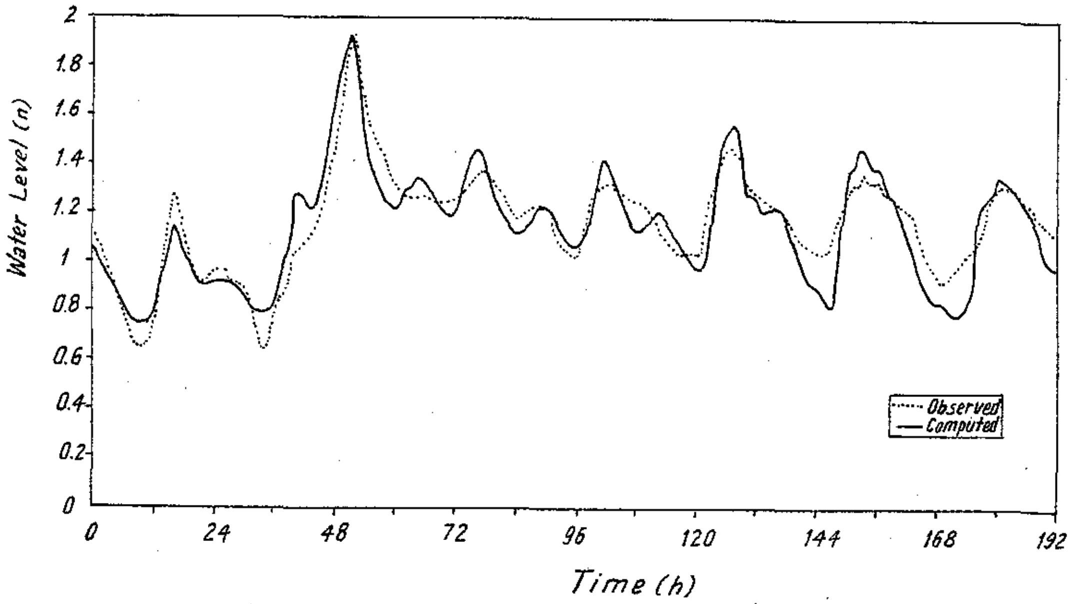


*Fig. 8b.* The Calibration of Discharge at Cau Hoi station from 28/8 to 04/9/1995

## Conclusions

Through this study followings are achieved. Firstly, it was found that the looped networks should be classified into simple and complex channel networks according to the type of junction. Secondly, it was also showed that the numerous unknowns in branches composing loops were reduced to a minimum number by the devised procedure similar to the Forward Sweep, where two top unknowns is a group of branches were used to express other unknowns in the same group. Finally, we could present two ways to solve unknowns for both cases. In case of the simple looped network, number of unknowns was reduced to only two, and they could be solved without using matrix calculation. In case of the complex looped network, it was decomposed into branch sets and the number of unknowns was reduced to twice of branch set number. They could be solved by a matrix calculation such as the Gauss' Elimination Method. Through the above process we succeeded in developing an effective method applicable to any type of channel network. A case study was done with satisfactory result.

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## VỀ PHƯƠNG PHÁP TÍNH TOÁN DÒNG KHÔNG ỔN ĐỊNH BẰNG SƠ ĐỒ PREISSMAN CHO HỆ THỐNG LÒNG DẪN CÓ LIÊN KẾT PHỨC TẠP


Nhiều đồng bằng của khu vực châu Á ở vùng hạ du các sông lớn là các vùng canh tác lúa nước, tuy nhiên chúng thường bị ngập lụt và vấn đề tiêu úng ở đây thường rất khó khăn. Ở các khu vực này các sông nhánh và các kênh tiêu thường tạo nên các mạng lưới trong đó dòng chảy bị ảnh hưởng bởi thủy triều và không đồng nhất. Để thiết kế các công trình chống lũ và nâng cao hiệu quả hệ thống tiêu, cần phải có kỹ thuật dự báo và tính toán chính xác mức nước trong sông kênh.

Trong bài báo này chúng tôi xây dựng một thuật toán có thể giải một cách hiệu quả hệ phương trình điều khiển mô tả dòng chảy không ổn định trong hệ thống kênh hở. Kỹ thuật tính toán dựa trên nguyên tắc sai phân hữu hạn ẩn theo sơ đồ do Preissmann đề nghị.

Kỹ thuật khử đuổi đã được phát triển để giải bài toán kênh đơn. Kênh phân nhánh đơn giản đã được đề cập và sau đó trình bày phương pháp tính toán cho trường hợp hệ thống kênh liên kết phức tạp để giảm thiểu tối đa khối lượng tính toán và bộ nhớ máy tính (trường hợp looped đơn giản và looped phức tạp). Nghiên cứu cũng sẽ giới thiệu tiêu chuẩn phân loại lưới kênh và các chỉ dẫn trong các trường hợp áp dụng.