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THE FORMS OF THE SHELL WITH ZERO BENDING STRESSES SUBJECTED TO HYDROSTATIC PRESSURE AND OTHER LOADS

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1. Introduction

The shells without bending stress usually are used in practice. The problems for determination of the forms of these shells when external loads are given, have been investigated in many works [1, 2].

In reality, these problems are inverse mathematical problems and for some simple cases of loading, analytical and numerical solutions are derived in [3, 4].

In this paper we consider the shell subjected to hydrostatic pressure and axially symmetrical load, which is parallel to axis of revolution. The differential equations for determination of meridian forms of the shell are obtained. Analytical and numerical solution of these equations are presented. The forms of the shell are shown by program written by FTN77 on PC.

These problems are solved according to geometrical linear theory of the shell.

2. Equations of linear membrane theory for shells of revolution under axially-symmetrical loading

The equation of equilibrium for a shell of revolution in an axisymmetric membrane state of stress are [5]:

$$\begin{aligned} &\frac{dT_s}{ds} + (T_\varphi - T_s)\frac{\sin\theta}{r} = -X, \\ &\frac{T_s}{R_1} + \frac{T_\varphi}{R_2} = Z, \end{aligned} \tag{2.1}$$

where T_s, T_φ - stress resultants; X, Z - the surface load components; R_1, R_2 - radius of curvatures of middle surface; r - radius of the hoop circle; θ - angle of the meridian and axis z.

If $\eta = \sec\theta$ then solution of (2.1) will be written [1]:

$$\begin{aligned} T_s &= \frac{1}{r}\left[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\right]\eta, \\ T_\varphi &= r\eta Z + \left[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\right]\frac{d\eta}{dr}\,. \end{aligned} \tag{2.2}$$

Note that C may be found from loading in boundary of the shell. If Q is axially symmetrical load, which is parallel to axis of revolution at $r = r_0$ then

$$C = T_s(r_0) \times r_0 \times \cos\eta_0 = -Q \times r_0.$$

From the condition of zero bending stresses, the curvature change is equal to zero, we have

$$\chi_s = -\frac{d}{ds}\Big(\frac{dw}{ds} - \frac{u}{R_1}\Big) = 0,$$
$$\chi_\varphi = \frac{\sin\theta}{r}\Big(\frac{dw}{ds} - \frac{u}{R_1}\Big) = 0,$$

hence

$$\frac{dw}{ds} - \frac{u}{R_1} = 0.$$

Substituting this relation into the compatibility deformation equation [5]

$$r\frac{d\varepsilon_\varphi}{ds} - (\varepsilon_\varphi - \varepsilon_s)\sin\theta - \Big(\frac{dw}{ds} - \frac{u}{R_1}\Big)\cos\theta = 0,$$

we obtain the condition of zero bending stresses in the form

$$r\frac{d\varepsilon_\varphi}{ds} - (\varepsilon_\varphi - \varepsilon_s)\frac{dr}{ds} = 0 \Rightarrow$$
$$\frac{d(r\varepsilon_\varphi)}{dr} = \varepsilon_s. \tag{2.3}$$

The deformation components can be expressed by stress resultants

$$\varepsilon_s = \frac{1}{Eh}(T_s - \nu T_\varphi),$$
$$\varepsilon_\varphi = \frac{1}{Eh}(T_\varphi - \nu T_s), \tag{2.4}$$

where E - the Young's modulus, ν - the poisson's ratio and h - the thickness of the shell.

3. Integro-differential equation for determination of shell forms

Substituting (2.4) and (2.2) into (2.3) we obtain integro-differential equation for determination of shell forms:

$$r^2\left[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\right]\frac{d^2\eta}{dr^2} +$$

$$r\left[\left[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\right] \times \Big(1 - \frac{rh'}{h}\Big) + r\left(rZ + \frac{d\Big[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\Big]}{dr}\right)\right]\frac{d\eta}{dr} +$$

$$\left[r^2 Z\Big(2 - \frac{rh'}{h} + \nu\Big) + \int\limits_{r_0}^{r}\Big[r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\Big]\Big(\frac{rh'}{h}\nu - 1\Big) - \right.$$

$$\left. r\nu\frac{d\Big[\int\limits_{r_0}^{r} r\Big(Z + \frac{X}{\sqrt{\eta^2-1}}\Big)dr + C\Big]}{dr} + r^3\frac{dZ}{dr}\right]\eta = 0. \tag{3.1}$$

