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GENERALIZED DIFFUSION THEORY OF HYDRODYNAMICAL PARTICLE MIGRATION IN SUSPENSIONS
Part 1: The case of equal densities

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Abstract. The general continuum theory has been developed for two- phase flows of fluid with deformable particles, where the micro- deformation of particles and the relative motion between phases have been taken into account [1-3].

This paper is concerned with using the simplest model from developed general theory for modeling of particle migration in suspensions- one of the most important and complicated aspects of particle- liquid two- phase flows, that has been observed and studied by many authors. For this purpose it is considered the motion of Newtonian fluid- rotating rigid spherical particles two- phase continuum with specialized nonlinear constitutive equations, when the particle and fluid have equal densities.

The obtained equation system has been used for studying quantitatively particle migration problem in the circular Couette flow.

Introduction

One of the most important and complicated aspects of particle- liquid two- phase flows is problem of particle migration. For example, many experiments show that for up- flow in a circular test section the bubbles tend to migrate toward the wall and thus the void fraction profile has a distinct peak near the wall. In contract, for down- flow the bubble tend to migrate toward the center of the pipe [4, 5]. The particle migration is observed also in gas- solid particle flow [6], and in concentrated suspensions [7, 8].

Several mechanisms have been proposed to explain theoretically the lateral migration phenomena. For examples, the particle migration in two- phase flows explained by the lift force due to shear stress was analytically derived by Saffman [9] and the lift force due to particle rotation derived by Rubinow and Keller [10]. These forces are extended and included in the equations of motion for every computational particle by applying the Euler- Lagrange approach for numerical simulation of gas- solid two- phase flow [11]. The constitutive equation for the particle flux, originally proposed by [12] in one dimension, is implemented in general two- dimensional flows with arbitrary geometry and boundary conditions, and is used for the numerical predictions of particle migration in transient circular and eccentric circular Couette flow [7, 8].

In [1-3], by analyzing the forces causing the migration of particles, it can be shown that to describe and predict particle migration phenomena, it is necessary to consider the nonlinear constitutive equations. For this purpose it is assumed that phenomenological coefficient of the non- linear part of the constitutive equations depend to first order on generalized diffusion flux. Moreover, these "generalized migration forces" are of a gyroscopic nature. This means that there is no contribution to the total dissipation of flow energy. Taking into account the macro-isotropy of a mixture, the no- dissipation property of nonlinear part of constitutive equations, and using the theory of isotropic tensors one constructed specialized nonlinear constitutive equations. This generalized developed theory was demonstrated in the case of rigid spherical particle fluid two-phase flow. The obtained equation system has been used to study quantitatively the particle migration in the infinite vertical circular cylinder in result of the constant pressure gradient along the cylinder axis and the gravity force.

In present paper this theory is demonstrated in the case of circular Couette flow.

2. Motion equation system of rotating particle- fluid two- phase flow

In the simplest case of suspension with rotating particle, when the mass densities of particle and fluid are equal and constant, the motion equations will have following form [1-3]:

$$\begin{aligned}
&\overline{\nabla}\cdot\overline{U}=0;\quad \overline{U}=\varphi\overline{U}_1+(1-\varphi)\overline{U}_2;\\
&\rho\frac{d\varphi}{dt}=-\overline{\nabla}\cdot\overline{J};\quad \overline{J}=\rho\varphi(\overline{U}_1-\overline{U});\\
&\frac{D\overline{J}}{Dt}=-\rho\varphi(1-\varphi)\Big[\overline{F}-\frac{1}{1-\varphi}(\overline{\nabla}\mu_1)_{p,T}\Big];\\
&\rho\frac{d\overline{U}}{dt}=\rho\overline{g}-\overline{\nabla}p-\overline{\nabla}\times\overline{\tau}_1+\overline{\nabla}\cdot\overline{\tau}_2;\\
&j\frac{d\overline{\omega}}{dt}+j(\overline{J}\cdot\overline{\nabla})\overline{\omega}=-\overline{\tau}_1+\overline{\nabla}\times\overline{\lambda}_1+\overline{\nabla}\cdot\overline{\lambda}_2;\quad j=\frac{4}{3}\pi R^3\rho\varphi;\\
&\frac{d}{dt}(\ldots)=\frac{\partial}{\partial t}(\ldots)+(\overline{U}\cdot\overline{\nabla})(\ldots);\\
&\frac{D}{Dt}(\ldots)=\frac{d}{dt}(\ldots)+[(\ldots)\cdot\overline{\nabla}]\overline{U}.
\end{aligned}\tag{2.1}$$

