FREE CONVECTION FLOW IN A VERTICAL ANNULUS WITH POWER LAW FLUID

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1. Introduction

In [1, 2] free convection flow in a vertical plate channel of finite height without and with wall thickness with power law fluid is investigated.

In [3] the flow in vertical cylinder is considered.

In this paper we consider free convection flow in a vertical annulus of finite height with different external temperatures (see Fig. 1). The problem is solved by a finite difference scheme. The calculation result when the height is much bigger than the diametter is compared with asymptotic solution. When the radii are very big the calculation results give good coincidence with the ones of plate channel in [1, 2].

2. Basic equations and establishing the problem

In Cylindrical coordinates the problem is governed by following equations in dimensionless form (see [2, 3]).

Continuity equation:

$$\frac{\partial\,\overline{r}\,\overline{v}_r}{\partial\,\overline{r}} + \frac{\partial\,\overline{r}\,\overline{v}_z}{\partial\,\overline{z}} = 0. \tag{2.1}$$

Momentum equation:

$$\overline{v}_r\,\frac{\partial\,\overline{v}_r}{\partial\,\overline{r}} + \overline{v}_z\,\frac{\partial\,\overline{v}_z}{\partial\,\overline{z}} = -\frac{d\overline{p}}{d\overline{z}} + \frac{1}{\overline{r}}\,\frac{\partial}{\partial\,\overline{r}}(\overline{r}\,\eta\,\overline{v}_{z,r}) + TG_{rg}. \tag{2.2}$$

Energy equation:

$$\overline{v}_r\,\frac{\partial\,\overline{T}}{\partial\,\overline{r}} + \overline{v}_r\,\frac{\partial\,\overline{T}}{\partial\,\overline{r}} = \frac{1}{\overline{r}}\,\frac{\partial}{\partial\,\overline{r}}\Big(\overline{r}\,\frac{\partial\,\overline{T}}{\partial\,\overline{r}}\Big)\cdot P_{rg}^{-1}, \tag{2.3}$$

$$\frac{1}{\overline{r}}\,\frac{\partial}{\partial\,\overline{r}}\Big(\overline{r}\,\frac{\partial\,\overline{T}}{\partial\,\overline{r}}\Big) + \Big(\frac{D}{H}\Big)^2\frac{\partial^2\,\overline{T}_1}{\partial\,\overline{z}^2} = 0, \tag{2.4}$$

for $r_1 \le r \le r_2$ and $r_3 \le r \le r_4$,

where H - channel height; D - channel width $= r_3 - r_2$;

$$\overline{z} = \frac{z}{H};\quad \overline{r}_i = \frac{r_i}{D};\quad U^* = v_k^{\frac{1}{2-n}}\,D^{\frac{1-2n}{2-n}}\,H^{\frac{n-1}{2-n}};\quad \overline{v}_z = \frac{v_z D}{HU^*}$$

$$\overline{v}_r = \frac{v_r}{U^*};\quad T_{av} = \frac{T_{e_1}+T_{e_2}}{2};\quad \overline{T} = \frac{T-T_\infty}{T_{av}-T_\infty};\quad \overline{T}_1 = \frac{T_1-T_\infty}{T_{av}-T_\infty};\quad \overline{p}' = \frac{p'D^2}{\rho U^{*2}H};$$

$$\eta = \left|\overline{v}_{z,r}\right|^{n-1};\quad P_{rg} = C_p\rho U^* D\lambda^{-1};\quad G_{rg} = g\beta(T_{av}-T_\infty)U^{*-2}H^{-1}D^2.$$

η - apparent viscosity, T_∞ - temperature of surroundings, T_{e_1} - given temperature at $r = r_1$, T_{e_2} - given temperature at $r = r_4$, T_1 - temperature inside the channel walls, $p' = p(z) - p(0) + g\rho z$, P_{rg}, G_{rg} - generalized Prandtl and Grashof number, v_k - kinematic viscosity, ρ - density, C_p - specific heat coefficient, λ - thermal conductivity, g - acceleration of gravity, β - thermal expansion coefficient.

