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STRENGTHENING BALANCE OF MOTION OF GYROSCOPE ON MOBILE BASIS

NGUYEN THANH MAU
Hanoi National University

In machine dynamics the balancing of gyroscopic system has a long tradition. In many textbooks the contributions to this topic are made regarding rigid gyroscope (see c.g. Huyen P., Mau N. T. [1-4]). The problems related to elastic multibody systems are more complicated. A good survey of recent results was presented by Anperin L. B. [5], Mau N. T. [6].

In a balancing problem usually there is asymptotic feature. However, in results of authors there are centenary elements. So a time expand, the balancing nature of gyroscopic system has been destroying.

In this paper author considers the balanced gyroscope with elastic symmetric axes on the mobile basis. The equations of motion of gyroscope have been represented in uncountable by degree units. The differential equation system is coming to the standard form, which can be solve by the method of separation motion. Computed result is the exponential function so centenary element destroyed.

1. The equation of motion of Gyro

Suppose that we have the gyroscope which satisfying the equation (1.4) in [1], where $v_1 = v_2 = 0$; $v_3 = \omega$ - const and $Lx_2 = -n\dot{\alpha} - \ell\alpha$; $Ly_2 = -n\dot{\beta} - \ell\beta$.

Similary to the former work: n - coefficient of liquid bearing friction;

α, β - inclined corner of outside and inside frame;

ℓ - coefficient of deformation symmetric axes. So generally, the system of equations (1.4) in the standard form will be written as:

$$\frac{dx}{dt} = W_1; \quad \frac{dy}{dt} = W_2$$

$$\begin{aligned}\mu A(y)\frac{dW_1}{dt} &= -\mu\big[d_2 W_1 W_2 \sin^2 y + W_2 \omega \cos y (d_2 \cos y - d_3) + C_1 \frac{\omega^2}{2} \sin 2x \cos^2 y \\ &+ \frac{\omega^2}{2}(d_4 - d_3 + d_0 \sin^2 y) \sin 2x\big] - (W_2 + \omega \sin x) \cos y - \chi n W_1 - \ell x\end{aligned} \tag{1.1}$$

$$\begin{aligned}\mu d_3 \frac{dW_2}{dt} &= -\mu\Big\{W_1\big[d_0 \omega \cos x - d_2(W_1 \sin y + \omega \cos x \cos y) \cos y \\ &+ d_2 \omega \cos x (W_1 \sin y + \omega \cos x \cos y) \sin y\Big\} + W_1 \cos y - \omega \cos x \sin y - \chi n W_2 - \ell y.\end{aligned}$$

Initial condition for the system (1.1) is

$$x(0) = x_0; \quad y(0) = 0; \quad \dot{x}(0) = \dot{y}(0) = 0.$$

Since the system (1.1) has balancing position $(0,0)$, we study the motion of gyroscope around this position. It is supposed that x and y are small.

2. Asymptotic solution

In the same way as the work [1], we will give the solution of the system (1.1) outside and inside boundary layer

a) At "0" order of small parameter μ, and outside boundary layer the solution will be

$$\begin{aligned} &\cos y^{(0)}\frac{dy^{(0)}}{dt}+\omega\sin x^{(0)}\cos y^{(0)}+n\chi\frac{dx^{(0)}}{dt}+\ell x^{(0)}=0.\\ &\cos y^{(0)}\frac{dx^{(0)}}{dt}-\omega\sin y^{(0)}\cos x^{(0)}-n\chi\frac{dy^{(0)}}{dt}-\ell y^{(0)}=0. \end{aligned} \tag{2.1}$$

Initial condition of (2.1) is:

$$x^{(0)}(t)\big|_{t=0}=x_0;\quad y^{(0)}(t)\big|_{t=0}=0.$$

The solution of the system (2.1) will be found by Poincare sum

$$\begin{aligned} x^{(0)}&=x_1^{(0)}(t)+\varepsilon x_2^{(0)}(t)+\varepsilon^2\ldots\\ y^{(0)}&=y_1^{(0)}(t)+\varepsilon y_2^{(0)}(t)+\varepsilon^2\ldots \end{aligned} \tag{2.2}$$

Replacing the formula (2.2) into (2.1) and keeping elements "0" order of small parameter ε, we obtain

$$\begin{aligned} &\frac{dy_1^{(0)}}{dt}+\omega x_1^{(0)}+n\chi\frac{dx_1^{(0)}}{dt}+\ell x_1^{(0)}=0,\\ &\frac{dx_1^{(0)}}{dt}-\omega y_1^{(0)}-n\chi\frac{dy_1^{(0)}}{dt}-\ell y_1^{(0)}=0. \end{aligned}$$

Multiplying the second equation on the i-supposed and adding one to the first, we receive equation:

$$(i+n\chi)\frac{d}{dt}(x_1^{(0)}-iy_1^{(0)})+(\omega+\ell)(x_1^{(0)}-iy_1^{(0)})=0.$$