If the thickness of the shell changes on rule

$$\frac{h}{h_0} = \Big(\frac{r}{r_0}\Big)^n,$$

where r_0 and h_0 are respectively the values of r and h at $r = r_0$, and n is real. The loading consists of hydrostatic pressure and loads parallel to axis z at the boundary of the shells [5]:

$$X = 0, \quad Z = q\Big(1 - \frac{s}{L}\Big) = q\Big(\frac{r - r_1}{r_0 - r_1}\Big), \quad C = -Qr_0,$$

where q - pressure at the bottom of the shell, s - coordinate of the any point of the meridian, L - the length of the shell meridian. If p is axial load at the boundary $r = r_1$ then from equilibrium condition we have relation

$$\begin{aligned} 2\pi r_0 Q &= 2\pi r_1 p = -\pi r_0^2 q \Rightarrow \\ C &= -r_0 Q = -pr_1 = qr_0^2/2. \end{aligned}$$

The basic equation (3.1) then reduces to:

$$r^2 f(r)\frac{d^2\eta}{dr^2} + rg(r)\frac{d\eta}{dr} + k(r)\eta = 0, \tag{3.2}$$

where

$$\begin{aligned} f(r) &= \frac{q}{r_0 - r_1}\Big(\frac{r^3}{3} - r_1\frac{r^2}{2} - \frac{r_0^3}{3} + \frac{r_1 r_0^2}{2}\Big) + C, \\ g(r) &= \Big[\frac{q}{r_0 - r_1}\Big(\frac{r^3}{3} - r_1\frac{r^2}{2} - \frac{r_0^3}{3} + \frac{r_1 r_0^2}{2}\Big) + C\Big](1-n) + 2qr^2\Big(\frac{r - r_1}{r_0 - r_1}\Big), \\ k(r) &= qr^2\Big(\frac{r - r_1}{r_0 - r_1}\Big)(2-n) + (n\nu - 1)\Big[\frac{q}{r_0 - r_1}\Big(\frac{r^3}{3} - r_1\frac{r^2}{2} - \frac{r_0^3}{3} + \frac{r_1 r_0^2}{2}\Big) + C\Big] + \frac{r^3 q}{r_0 - r_1}. \end{aligned}$$

The equation (3.2) is differential equation of Fock type with regularity point $r = 0$.

If $d = \rho_1 - \rho_2$ is not natural, solution is found in the form:

$$\eta_1 = r^{\rho_1}\sum_{i=0}^{\infty} C_i r^i, \qquad \eta_2 = r^{\rho_2}\sum_{i=0}^{\infty} C_i r^i, \tag{3.3}$$

ρ_1 and ρ_2 are solutions of the characteristic equation:

$$\rho(\rho - 1)f(0) + \rho g(0) + k(0) = 0,$$

where

$$\begin{aligned} f(0) &= \frac{q}{r_0 - r_1}\Big(\frac{r_1 r_0^2}{2} - \frac{r_0^3}{3}\Big) + C, \\ g(0) &= (1-n)f(0), \\ k(0) &= (n\nu - 1)f(0), \\ \rho_{1,2} &= \frac{1}{2}\Big[n \pm \sqrt{n^2 - 4(n\nu - 1)}\Big]. \end{aligned}$$

The coefficients C_i in the (3.3) are determined by substituting (3.3) into (3.2) and giving zero coefficient with the same degree r^k, we have

$$\begin{aligned} C_0 &= 1, \quad C_1 = 0, \\ C_2 &= \frac{r_1 q}{2f(0)(r_0 - r_1)}, \\ C_m &= \frac{1}{f(0)}\Big[C_{m-2}\frac{r_1 q}{2(r_0 - r_1)} - C_{m-3}\frac{q}{3(r_0 - r_1)}\Big] \quad \text{with } m \geq 3. \end{aligned}$$

Coefficients in the series (3.3) then can be reduced to function:

$$\eta_1 = \frac{r^{\rho_1}}{1 - \frac{q}{f(0)(r_0 - r_1)}\left(\frac{r_1 r^2}{2} - \frac{r^3}{3}\right)}, \qquad \eta_2 = \frac{r^{\rho_2}}{1 - \frac{q}{f(0)(r_0 - r_1)}\left(\frac{r_1 r^2}{2} - \frac{r^3}{3}\right)}. \tag{3.4}$$

General solution is:

$$\eta = A\eta_1 + B\eta_2, \tag{3.5}$$

A, B - constants, which are given from boundary conditions of η in the $r = r_1$ and $r = r_0$.

