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INTERACTION OF THE ELEMENTS CHARACTERIZING THE QUADRATIC NONLINEARITY AND FORCED EXCITATION WITH THE OTHER EXCITATIONS

NGUYEN VAN DAO
Vietnam National University, Hanoi

Introduction

In nonlinear systems, the first order of smallness terms of quadratic nonlinearity and the forced excitation with nonresonance frequency and the second order of smallness terms of linear friction, cubic nonlinearity, forced and parametric excitation with resonance frequencies have no effect on the oscillation in the first approximation. However, they do interact one with another in the second approximation and new nonlinear phenomena occur. The study of these phenomena, using the asymptotic method of nonlinear mechanics [1] with a digital computer, is our aim.

1. Interaction between the elements of quadratic nonlinearity and forced excitation themselves

Let us consider a nonlinear system governed by the differential equation

$$\ddot{x} + x = \varepsilon\left[\alpha x^2 + q\cos 2\varphi(\tau)\right] - \varepsilon^2(2h\dot{x} + \beta x^3), \tag{1.1}$$

where the dots indicate differentiation with respect to time, α, q, h and β are constants, $\tau = \varepsilon t$ and ε is a small dimensionless parameter characterizing the smallness of the terms behind it. The parameter ε is introduced artificially and used as a book-keeping device and will be set equal to unity in the final solution. The quadratic term may be due to curvature or and asymmetric material nonlinearity. The function $\varphi(\tau)$ is supposed to be a form

$$\frac{d\varphi}{dt} = \nu(\tau), \quad \tau = \varepsilon t, \tag{1.2}$$

where $\nu(\tau)$ is close to the natural frequency i.e. to unity:

$$\nu^2(\tau) = 1 + \varepsilon^2\Delta(\tau). \tag{1.3}$$

The equation (1.1) can be rewritten as:

$$\ddot{x} + \nu^2(\tau)x = \varepsilon\left[\alpha x^2 + q\cos 2\varphi(\tau)\right] - \varepsilon^2(-\Delta x + 2h\dot{x} + \beta x^3). \tag{1.4}$$

A solution of this equation is sought by using the asymptotic method of nonlinear oscillation [1]

$$x = a\cos\theta + \varepsilon u_1(a,\psi,\theta) + \varepsilon^2 u_2(a,\psi,\theta) + \varepsilon^3\ldots, \quad \theta = \varphi + \psi, \tag{1.5}$$

$$\frac{da}{dt} = \varepsilon A_1(a,\psi) + \varepsilon^2 A_2(a,\psi) + \ldots, \quad \frac{d\psi}{dt} = \varepsilon B_1(a,\psi) + \varepsilon^2 B_2(a,\psi) + \ldots,$$

where $u_i(a,\psi,\theta)$ are periodic functions with period 2π with respect to both variables ψ and θ and do not contain the first harmonics $\sin\theta$, $\cos\theta$. The functions $A_i(a,\psi)$, $B_i(a,\psi)$ are periodic with respect to the variable ψ. These functions will be determined in the process of approximation calculations.

Substituting the expressions (1.5) into equation (1.4) and comparing the coefficients of ε^1 we obtain

$$-2\nu(\tau)A_1\sin\theta - 2a\nu(\tau)B_1\cos\theta + \nu^2(\tau)\Big(\frac{\partial^2 u_1}{\partial\theta^2} + u_1\Big) = \alpha a^2\cos^2\theta + q\cos 2\varphi(\tau). \tag{1.6}$$

Comparing the harmonics in (1.6) gives:

$$A_1 = B_1 = 0, \tag{1.7}$$

$$u_1 = \frac{\alpha a^2}{2\nu^2(\tau)} - \frac{1}{6\nu^2(\tau)}(\alpha a^2 + 2q\cos 2\psi)\cos 2\theta - \frac{q}{3\nu^2(\tau)}\sin 2\psi\cos 2\theta. \tag{1.8}$$