In (2.1)

$\overline{U}_1$ and $\overline{U}_2$ - mean velocity of particles and fluid,

$\overline{U}$ - mean velocity of suspension,

φ - volume concentration of particles,

$\overline{J}$ - diffusion flux of particles,

$\overline{\omega}$ - mean rotation velocity of particles,

j - particle moment of inertia,

μ_1 - generalized chemical potential of particle,

$\overline{F}$ - generalized diffusion force,

p - thermodynamical pressure,

$\overline{\tau}_1$ - vector equivalent to antisymmetric part of viscous stress tensor,

$\overline{\tau}_2$ - symmetrical part of viscous stress tensor,

$\overline{\lambda}_1$ - vector, equivalent to antisymmetric part of moment stress tensor,

$\overline{\lambda}_2$ - its symmetrical part.

The specialized nonlinear constitutive equations can be written in the form [1-3]

$$
\begin{aligned}
&\overline{F} = \alpha_1 \overline{J} + \frac{1}{2}\alpha_2 \overline{\nabla} \times \overline{\omega} - \beta_1 \Big(\overline{\omega} - \frac{1}{2}\overline{\nabla} \times \overline{U}\Big) \times \overline{J},\\
&\overline{\lambda}_1 = \alpha_2 \overline{J} + \frac{1}{2}\alpha_3 \overline{\nabla} \times \overline{\omega},\\
&\overline{\tau}_1 = \alpha_4 \Big(\overline{\omega} - \frac{1}{2}\overline{\nabla} \times \overline{U}\Big),\\
&\overline{\tau}_2 = \frac{1}{2}\alpha_5 [\overline{\nabla}\,\overline{U} + (\overline{\nabla}\,\overline{U})^T],\\
&\overline{\lambda}_2 = \alpha_6 (\overline{\nabla} \cdot \overline{\omega}) \overline{I} + \frac{1}{2}\alpha_7 [(\overline{\nabla}\,\overline{\omega}) + (\overline{\nabla}\,\overline{\omega})^T - \frac{2}{3}\overline{\nabla} \cdot \overline{\omega}\, \overline{I}],
\end{aligned}
\tag{2.2}
$$

where $\alpha_1, \ldots, \alpha_7$, β_1 - constitutive coefficients.

The equation system (2.1) together with constitutive equations (2.2) can be solved if there are determined the boundary conditions for the mean velocity $\overline{U}$, rotation velocity $\overline{\omega}$ and volume concentration φ. In this paper we suppose that on the flow rigid boundary we have following conditions:

$$
\overline{U} = 0; \quad \overline{\omega} = t\frac{1}{2}\overline{\nabla} \times \overline{U}, \quad \overline{n} \cdot \overline{J} = 0. \tag{2.3}
$$

In (2.3) the parameter t is characterizing the influence of the flow boundary on the particle rotation. When $t = 0$ particle can not be rotated at boundary and when $t = 1$ particle is rotated with velocity equal the external rotation of flow.

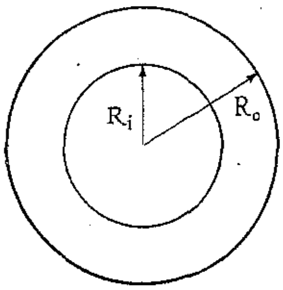

Fig. 1

3. Steady numerical solution

As a simplest demonstration of the theory, now we are considering the steady circular Couette flow [Fig. 1].

In this case the equation system (2.1) - (2.2) can be shown to have the form:

$$
\begin{aligned}
&\frac{1}{r}\frac{d}{dr}\Big[a_1 r \frac{dU_\theta}{dr}\Big] + \frac{d}{dr}\Big\{a_2\Big[\omega_x - \frac{1}{2r}\frac{d}{dr}(rU_\theta)\Big]\Big\} = 0,\\
&\frac{1}{r}\frac{d}{dr}\Big[a_3 r \frac{d\omega_x}{dr}\Big] + a_2\Big[\omega_x - \frac{1}{2r}\frac{d}{dr}(rU_\theta)\Big] = 0,\\
&-a_4\frac{d\varphi}{dr} - a_5\Big[\omega_x - \frac{1}{2r}\frac{d}{dr}(rU_\theta)\Big]\frac{d\omega_x}{dr} = 0,
\end{aligned}
\tag{3.1}
$$

where we put:

$$
a_1 = \frac{1}{2}\alpha_5; \quad a_2 = \alpha_4; \quad a_3 = \frac{1}{2}(\alpha_7 - \alpha_3); \quad a_4 = -\frac{1}{(1-\varphi)}\frac{\partial \mu_1}{\partial \varphi}; \quad a_5 = \frac{\alpha_2 \beta_1}{2\alpha_1}. \tag{3.2}
$$