Boundary conditions

At $\bar{r} = \bar{r}_2$

$$\bar{v}_r(\bar{r}_2,\bar{z}) = \bar{v}_z(\bar{r}_2,\bar{z}) = 0, \quad \overline{T}_1(\bar{r}_2,\bar{z}) = \overline{T}(\bar{r}_2,\bar{z}), \quad \lambda_1\frac{\partial \overline{T}_1}{\partial \bar{r}}(\bar{r}_2,\bar{z}) = \lambda\frac{\partial \overline{T}}{\partial \bar{r}}(\bar{r}_2,\bar{z}). \tag{2.5}$$

At $\bar{r} = \bar{r}_3$

$$\bar{v}_r(\bar{r}_3,\bar{z}) = \bar{v}_z(\bar{r}_3,\bar{z}) = 0, \quad \overline{T}_1(\bar{r}_3,\bar{z}) = \overline{T}(\bar{r}_3,\bar{z}), \quad \lambda_1\frac{\partial \overline{T}_1}{\partial \bar{r}}(\bar{r}_3,\bar{z}) = \lambda\frac{\partial \overline{T}}{\partial \bar{r}}(\bar{r}_3,\bar{z}). \tag{2.6}$$

$$\text{At} \quad \bar{r} = \bar{r}_1 \quad \overline{T}_1(\bar{r}_1,\bar{z}) = \overline{T}_{e_1} \le 1. \tag{2.7}$$

$$\text{At} \quad \bar{r} = \bar{r}_4 \quad \overline{T}_1(\bar{r}_1,\bar{z}) = \overline{T}_{e_2} \ge 1. \tag{2.8}$$

$$\text{At} \quad \bar{z} = 0 \quad p'(0) = \bar{v}_r(\bar{r},0) = \overline{T}(\bar{r},0) = 0. \tag{2.9}$$

$$\bar{v}_z(\bar{r},0) = v_{z_0}$$

$$\text{At} \quad \bar{z} = 1 \quad \bar{p}'(1) = 0. \tag{2.10}$$

Because of smallness of D in comparison with H: $(D/H) \ll 1$ the second term in (2.4) can be negleted. This leads to the following equation.

$$\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \overline{T}}{\partial \bar{r}}\right) = 0, \quad \bar{r}_1 \le \bar{r} \le \bar{r}_2 \quad \text{and} \quad \bar{r}_3 \le \bar{r} \le \bar{r}_4. \tag{2.11}$$

In addition, from the continuity equation and condition

$$\bar{v}_r(\bar{r}_2,\bar{z}) = \bar{v}_r(\bar{r}_3,\bar{z}) = 0$$

it follows

$$\int_{r_2}^{r_3} \bar{v}_z\,\bar{r}dr = \frac{1}{2}v_{z0}(\bar{r}_3^2 - \bar{r}_2^2). \tag{2.12}$$

The unknowns of system (2.1) - (2.7) are $\bar{v}_r$, $\bar{v}_z$, $\overline{T}z$, $\overline{T}_1$, $\bar{p}$, v_{z0}. Two qualities of particular interest are the average velocity along the channel $\bar{v}_z$, and the total heat transfer from the wall Q, which is characterized by average Nusselt number $\overline{N}_{uD}$.

3. Numerical solutions

Further we'll drop all the signs "$-$" for convenience. First, we can exclude T_1 by integrating (2.1) combining with boundary conditions in (2.5) $\div$ (2.8) and we get following boundary conditions for T

$$\begin{aligned} \Psi_{e_1}(T - T_{e_1}) &= \frac{\partial T}{\partial r} \quad \text{at} \quad r = r_2, \\ \Psi_{e_2}(T_{e_2} - T) &= \frac{\partial T}{\partial r} \quad \text{at} \quad r = r_3, \end{aligned} \tag{3.1}$$

where

$$\Psi_{e_1}=\frac{\lambda_1}{r_2\ln(r_2/r_1)}\ ;\quad \Psi_{e_2}=\frac{\lambda_1}{r_3\ln(r_4/r_3)}\ .$$