Comparing the coefficients of ε^2 in (1.4) we get

$$\begin{aligned} &-2\nu(\tau)A_2\sin\theta - 2a\nu(\tau)B_2\cos\theta + \nu^2(\tau)\Big(\frac{\partial^2 u_2}{\partial\theta^2} + u_2\Big) = \\ &= 2a\alpha u_1\cos\theta + \Delta a\cos\theta + 2ha\nu\sin\theta - \beta a^3\cos^3\theta, \end{aligned} \tag{1.9}$$

which gives

$$\begin{aligned} -2\nu(\tau)A_2 &= 2h\nu(\tau)a - \frac{\alpha q}{3\nu^2(\tau)}a\sin 2\psi, \\ -2a\nu(\tau)B_2 &= \Delta a + \Big(\frac{5\alpha^2}{6\nu^2(\tau)} - \frac{3}{4}\beta\Big)a^3 - \frac{\alpha q}{3\nu^2(\tau)}a\cos 2\psi. \end{aligned} \tag{1.10}$$

So, in the second approximation we have

$$x = a\cos\theta + \varepsilon\Big[\frac{\alpha a^2}{2} - \frac{1}{6}(\alpha a^2 + 2q\cos 2\psi)\cos 2\theta - \frac{q}{3}\sin 2\psi\sin 2\theta\Big], \tag{1.11}$$

$$\begin{aligned} \frac{da}{dt} &= \varepsilon^2\Big[-ha + \frac{\alpha q}{6\nu(\tau)}a\sin 2\psi\Big], \\ a\frac{d\psi}{dt} &= -\frac{\varepsilon^2\Delta}{2\nu}a + \varepsilon^2\Big[\frac{\gamma}{\nu(\tau)}a^3 + \frac{\alpha q}{6\nu(\tau)}a\cos 2\psi\Big], \end{aligned} \tag{1.12}$$

where

$$\gamma = \frac{3\beta}{8} - \frac{5\alpha^2}{12}, \quad \nu(\tau) = \nu_0 + \varepsilon\mu t, \quad \nu_0 \approx 1. \tag{1.13}$$

Stationary Oscillation:

Supposing that $\nu(\tau) = \omega = \text{const}$ and considering the stationary oscillation with constant amplitude a and phase ψ we have:

$$\frac{\alpha q}{6}\sin 2\psi = h\omega, \quad \frac{\alpha q}{6}\cos 2\psi = \frac{\Delta}{2} - \gamma a^2, \quad a \neq 0. \tag{1.14}$$

Eliminating the phase ψ we get:

$$W(a^2,\omega) = 0, \tag{1.15}$$

where

$$W(a^2,\omega) = \frac{-\alpha^2 q^2}{36} + \left(\frac{\Delta}{2} - \gamma a^2\right)^2 + h^2\omega^2, \quad \varepsilon^2\Delta = \omega^2 - 1 \approx 2(\omega - 1), \quad \gamma = \frac{3}{8}\beta - \frac{5\alpha^2}{12}. \tag{1.16}$$

From equation (1.15) it follows that:

$$\gamma a^2 \approx \omega - 1 \pm \sqrt{\frac{\alpha^2 q^2}{36} - h^2\omega^2}. \tag{1.17}$$

The dependence of the amplitude a on the external frequency ω is presented in figure 1 for the parameters: $\varepsilon^2\alpha q = 0.063$, $\varepsilon^2 h = 0.01$, $\varepsilon^2\gamma = 0.08$.

The stability of nontrivial stationary solutions ($a \neq 0$) of the equation (1.12) when ω is constant can be studied by using the corresponding variational equations, which lead to the condition: [1]

$$\frac{\partial W}{\partial a_0} > 0. \tag{1.18}$$

Because function W (1.16) is positive outside and negative inside the resonance curve, the stable branch of the resonance curve is the upper branch, which corresponds to the upper sign before the radical in (1.17). Thus, between the two forms of oscillations corresponding to definite values of ω, the form with large amplitude is stable and the form with small amplitude is unstable.