The solution of this equation can be represented in the form:

$$\varphi(t)=\varphi(0)e^{-\omega_0(n\chi-i)t}.$$

where:

$$\varphi(t)=x_1^{(0)}-iy_1^{(0)};\quad \omega_0=\frac{\omega+\ell}{1+n^2\chi^2}\,.$$

Accordingly result presented by aspect:

$$x_1^{(0)}(t)=x_0e^{-\omega_0t}\cos\omega_0t,\quad y_1^{(0)}(t)=-x_0e^{-\omega_0t}\sin\omega_0t. \tag{2.3}$$

At the "first" order of small parameter μ we have the following system:

$$\begin{aligned} &\frac{dx^{(1)}}{dt}=W_1^{(1)};\quad \frac{dy^{(1)}}{dt}=W_2^{(1)},\\ &W_2^{(1)}\cos y^{(0)}+\chi nW_1^{(0)}+\ell x^{(1)}+\omega x^{(1)}\cos y^{(0)}\cos x^{(0)}-y^{(0)}\sin y^{(0)}(W_1^{(0)}+\omega\sin x^{(0)})\\ &=-(d_0\cos^2 y^{(0)}+a_2+C_1\sin^2 y^{(0)})\frac{dW_1^{(0)}}{dt}-W_1^{(0)}W_2^{(0)}d_2\sin 2y^{(0)}\\ &-W_2^{(0)}(d_2\cos 2y^{(0)}-d_3)\omega\cos x^{(0)}-\omega^2\sin 2x^{(0)}(C_2\cos^2 y^{(0)}+d_0\sin y^{(0)}+d_4-d_3),\\ &-W_1^{(1)}\cos y^{(0)}+\omega x^{(1)}\sin x^{(0)}\sin y^{(0)}+n\chi W_1^{(0)}+\ell y^{(1)}+y^{(1)}(W_1^{(0)}\sin y^{(0)}+\omega\cos x^{(0)}\cos y^{(0)})\\ &=-d_3\frac{dW_2^{(0)}}{dt}+(W_2^{(0)})^2d_2\sin y^{(0)}-W_2^{(0)}\omega\cos x^{(0)}(d_0+d_2\sin y^{(0)})-\frac{\omega^2}{2}d_2\cos^2 x^{(0)}\sin y^{(0)} \end{aligned} \tag{2.4}$$

Initial condition is:

$$x^{(1)}(t)\big|_{t=0} = 0; \quad y^{(1)}(t)\big|_{t=0} = 0.$$

The solution of the system (2.4) will be found in the form:

$$\begin{aligned} x^{(1)}(t) &= x_1^{(1)}(t) + \varepsilon x_2^{(1)}(t) + \varepsilon^2 \dots \\ y^{(1)}(t) &= y_1^{(1)}(t) + \varepsilon y_2^{(1)}(t) + \varepsilon^2 \dots \end{aligned} \tag{2.5}$$

In the same way, at the "zero" order of parameter ε, we have

$$\begin{aligned} &n\chi \frac{dx_1^{(1)}}{dt} + \omega x^{(1)} + \ell x^{(1)} + \frac{dy_1^{(1)}}{dt} = -(d_0 + a_2)\frac{dW_1^{(0)}}{dt} - \omega W_2^{(0)}(d_0 - d_3) - \omega^2 x^{(0)}(C_1 + d_4 - d_3), \\ &-\frac{dx_1^{(1)}}{dt} + \omega y_1^{(1)} + \ell y^{(1)} + n\chi \frac{dy_1^{(1)}}{dt} = -d_3 \frac{dW_2^{(0)}}{dt} - \omega d_0 W_1^{(0)} - \omega d_2 y^{(0)} \end{aligned} \tag{2.6}$$

with the initial condition

$$x_1^{(1)}(0) = y_1^{(1)}(0) = 0.$$

Analogously, we have:

$$(n\chi - i)\frac{d\psi(t)}{dt} + (\omega + \ell)\psi(\ell) = f(t),$$

where

$$\begin{aligned} \psi(t) &= x_1^{(1)} - i y_1^{(1)}, \\ f(t) &= -(d_0 + a_2)\frac{dW_1^{(0)}}{dt} - \omega W_2^{(0)}(d_2 - d_3) - \omega^2 x^{(0)}(C_1 + d_4 - d_3) \\ &\quad - i\Big(d_3 \frac{dW_2^{(0)}}{dt} - \omega d_0 W_1^{(0)} - \omega d_2 y^{(0)}\Big). \end{aligned}$$