Affter T founded T_1 can be calculated as

$$T_1=-\frac{T(r_3,z)-T_{e_2}}{\ln(r_4/r_3)}\ln(r)+T_{e_2}+\frac{T(r_3,z)-T_{e_2}}{\ln(r_4/r_3)}\ln(r_4)\quad \text{for } \bar{r}_3\le\bar{r}\le\bar{r}_4$$

and

$$T_1=\frac{T(r_2,z)-T_{e_1}}{\ln(r_2/r_1)}\ln(r)+T_{e_1}-\frac{T(r_2,z)-T_{e_1}}{\ln(r_2/r_1)}\ln(r_1)\quad \text{for } r_1\le r\le r_2$$

(2.1)-(2.3), (2.10), (2.12), (3.1) is a closed system for v_r, v_z, T, p', v_{z0}. We solve this system by a finite difference method. The finite difference equations are (see Fig. 2)

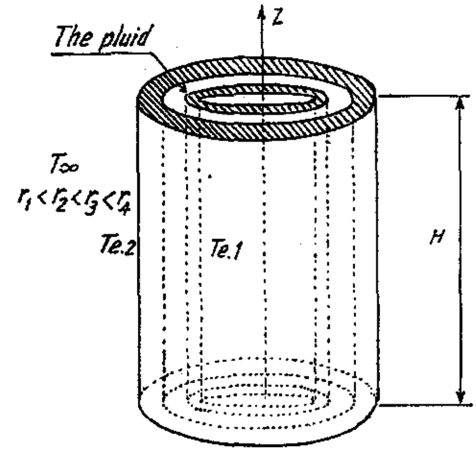


Fig. 1

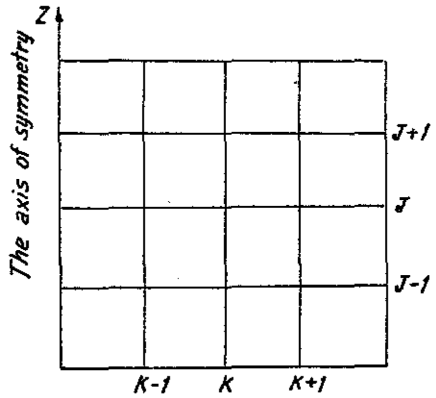


Fig. 2

$$\frac{\left(r\overset{s+1}{v}_r\right)^{j+1}_{k+1}-\left(r\overset{s+1}{v}_r\right)^{j+1}_k}{\Delta r}+\frac{\left(r\overset{s+1}{v}_z\right)^{j+1}_{k+1}+\left(r\overset{s+1}{v}_z\right)^{j+1}_k-\left(rv_z\right)^j_{k+1}-\left(rv_z\right)^j_k}{2\Delta z}=0 \qquad (3.2)$$

$$\left(\overset{s}{v}_z\right)^{j+1}_k\frac{\left(\overset{s+1}{v}_z\right)^{j+1}_k-\left(v_z\right)^j_k}{\Delta z}+\left(\overset{s}{v}_r\right)^{j+1}_k\frac{\left(\overset{s+1}{v}_r\right)^{j+1}_{k+1}-\left(\overset{s+1}{v}_r\right)^{j+1}_{k-1}}{2\Delta r}=-\frac{\left(\overset{s+1}{p}{}'\right)^{j+1}-p'^j}{\Delta z}+G_{rg}\left(\overset{s+1}{T}\right)^{j+1}_k$$
$$+\frac{\overset{s}{\eta}{}^{j+1}_{k+(1/2)}\left(\left(\overset{s+1}{v}_z\right)^{j+1}_{k+1}-\left(\overset{s+1}{v}_z\right)^{j+1}_k\right)-\left(\overset{s}{\eta}\right)^{j+1}_{k-(1/2)}\left(\left(\overset{s+1}{v}_z\right)^{j+1}_k-\left(\overset{s+1}{v}_z\right)^{j+1}_{k-1}\right)}{(\Delta r)^2} \qquad (3.3)$$

$$\left(\overset{s}{v}_z\right)^{j+1}_k\frac{\left(\overset{s+1}{T}\right)^{j+1}_k-\left(\overset{s+1}{T}\right)^j_k}{\Delta z}+\left(\overset{s}{v}_r\right)^{j+1}_k\frac{\left(\overset{s+1}{T}\right)^{j+1}_{k+1}-\left(\overset{s+1}{T}\right)^{j+1}_{k-1}}{2\Delta r}=$$
$$=P_{rg}^{-1}\frac{\left(\overset{s+1}{T}\right)^{j+1}_{k+1}-2\left(\overset{s+1}{T}\right)^{j+1}_k+\left(\overset{s+1}{T}\right)^{j+1}_{k-1}}{(\Delta r)^2} \qquad (3.4)$$