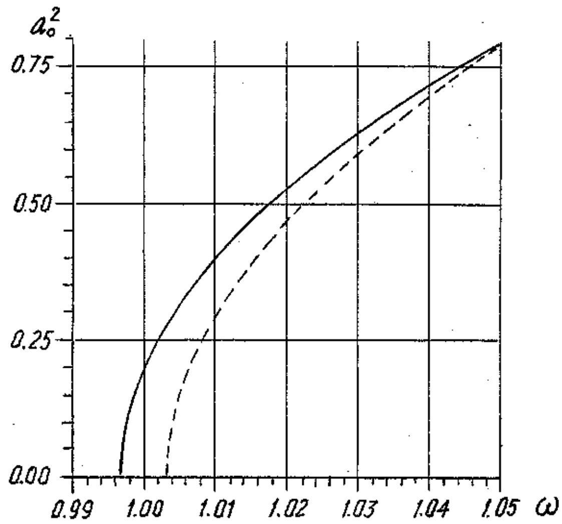

Fig. 1

Following Chapter 4 of [1], the trivial solution $a = 0$ of the equation (1.12) is stable if the value ω does not lie in that interval of the axis ω, from which the resonance curve is rising. In figure 1 the stable branches are shown by heavy lines, while the unstable ones are shown by dotted lines.

The passage of the system under consideration through resonance when $\nu(\tau)$ is not a constant, but changes by the law: $\nu(\tau) = \nu_0 + \varepsilon\mu t$, can be examined by integration of the differential equations (1.12). The parameters are chosen as $t_0 = 0$, $a_0 = 0.009$, $\psi_0 = 0$, $\varepsilon^2 h = 0.001$, $\varepsilon^2\gamma = 0.01$, $\varepsilon^2\alpha q = -0.024$, $\nu_0 = 1$, $\mu = 10^{-5}$ (curve 1, Fig. 2); $\mu = 2 \cdot 10^{-5}$ (curve 2, Fig. 2); $\mu = -10^{-5}$ (curve 1, Fig. 3); $\mu = -2 \cdot 10^{-5}$ (curve 2, Fig. 3).

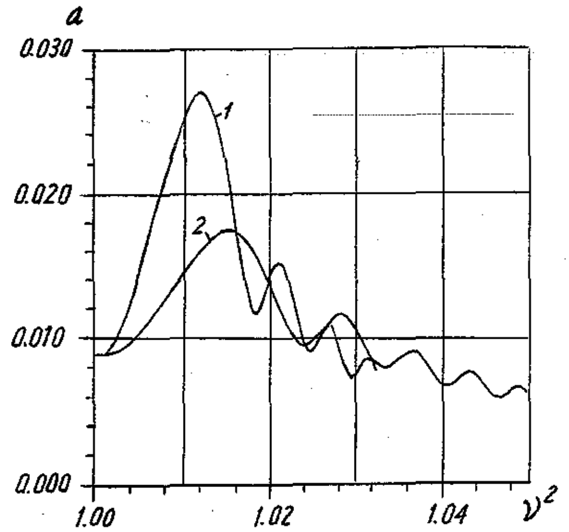

Fig. 2

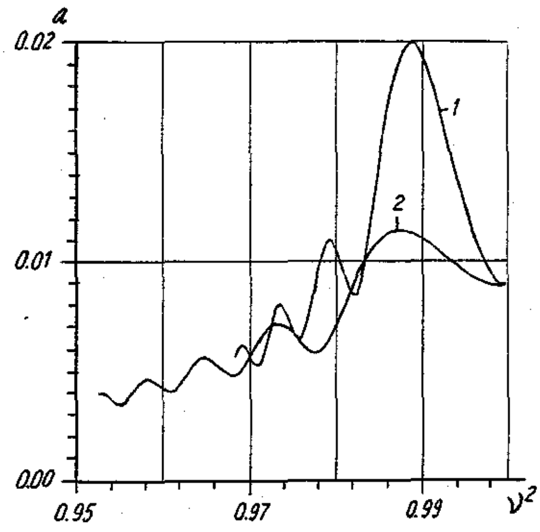

Fig. 3

From the expression (1.12) and (1.13) one can see that the quadratic nonlinearity (α) is always to softenize the system under consideration regardless of the sign of α. Moreover, two elements characterizing quadratic nonlinearity αx^2 and forced excitation $q\cos 2\varphi(\tau)$ combine together and act just like a parametric excitation with an intensity αq.