If $d = \rho_1 - \rho_2$ is natural, solution has the form:

$$\eta_1 = \frac{r^{\rho_1}}{1 - \frac{q}{f(0)(r_0 - r_1)\left(\frac{r_1 r^2}{2} - \frac{r^3}{3}\right)}}, \qquad \eta_2 = \eta_1 \ln r + a r^{\rho_1 - d}. \tag{3.6}$$

If z denotes the distance along the axis of revolution, then a relationship between r and z may be obtained by integrating the equation with $r_0 > r$:

$$\frac{dr}{dz} = -\sqrt{1 - \eta^2}\,. \tag{3.7}$$

The general shapes of shell obtained from equation (3.4) - (3.7) are illustrated in numerical results Tab. 1 and Fig. 1). The type of curve depends on the n, A, B, q, r_1, r_0. These numerical results are given by program written by FTN77 on PC for calculating η and integrating (3.7).

Tab. 1

R_i	Z_1	Z_2	Z_3	Z_4
14	0	0	0	0
13	2.125	2.334	1.244	1.204
12	4.089	4.853	2.540	2.387
11	5.904	7.371	3.848	3.532
10	7.552	9.765	5.136	4.628
9	9.06	11.987	6.386	5.671
8	10.444	14.032	7.587	6.661
7	11.718	15.910	8.732	7.598
6	12.894	17.638	9.820	8.484

Example. **The shells with various boundary values of $\eta(r_0)$ and $\eta(r_1)$**

Let us consider the shells with $r_0 = 14\,m$, $r_1 = 6\,m$, $q = -100\,T/m^2$, $n = 2$, $\nu = 0.33$, $Q = -r_0^* q$ is the axial load at boundary r_1 and given from equilibrium condition of the shell. The boundary values of $\eta(r_0)$ and $\eta(r_1)$ are following:

1. The meridian form Z_1 according to $\eta(r_0) = 1.1$; $\eta(r_1) = 1.3$
2. The meridian form Z_2 according to $\eta(r_0) = 1.1$; $\eta(r_1) = 1.5$
3. The meridian form Z_3 according to $\eta(r_0) = 1.3$; $\eta(r_1) = 1.5$
4. The meridian form Z_4 according to $\eta(r_0) = 1.3$; $\eta(r_1) = 1.7$

The radius of the cross circles is given in column R of the Table 1. The distances z (from shell bottom to circle R_i) are presented in each columns Z_1, Z_2, Z_3, Z_4. The meridian forms of these shell are shown on figure 1.

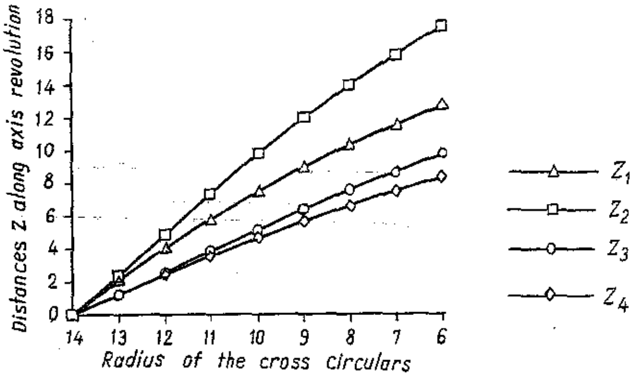

Fig. 1. The meridian forms of the shells with various values of eta

Conclusion

The equation for determining meridian form of the shell is given. The solution of equation is obtained for the shells subjected to hydrostatic pressure and axially symmetrical load, which is parallel to axis of revolution. Many examples are considered to illustrate the method of determination of the shell forms. Using this method, the other meridian curves may be obtained by executing given program with other values of n, A, B, q, r_1, r_0.

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DẠNG VỎ KHÔNG CHỊU ỨNG SUẤT UỐN DƯỚI TÁC DỤNG CỦA ÁP LỰC THỦY TĨNH VÀ CÁC DẠNG TẢI KHÁC

Trong bài báo đã đưa ra phương trình vi phân xác định dạng đường sinh của vỏ tròn xoay không chịu uốn dưới tác dụng của áp lực thủy tĩnh và tải đối xứng tác dụng song song với trục quay trên biên vỏ. Nghiệm phương trình đã được tìm ra dưới dạng nửa giải tích. Kết quả số cụ thể của các điểm trên đường sinh cho các trường hợp góc của đường sinh tại biên vỏ khác nhau và hình dạng đường sinh vỏ đã được hiển thị trên đồ thị nhờ chương trình được viết bằng ngôn ngữ FTN77. Các dạng khác của vỏ có thể được tìm bằng phương pháp và chương trình này với các điều kiện khác về áp lực và điều kiện hình học trên biên vỏ. Kết quả nghiên cứu có thể áp dụng trong việc thiết kế các vỏ tròn xoay không chịu uốn.