where s - iteration number, $\eta_{k+(1/2)}$, $\eta_{k-(1/2)}$ is taken equal to $\left|\frac{(v_z)_{k+1}-(v_z)_k}{\Delta r}\right|^{n-1}$; $\left|\frac{(v_z)_{k+1}-(v_z)_{k-1}}{\Delta r}\right|^{n-1}$. This is a non-linear system. The truncation errors is of $0(\Delta z,\Delta r^2)$.

We solve this system by iterating on index s. Let's assume that all quantities at j-row and quantities with index s at $j+1$-row are known. From (3.3), (3.4) using the Thomas algorithm we can obtain (drop index $s+1$ and $j+1$ at v_z and p' for convenience).

$$A_k(v_z)_{k-1}+B_k(v_z)_k+C_k(v_z)_{k+1}+p'=D_k;\quad k=\overline{2,N-1};\quad (v_z)_1=(v_z)_2;\quad (v_z)_N=0 \quad (3.5)$$

$$\int_{r_2}^{r_3} rv_z\,dr=\frac{1}{2}v_{z0}(r_3^2-r_2^2) \quad (3.6)$$

(3.5), (3.6) are $N+1$ equations for $(N+1)$ unknowns p', $(v_z)_1$, $(v_z)_2$,...,$(v_z)_N$. We solve this system as follows:

Let p_1, p_2, $p_1 \neq p_2$ - two arbitrary values. Using the Thomas algorithm we can find two solutions $v_z^{(1)}$, $v_z^{(2)}$:

$$v_z^{(1)}=\left((v_z)_1^{(1)},(v_z)_2^{(1)},\ldots,(v_z)_N^{(1)}\right),\quad v_z^{(2)}=\left((v_z)_1^{(2)},(v_z)_2^{(2)},\ldots,(v_z)_N^{(2)}\right),$$

of system (3.5). Because of the linearity $\alpha p_1+(1-\alpha)p_2$, $\alpha v_z^{(1)}+(1-\alpha)v_z^{(2)}$; $\forall\alpha$ are solutions of (3.5), too. Substitution into (3.6) gives

$$\alpha=\frac{\frac{1}{2}v_{z0}(r_3+r_2)-\int_{r_2}^{r_3} rv_z^{(2)}dr}{\int_{r_2}^{r_3} r(v_z^{(1)}-v_z^{(2)})dr}$$

4. Discussion of the results

A. The case without channel thickness

a. Asymptotic solution. When $(H/D)\to\infty$ then far from the entrance the problem is one - dimensional and we can find the solution easily:

$$T=a\ln(r)+b \quad (4.1)$$

$$a=\frac{T_{e_2}-T_{e_1}}{\ln(r_3/r_2)} \quad (4.2)$$

$$b=\frac{T_{e_1}\ln(r_3)-T_{e_2}\ln(r_2)}{\ln(r_3/r_2)} \quad (4.3)$$

$$v_z=(G_{rg}|b^*|/2)^{1/n}\text{sign}\,|b^*|\int_{r_2}^{r}|\omega|^{1/n}\,\text{sign}|\omega|dr=(G_{rg}|b^*|/2)^{1/n}\text{sign}|b^*|\int_{r_2}^{r}W\,dr \quad (4.4)$$

where

$$b^*=b-0.5a \quad (4.5)$$

$$\omega(r)=-r-(a/b^*)r\ln(r)+(c/r) \quad (4.6)$$

$$W=|\omega|^{1/n}\,\text{sign}\,(\omega)$$

Constant c is chosen to satisfy the condition $\int_{r_2}^{r_3} W dr = 0$

$$v_{z0} = \frac{2}{r_2+r_3}\int_{r_2}^{r_3} r v_z dr = -\frac{(G_{rg}|b^*|/2)\text{sign}|b^*|}{r_2+r_3}\int_{r_2}^{r_3} r^2 W dr \qquad (4.7)$$

$$\overline{N}_{uD} = \frac{P_{rg}b^*}{r_2+r_3}(G_{rg}|b^*|/2)^{1/n}\int_{r_2}^{r_3}[(a/b^*)r^2\ln(r)+r^2]W dr \qquad (4.8)$$

if $T_{e_1} = T_{e_2}$ (symmetric external temperatures) then $T_{e_1} = T_{e_2} = 1$; $a = 0$; $b = 1$