The system of equations (1.12) has a trivial solution $a = 0$, which corresponds to a pure forced oscillation under the action of an external excitation $\varepsilon q \cos 2\varphi$:

$$x = -\varepsilon \frac{q}{3} \cos 2\varphi. \tag{1.19}$$

2. Interaction between the elements of quadratic nonlinearity and forced excitations

The system under consideration in this paragraph is governed by d.e.

$$\ddot{x} + \omega^2 x = \varepsilon(\alpha x^2 + q\cos 2\omega t) + \varepsilon^2\big[\Delta x - 2h\dot{x} - \beta x^3 + r\cos(\omega t - \eta)\big], \quad \omega^2 = 1 + \varepsilon^2 \Delta. \tag{2.1}$$

Here, the nonresonance forced excitation (q) is of the first order of smallness, while the resonance forced excitation (r) is of second order of smallness. These excitations have no effect on the oscillation in the first approximation, but they interact one with another in the second approximation. Similarly to the previous paragraph, the solution of the equation (2.1) is found in the series (1.5). The equations (1.11)-(1.13) now take the form:

$$x = a\cos\theta + \varepsilon\Big[\frac{\alpha a^2}{2} - \frac{1}{6}(\alpha a^2 + 2q\cos 2\psi)\cos 2\theta - \frac{q}{3}\sin 2\psi \sin 2\theta\Big], \quad \theta = \omega t + \psi, \tag{2.2}$$

$$\begin{aligned} \frac{da}{dt} &= \frac{\varepsilon^2}{2\omega}\Big[-2ha\omega + \frac{\alpha q}{3} a \sin 2\psi - r\sin(\psi + \eta)\Big], \\ a\frac{d\psi}{dt} &= \frac{\varepsilon^2}{2\omega}\Big[-\Delta a + 2\gamma a^3 + \frac{\alpha q}{3} a\cos 2\psi - r\cos(\psi + \eta)\Big], \end{aligned} \tag{2.3}$$

$\gamma = \frac{3\beta}{8} - \frac{5\alpha^2}{12}$.

The stationary solutions of equations (2.3) are determined by the relations:

$$f_0 = 0, \quad g_0 = 0,$$
$$f_0 = 2h\omega a - \frac{\alpha q}{3} a \sin 2\psi + r\sin(\psi + \eta),$$
$$g_0 = \Delta a - 2\gamma a^3 - \frac{\alpha q}{3} a \cos 2\psi + r\cos(\psi + \eta),$$

or equivalently:

$$f_0 \cos\psi - g_0 \sin\psi = 0, \quad f_0 \sin\psi + g_0 \cos\psi = 0.$$

From here we obtain

$$\begin{aligned} 2h\omega a\cos\psi - (\Delta a - 2\gamma a^3 + \frac{\alpha q}{3} a)\sin\psi + r\sin\eta = 0, \\ 2h\omega a\sin\psi + (\Delta a - 2\gamma a^3 - \frac{\alpha q}{3} a)\cos\psi + r\cos\eta = 0. \end{aligned} \tag{2.4}$$