The velocity $\overline{U}$, the rotation velocity $\overline{\omega}$ and the volume concentration φ will be found from the following conditions:

$$
\begin{aligned}
&U_\theta(R_i) = \Omega_0 R_i, \quad U_\theta(R_0) = 0,\\
&\omega_x(R_i) = t\Omega_x(R_i), \quad \omega_x(R_0) = t\Omega_x(R_0),\\
&\frac{2}{R_0^2 - R_i^2}\int_{R_i}^{R_0} \varphi r dr = \varphi_0,
\end{aligned}
\tag{3.3}
$$

where:

Ω_0 - rotation velocity of the inner cylinder,

φ_0 - average volume concentration,

$\Omega_x = \dfrac{1}{2r}\dfrac{d}{dr}(rU_\theta)$ - external rotation velocity.

The system of equations (3.1) - (3.3) is solved numerically with different values of parameters t, φ_0 and Ω_0; and with following parameters $a_1, \ldots, a_5$:

$$a_1 = 0.005;\ a_2 = 0.0005;\ a_3 = 0.001;\ a_4 = 0.00015;\ a_5 = 0.005.$$

Results of the calculation in the case, where $\hat{\Omega}_0 = 1.5$; $\varphi_0 = 0.5$ and t is taken equal to 0; 0.5 and 1, are presented in Fig. 2. It is can be seen that boundary condition for particle rotation velocity plays important role in the process of particle migration. Only by the experiments one can show that is the real value for parameter t.

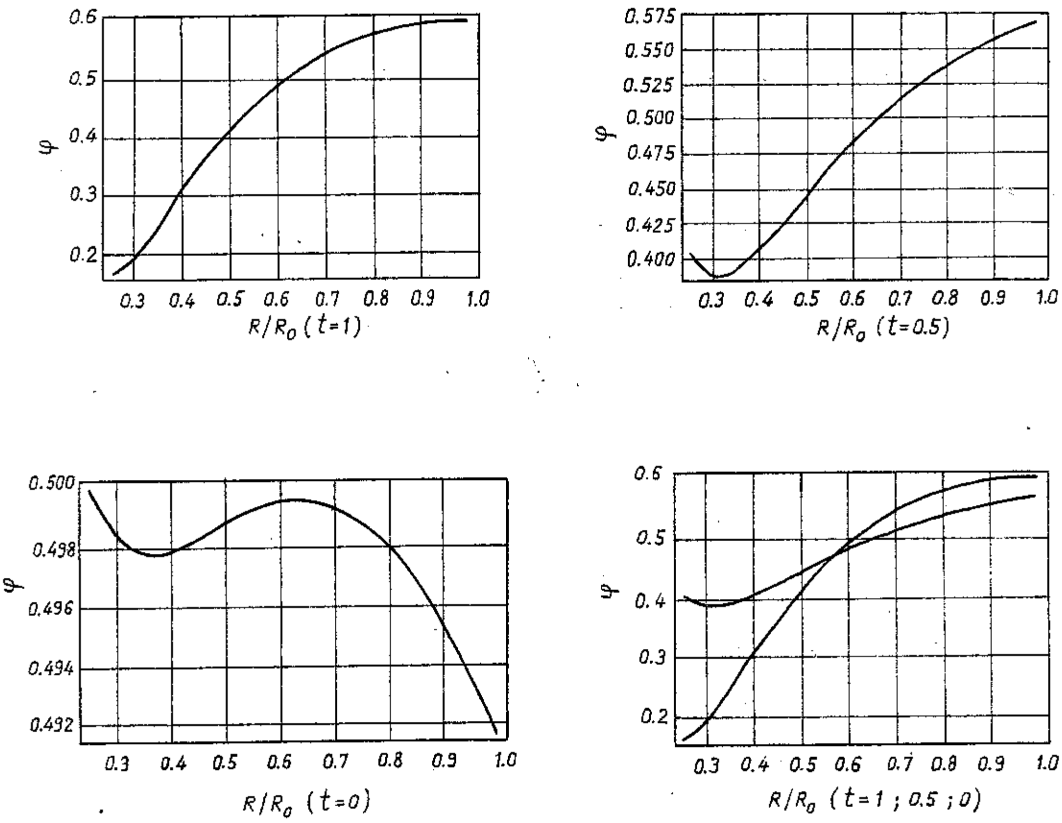

Fig. 2

In Fig. 3 the velocity, particle rotation velocity and external flow rotation profiles for the case where $\Omega_0 = 1.5$, $\varphi_0 = 0.5$ and t is taken equal 0; 0.5 and 1 are plotted.