(4.1) becomes: $T = 1$ (4.9)

(4.6) becomes: $\omega = (c/r) - r$ (4.10)

For comparison we take $P_{rg} = 100$; $G_{rg} = 4.795 \times 10^{-2}$; $n = 0.66$; $\lambda_1 = 4$; $r_1 = 1$; $T_{e_2} = 1.5$; $T_{e_1} = 0.5$.

The formulae (4.7), (4.8) give $v_{z0} = 5.77 \times 10^{-4}$; $\overline{N}_{uD} = 3.19 \times 10^{-2}$

Numerical results are $v_{z0} = 5.70 \times 10^{-4}$; $\overline{N}_{uD} = 3.15 \times 10^{-2}$

The differences are smaller 1.2%

b. Numerical example. The fluid under consideration is a 1000 wppm solution of water and CMC (carboxy methyl cellulose). The input data are as follows (with dimensions) (see [2])

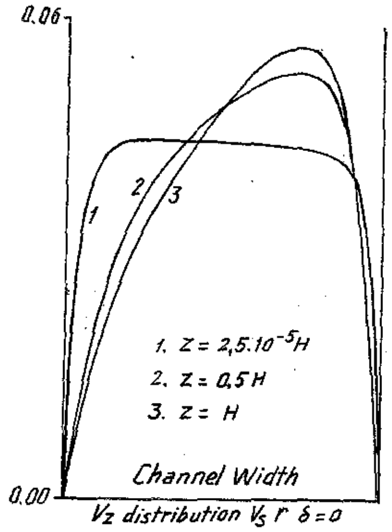
$T_\infty = 15°C$	$T_{e_1} = 20°C$	$T_{e_2} = 30°C$
$D = 2cm$	$H = 20cm$	$p = 1000kg/m^3$
$C_p = 4.18 \times 10^3 j/kgK$	$\lambda = 0.597W/mK$	$v_k = 7.35 \times 10^{-8} m^2/s^{2-n}$
$\beta = 1.8 \times 10^{-4}\ 1/K$	$n = 0.66$	

The calculation results are

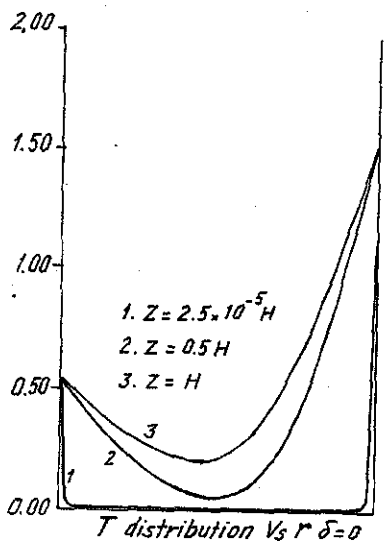
$v_{z0} = 4.34 \times 10^{-2}$ (that's 1.36×10^{-1}cm/s)

$N_{uD} = 4.18$

The distribution of T, v_z are shown in Fig. 3, 4



V_z distribution V_s r $\delta = 0$
Fig. 3



T distribution V_s r $\delta = 0$
Fig. 4

To compare with plate channel we take $r_1 = 5000$

$T_{e_1} = T_{e_2}$ our results are: $v_{z0} = 4.12 \times 10^{-2}$.