Note: The equations (2.4) belong to the form

$$Y\sin\psi + Z\cos\psi = C. \tag{1}$$

The functions $\sin\psi$ and $\cos\psi$ satisfy the relationship

$$\sin^2\psi + \cos^2\psi = 1. \tag{2}$$

From equation (1) we have

$$Y^2\sin^2\psi = C^2 + Z^2\cos^2\psi - 2ZC\cos\psi. \tag{3}$$

Eliminating $\sin\psi$ between last two equations we get

$$(Y^2 + Z^2)\cos^2\psi - 2ZC\cos\psi + C^2 - Y^2 = 0.$$

From here we obtain

$$\cos\psi = \frac{1}{Y^2 + Z^2}\left[ZC \pm \sqrt{Z^2C^2 - (Y^2 + Z^2)(C^2 - Y^2)}\right].$$

Therefore, the condition for reality of $\cos\psi$ is that the under radical expression should not be negative:

$$Y^2 + Z^2 \geq C^2. \tag{2.5}$$

Applying the condition (2.5) to equations (2.4) we have

$$a^2\left\{4h^2\omega^2 + \left(\Delta - 2\gamma a^2 + \frac{\alpha q}{3}\right)^2\right\} \geq r^2\sin^2\eta, \tag{2.6}$$

$$a^2\left\{4h^2\omega^2 + \left(\Delta - 2\gamma a^2 - \frac{\alpha q}{3}\right)^2\right\} \geq r^2\cos^2\eta. \tag{2.7}$$

System without friction ($h = 0$)

In this case equations (2.4) take the form

$$\begin{aligned}\Big[2\gamma a^2 - \Big(\Delta + \frac{\alpha q}{3}\Big)\Big] a \sin\psi &= -r\sin\eta,\\ \Big[2\gamma a^2 - \Big(\Delta - \frac{\alpha q}{3}\Big)\Big] a \cos\psi &= r\cos\eta.\end{aligned} \tag{2.8}$$

a) If $2\gamma a^2 - \Big(\Delta + \dfrac{\alpha q}{3}\Big) \neq 0$ and $2\gamma a^2 - \Big(\Delta - \dfrac{\alpha q}{3}\Big) \neq 0$ then eliminating the phase ψ from (2.8) we get the equation of the resonance curve C_1:

$$W(\omega^2, a^2) = 0. \tag{2.9}$$

where

$$W(\omega^2, a^2) = \frac{r^2\sin^2\eta}{\Big[2\gamma a^2 - \Big(\Delta + \frac{\alpha q}{3}\Big)\Big]^2} + \frac{r^2\cos^2\eta}{\Big[2\mu a^2 - \Big(\Delta - \frac{\alpha q}{3}\Big)\Big]^2} - a^2. \tag{2.10}$$

b) If $2\gamma a^2 - \Big(\Delta + \dfrac{\alpha q}{3}\Big) = 0$, i.e. if we have the resonance curve C_2:

$$2\gamma a^2 = \omega^2 - 1 + \frac{\alpha q}{3}\,. \tag{2.11}$$

then

$$o.a.\sin\psi = -r\sin\eta, \quad \frac{2\alpha q}{3} a\cos\psi = r\cos\eta.$$

From here we obtain:

$$\sin\eta = 0 \Rightarrow \eta = 0,\ \pi; \quad \cos\eta = \pm 1 \Rightarrow \psi = \arccos\frac{\pm 3r}{2\alpha q a} \Rightarrow a^2 \geq \frac{9r^2}{4\alpha^2 q^2}\,. \tag{2.12}$$

c) If $2\gamma a^2 - \Big(\Delta - \dfrac{\alpha q}{3}\Big) = 0$, i.e. if we have the resonance curve C_3:

$$2\gamma a^2 = \omega^2 - 1 - \frac{\alpha q}{3}\,, \tag{2.13}$$

then

$$o.a\cos\psi = r\cos\eta, \quad \frac{2\alpha q}{3} a\sin\psi = r\sin\eta.$$

From here we obtain:

$$\cos\eta = 0 \Rightarrow \eta = \frac{\pi}{2},\ \frac{3\pi}{2}; \quad \sin\eta = \pm 1 \Rightarrow \psi = \pm\arcsin\frac{3r}{2\alpha q a} \Rightarrow a^2 \geq \frac{9r^2}{4\alpha^2 q^2}\,. \tag{2.14}$$