The solution of the system (2.6) has the form

$$x_1^{(1)}(t) = x_0 t\ g(\omega_0 t)e^{-\omega_0 t}, \quad y_1^{(1)}(t) = x_0 t\ h(\omega_0 t)e^{-\omega_0 t}. \tag{2.7}$$

where: $g(\omega_0 t)$; $h(\omega_0 t)$ are harmonic functions. Now, we study the motion of Gyro at the "first" order of parameter μ . It is necessary to define the solution of the system (2.4) to the "first" order of parameter ε. In other word, we consider the system quasi receiving at the "first" order of parameter ε:

$$\begin{aligned} &\frac{dy_2^{(1)}}{dt} + n\chi \frac{dx_2^{(1)}}{dt} + \omega x_2^{(1)} + \ell x_2^{(1)} = 0, \\ &n\chi \frac{dy_2^{(1)}}{dt} + \omega y_2^{(1)} + \ell y_2^{(1)} - \frac{dx_2^{(1)}}{dt} = 0. \end{aligned} \tag{2.8}$$

with initial condition is: $x_2^{(1)}(0) = y_2^{(1)}(0) = 0$. It is easy to see that: the solution of (2.8) is zero function:

$$x_2^{(1)}(t) = 0; \quad y_2^{(1)}(t) = 0.$$

Now, we give the solution of system (1.1) inside boundary layer.

3. Solution inside the boundary layer

The system (1.1) inside boundary layer can be written as:

$$\frac{dx}{d\tau} = \mu W_1; \quad \frac{dy}{d\tau} = \mu W_2; \quad \mu\tau = t,$$

$$A(y)\frac{dW_1}{d\varepsilon} = -\mu\big[d_2 W_1 W_2 \sin 2y + \omega W_1 b \cos x(d_2 \cos y - d_3) + \frac{\omega^2}{2}\sin 2x C_1 \cos^2 y + d_0 \sin^2 y + d_4 - d_3)\big] - (W_2 + \omega \sin x)\cos y - \chi n W_1 - \ell x, \tag{3.1}$$

$$d_3\frac{dW_2}{d\tau} = -\mu\Big\{W_1[d_0\omega \cos x - d_2(W_1 \sin y + \omega \cos x \cos y)] + d_2\omega \cos x(W_1 \sin y + \omega \cos x \cos y)\sin y\Big\} + W_1 \cos y - \omega \cos x \sin y - n\chi W_2 - \ell y.$$

At "zero" order of μ we have

$$\frac{dx^{(0)}}{d\tau} = 0; \quad \frac{dy^{(0)}}{d\tau} = 0, \tag{3.2}$$

$$A^{(0)}(y^{(0)})\frac{dW_1^{(0)}}{d\tau} = -(W_2^{(0)} + \omega \sin x^{(0)})\cos y^{(0)} - n\chi W_1^{(0)} - \ell x^{(0)},$$

$$d_3\frac{dW_2^{(0)}}{d\tau} = -W_1^{(0)}\cos y^{(0)} - \omega \cos x^{(0)} \sin y^{(0)} - n\chi W_2^{(0)} - \ell y^{(0)}.$$

The initial condition for the system (3.2) is:

$$y^{(0)}(0) = x^{(0)}(0) = 0, \quad W_1^{(0)}(0) = \chi n \chi_0 \omega_0, \quad W_2^{(0)}(0) = \chi_0 \omega_0.$$

It is easy to show that: $x^{(0)}(\tau) = y^{(0)}(\tau) = 0$. So accordingly we have

$$(d_0 + a_2)\frac{dW_1^{(0)}}{d\tau} + n\chi W_1^{(0)} + W_2^{(0)} = 0,$$
$$W_1^{(0)} + d_3\frac{dW_2^{(0)}}{d\tau} + n\chi W_2^{(0)} = 0. \tag{3.3}$$

The solution of the system (3.3) is represented in the form

$$W_1^{(0)}(\tau) = e^{-K_3\tau}\big[C_1(\frac{\chi n a_2}{2a_0}\cos K_2\tau + \frac{K_1}{2d_0}\sin K_2\tau) + C_2(\frac{n\chi a_2}{2d_0}\sin K_2\tau - \frac{K_1}{2d_0}\sin K_2\tau),$$
$$W_2^{(0)}(\tau) = e^{-K_3\tau}(C_1\cos K_2\tau + C_2 \sin K_2\tau), \tag{3.4}$$

where

$$C_1 = x_0\omega_0; \quad C_2 = -\frac{1}{K_1}(2d_0x_0n\chi\omega_0 - a_2\omega_0 n\chi x_0) = -\frac{\omega_0 n\chi x_0}{K_1}(2d_0 - a_2)$$

and

$$K_1 = \sqrt{4d_3(d_0 - a_2) - \chi^2 n^2 a_2^2}; \quad K_3 = \frac{n\chi}{2d_3}; \quad K_2 = \frac{K_1}{2d_3(d_0 + a_2)}.$$

At "first" order of μ we obtain

$$\frac{dx^{(1)}}{d\tau} = W_1^{(0)}, \quad \frac{dy^{(1)}}{d\tau} = W_2^{(0)}.$$

From this system of equations we have

$$x^{(1)}(\tau) = \int_0^\tau W_1^{(0)}(\tau)d\tau; \quad y^{(1)}(\tau) = \int_0^\tau W_2^{(0)}d\tau$$