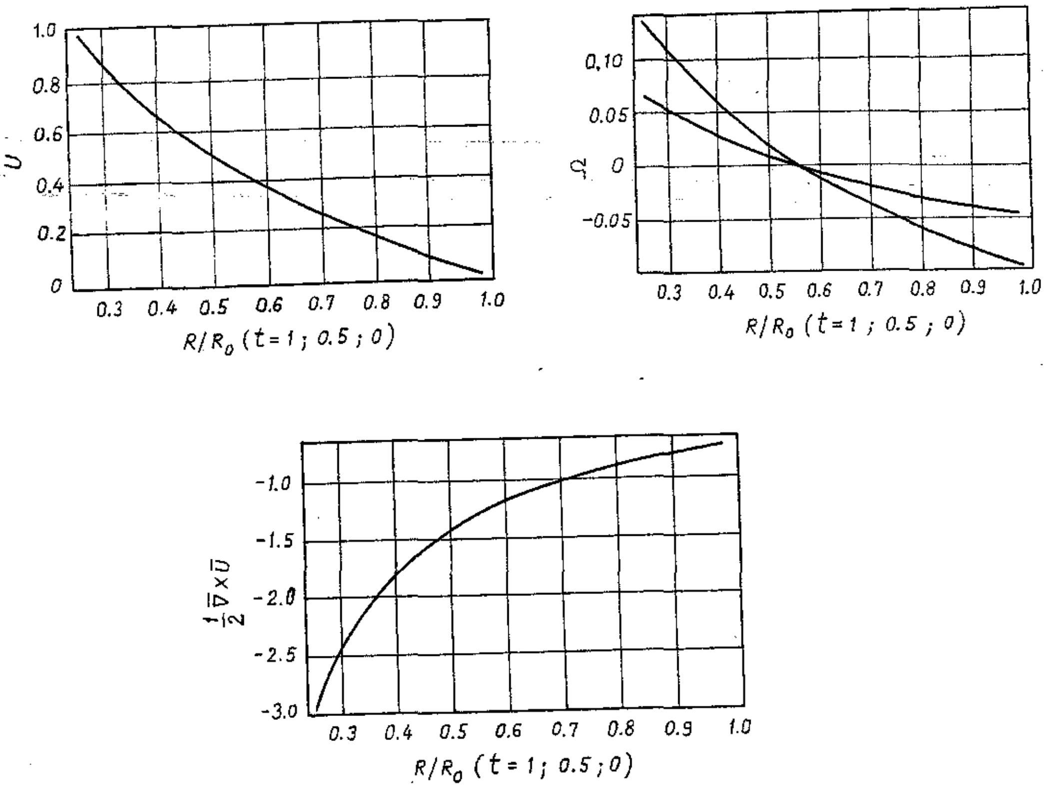

Fig. 3

4. Conclusion

It has been shown that the developed generalized diffusion theory of rotating particle - fluid two- phase flow can be used to predict the particle migration in suspensions. In the case of circular Couette flow, it can be seen that the rotation velocity of particle, and especially its boundary condition play most important role in the particle migration. Obviously the obtained results are exceptionally qualitative, because many phenomenological parameters have to be found.

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LÝ THUYẾT KHUYẾCH TÁN SUY RỘNG CỦA SỰ DỊCH CHUYỂN THỦY ĐỘNG LỰC CÁC HẠT TRONG HỖN HỢP LỎNG-RẮN

Lý thuyết tổng quát các dòng chảy 2 pha chất lỏng mang các hạt có thể biến dạng được có :hú ý tới biến dạng vi mô và chuyển động tương đối giữa các pha đã được xây dựng trong [1-3].

Bài báo này đề cập đến việc sử dụng mô hình đơn giản nhất của lý thuyết đã được phát triển ,rên để mô phỏng quá trình dịch chuyển của các hạt trong hỗn hợp lỏng-rắn, một trong những rấn đề quan trọng và phức tạp nhất của dòng chảy nhiều pha. Để đạt được mục đích đó, đã xét :huyển động của chất lỏng Newton mang các hạt cầu cứng có thể quay được với các phương trình :ác định phi tuyến đặc biệt. Hệ phương trình thu nhận được đã được sử dụng để nghiên cứu lịnh tính sự dịch chuyển các hạt trong dòng chảy Couette giữa 2 hình trụ.