So, if $\eta = 0,\ \dfrac{\pi}{2},\ \pi,\ \dfrac{3\pi}{2}$, the resonance curves C_2, C_3 do not exist. If $\eta = 0,\ \pi$, then beside the resonance curve C_1 there is still semi-straight line C_2 in the (a^2, ω^2)-plane with $a^2 \geq \dfrac{9r^2}{4\alpha^2 q^2}$. If $\eta = \dfrac{\pi}{2},\ \dfrac{3\pi}{2}$ then beside the resonance curve C_1 there is still semi-straight line C_3 in the (a^2, ω^2)-plane with $a^2 \geq \dfrac{9r^2}{4\alpha^2 q^2}$.

System with friction ($h \neq 0$)

Solving the system of equations (2.4) relatively $\sin\psi$ and $\cos\psi$ we have:

a) For the case $D \neq 0$:

$$a \sin \psi = \frac{D_1}{D}, \quad a \cos \psi = \frac{D_2}{D}, \tag{2.15}$$

where

$$\begin{aligned} D &= 4\omega^2 h^2 + (\Delta - 2\gamma a^2)^2 - \frac{\alpha^2 q^2}{9}, \\ D_1 &= -r\Big[2h\omega \sin \eta + \Big(\Delta - 2\gamma a^2 + \frac{\alpha q}{3}\Big) \cos \eta\Big], \\ D_2 &= -r\Big[2h\omega \cos \eta - \Big(\Delta - 2\gamma a^2 - \frac{\alpha q}{3}\Big) \sin \eta\Big]. \end{aligned} \tag{2.16}$$

Eliminating ψ from (2.15) gives the following equation for amplitude (a) and frequency (ω)

$$a^2 = \frac{D_1^2 + D_2^2}{D^2}. \tag{2.17}$$

b) If $D = 0$ we have

$$2\gamma a^2 = \omega^2 - 1 \pm \sqrt{\frac{\alpha^2 q^2}{9} - 4\omega^2 h^2}, \tag{2.18}$$

and $\sin \psi$, $\cos \psi$ exist only when $D_1 = D_2 = 0$, or equivalently

$$D_1 \cos \eta - D_2 \sin \eta = 0, \quad D_1 \sin \eta + D_2 \cos \eta = 0.$$

From here we obtain:

$$2\gamma a_*^2 = \omega_*^2 - 1 + \frac{\alpha q}{3} \cos 2\eta, \quad \omega_* = -\frac{\alpha q}{6h} \sin 2\eta. \tag{2.19}$$

The formula (2.6) with taking into account $D = 0$ and (2.19) gives a restriction to a_*:

$$a_*^2 \geq \frac{9r^2}{4\alpha^2 q^2}. \tag{2.20}$$

3. Interaction of the elements of first degree of smallness quadratic nonlinearity and forced excitation with the self-excitation of second degree of smallness

Let us consider a nonlinear system described by the following differential equation:

$$\begin{aligned} \ddot{x} + \omega^2 x &= \varepsilon(\alpha x^2 + q \cos 2\omega t) + \varepsilon^2\big[\Delta x + D(1 - \delta x^2)\dot{x} - \beta^3 x\big], \\ \omega^2 &= 1 - \varepsilon^2 \Delta. \end{aligned} \tag{3.1}$$

Where D, δ are positive constants. The other parameters are the same as in the previous paragraphs.