$N_{uD} = 3.39$. The difference from [2] is 8.4%

B. The case with wall thickness

a. Asymptotic solution. The one - dimensional solution are

$$T = a \ln(r) + b$$

where

$$a = \frac{T_{e_2} - T_{e_1}}{\dfrac{1}{r_3 \Psi_{e_2}} + \dfrac{1}{r_2 \Psi_{e_1}} + \ln(r_3/r_2)}, \quad b = \frac{T_{e_1} \ln(r_3) - T_{e_2} \ln(r_2) + \dfrac{T_{e_1}}{r_3 \Psi_{e_2}} + \dfrac{T_{e_2}}{r_2 \Psi_{e_1}}}{\dfrac{1}{r_3 \Psi_{e_2}} + \dfrac{1}{r_2 \Psi_{e_1}} + \ln(r_3/r_2)}$$

The formulae for v_{z0}, N_{uD} remain the same as above.

b. Numerical example. Let δ - the dimensionless thickness ($\delta = r_2 - r_1 = r_4 - r_3$)

Take $\delta = 0.125$ and $\delta = 0.025$. The other data are the same. Results:

$$v_{z0} = 4.32 \times 10^{-2}; \quad N_{uD} = 4.11 \quad \text{for} \quad \delta = 0.125$$

The distribution of v_z, T is shown in Fig. 5, 6

$$V_{z0} = 4.32 \times 10^{-2}; \quad N_{uD} = 4.70 \quad \text{for} \quad \delta = 0.025$$

if $r_1 = 5000$, $T_{e_2} = T_{e_1}$ then $v_{z0} = 3.73 \times 10^{-2}$

$N_{uD} = 3.45$. The difference from [1] for plate channel are 1.4%

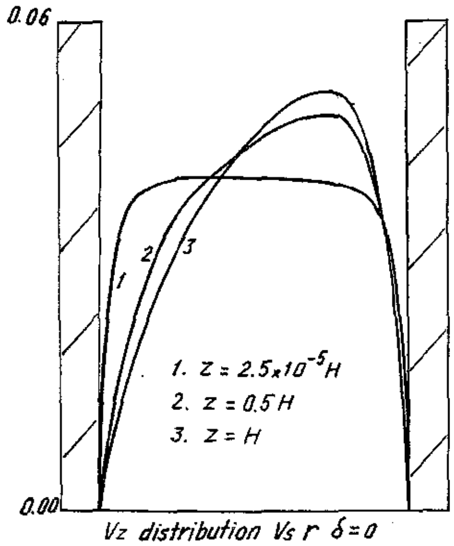


Fig. 5

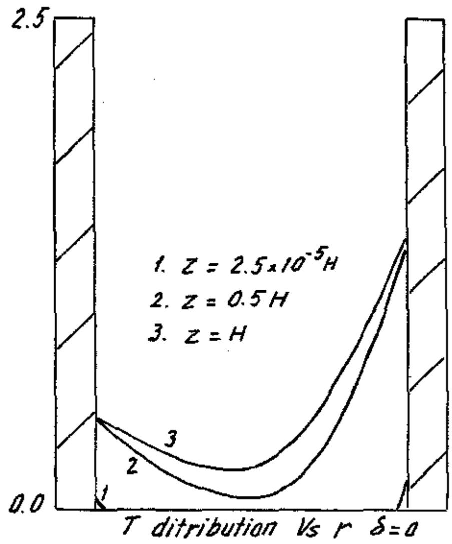


Fig. 6

5. Conclusion

More detailed calculation leads to following conclusions:

+ Influence of radius value on convection flow are very small so convection flow in plate channel and in annulus with same width is almost the same.

+ The wall thickness reduce the convection intensity

+ The convection (presented by v_{z0} and N_{uD}) in case of asymmetric external temperatures is stronger than in case of symmetric external temperatures with the same average.

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CHUYỂN ĐỘNG ĐỐI LƯU NHIỆT TỰ DO TRONG KHE TRỤ THẲNG ĐỨNG CỦA CHẤT LỎNG QUY LUẬT MŨ

Trong bài báo các tác giả nghiên cứu chuyển động đối lưu nhiệt tự do của chất lỏng quy luật mũ trong kênh nằm giữa hai ống trụ thẳng đứng, có chiều cao hữu hạn. Nhiệt độ hai thành cho trước và khác nhau. Bài toán được giải bằng sơ đồ sai phân hữu hạn. Kết quả tính toán được so sánh với nghiệm tiệm cận và trường hợp kênh phẳng. Có phân tích ảnh hưởng của bán kính trụ cũng như bề dày thành đến dòng đối lưu.