The approximate solution of the equation (3.1) will be found in the form (2.2) with the amplitude (a) and phase (ψ) satisfying the relations:

$$\begin{aligned} \frac{da}{dt} &= \frac{\varepsilon^2}{2\omega}\Big[D a \omega \Big(1 - \frac{\delta}{4} a^2\Big) + \frac{\alpha q}{3} a \sin 2\psi\Big], \\ a\frac{d\psi}{dt} &= \frac{\varepsilon^2}{2\omega}\Big(- \Delta a + 2\gamma a^3 + \frac{\alpha q}{3} a \cos 2\psi\Big), \end{aligned} \tag{3.2}$$

where

$$\gamma = \frac{3\beta}{8} - \frac{5\alpha^2}{12} \cdot$$

Equation (3.2) have a trivial solution $a = 0$. The non-trivial ($a \neq 0$) stationary amplitude a_0 and phase ψ_0 are determined from the equations:

$$\begin{aligned} \frac{\alpha q}{3} \sin 2\psi_0 &= -D\omega\Big(1 - \frac{\delta}{4}a_0^2\Big), \\ \frac{\alpha q}{3} \cos 2\psi_0 &= \Delta - 2\gamma a_0^2. \end{aligned} \tag{3.3}$$

Eliminating the phase ψ_0 gives:

$$W(a_0, \omega^2) = 0, \tag{3.4}$$

$$W(a_0, \omega^2) = (\Delta - 2\gamma a_0^2)^2 + D^2\omega^2\Big(1 - \frac{\delta}{4}a_0^2\Big)^2 - \frac{\alpha^2 q^2}{9} \cdot \tag{3.5}$$

From the last two equations we obtain approximately

$$\omega^2 = 1 + 2\varepsilon^2\gamma a_0^2 \pm \varepsilon^2\sqrt{\frac{\alpha^2 q^2}{9} - D^2\Big(1 - \frac{\delta}{4}a_0^2\Big)^2} \cdot \tag{3.6}$$

This formula is plotted in the figure 4 for the parameters: $\varepsilon^2 D = 10^{-3}$, $\delta = 40$, $\varepsilon\frac{\alpha q}{3} = 10^{-3}$ and $\varepsilon^2\gamma = -0.005$ (curve 1), $\varepsilon^2\gamma = 0.01$ (curve 2) and $\varepsilon^2\gamma = 0.025$ (curve 3)

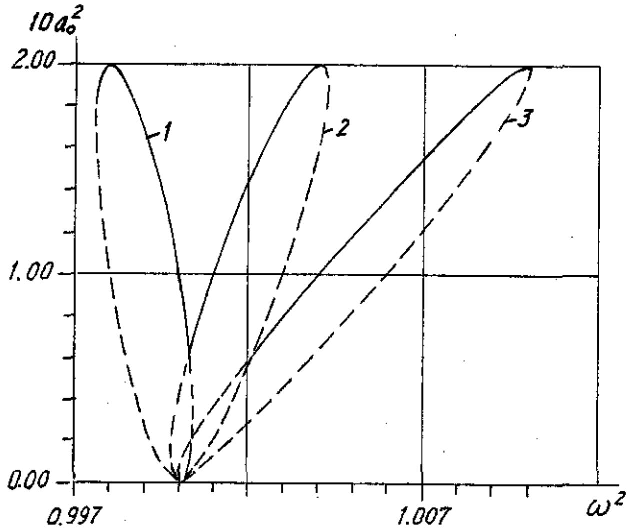

Fig. 4

Denoting the right hand sides of the equations (3.2) by X and Y, respectively, we have:

$$\begin{aligned} \Big(\frac{\partial X}{\partial a}\Big)_0 &= -\frac{\varepsilon^2}{4}D\delta a_0^2, \quad \Big(\frac{\partial X}{\partial \psi}\Big)_0 = \frac{\varepsilon^2 a_0}{\omega}(\Delta - 2\gamma a_0^2) \\ \Big(\frac{\partial Y}{\partial a}\Big)_0 &= \frac{2\varepsilon^2}{\omega}\gamma a_0^2, \quad \Big(\frac{\partial Y}{\partial \psi}\Big)_0 = \varepsilon^2 a_0 D\Big(1 - \frac{\delta}{4}a_0^2\Big), \end{aligned} \tag{3.7}$$

where the subscript "0" means that the derivatives are calculated at stationary values a_0, ψ_0 (3.3). The stability conditions of stationary oscillations are

$$
\begin{aligned}
&a_0\Big(\frac{\partial X}{\partial a}\Big)_0 + \Big(\frac{\partial Y}{\partial \psi}\Big)_0 = \varepsilon^2 a_0 D\Big(1 - \frac{\delta}{2}a_0^2\Big) < 0,\\
&\Big(\frac{\partial X}{\partial a}\Big)_0 \cdot \Big(\frac{\partial Y}{\partial \psi}\Big)_0 - \Big(\frac{\partial X}{\partial \psi}\Big)_0 \Big(\frac{\partial Y}{\partial a}\Big)_0 = \frac{\varepsilon^4}{2\omega^2} a_0^3 \cdot \frac{\partial W}{\partial a_0^2}.
\end{aligned}
\tag{3.8}
$$

Hence $a_0 > 0$, the stability conditions take the form

$$
a_0^2 > \frac{2}{\delta}, \quad \frac{\partial W}{\partial a_0^2} > 0. \tag{3.9}
$$

To study the stability of the zero solution $a = 0$ of equations (3.2) we introduce the variable u, v connected with a and ψ by the relations:

$$
u = a\cos\psi, \quad v = a\sin\psi. \tag{3.10}
$$

We have

$$
\begin{aligned}
\frac{da}{dt} &= \frac{\varepsilon^2}{2\omega}\Big[D\omega u + \Big(\Delta + \frac{\alpha q}{3}\Big)v\Big] + \ldots\\
\frac{dv}{st} &= \frac{\varepsilon^2}{2\omega}\Big[\Big(-\Delta + \frac{\alpha q}{3}\Big)u + D\omega v\Big] + \ldots
\end{aligned}
\tag{3.11}
$$

where the non-written terms contain u and v with higher degrees of smallness. The origin $u = v = 0$ ($a = 0$) of the system of equations (3.11) is unstable, because the characteristic equation of the linear terms of (3.11) has the roots with positive real part.

In the figure 4 the stable branches of resonance curves are shown by heavy lines, while the unstable ones-by dotted lines.

4. Conclusion

In the nonlinear system under consideration, the elements characterizing the first degree of smallness quadratic nonlinearity and nonresonance forced excitation (for brief, N-F-elements) have no effect on the oscillation in the first approximation. However, they interact one with another in the second approximation and appear as a parametric excitation with modulation of the product of their intensity (α, q). This means that each element (α and q) standing alone has no effect on the system and these elements have equal role. The resonance curve (Fig. 1) is bent to the right and cuts the frequency-axis at two points. This curve is typical for a nonlinear system with parametric excitation. The passage of the system under consideration through resonance has been examined (Fig. 2, 3).

In the second paragraph their interaction between these elements and the second degree of smallness resonance forced excitation has been studied. Some typical results for the interaction between parametric and forced excitations have been obtained. The interaction between N-F-elements and self-excitation is given in the paragraph 3. The resonance curves have oval forms (Fig. 4) and are bent either to the left or to the right, depending on the sign of the parameter γ.

Acknowledgments

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TƯƠNG TÁC CỦA CÁC PHẦN TỬ ĐẶC TRƯNG CHO PHI TUYẾN CẤP HAI VÀ KÍCH ĐỘNG CƯỠNG BỨC VỚI CÁC KÍCH ĐỘNG LOẠI KHÁC

Trong các hệ phi tuyến, những số hạng phi tuyến cấp hai và kích động cưỡng bức không cộng hưởng có bậc bé ε và các số hạng ma sát tuyến tính, phi tuyến cấp ba, các kích động thông số và cưỡng bức cộng hưởng có bậc bé ε^2 sẽ không có tác dụng trong xấp xỉ thứ nhất, song chúng tác động qua lại với nhau trong xấp xỉ thứ hai và những hiện tượng phi tuyến mới sẽ xuất hiện. Việc nghiên cứu các hiện tượng này là mục tiêu của bài báo. Phương pháp tiệm cận của cơ học phi tuyến kết hợp với máy tính đã cho phép giải bài toán đặt ra.