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SOLUTION TO THE PROBLEM OF THE ELASTOPLASTIC STABILITY OF THIN RECTANGULAR PLATES IN TWO CASES OF BOUNDARY CONDITION

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1. Introduction

In this paper, the theory of the elastoplastic process is applied to derive the governing equations of stability problems of thin rectangular plates subjected to complex loading processes. The solution presented in the paper belongs to the two following cases of boundary condition

1) The considered plate has all four edges clamped stiffly.

2) The considered plate has two opposite edges clamped stiffly while the two others are simply supported.

The plates with four edges simply supported has been considered in [4].

2. Governing equations of the problem

Let's consider a rectangular plate with the thickness h and the lengths of the edges a, b. The coordinate system $Oxyz$ is chosen such that the middle surface of the plate coincides with the plane Oxy and the four edges can be described by $x = 0$, $x = a$, $y = 0$, $y = b$.

The external forces acting on the plate are biaxial compression forces of intensity p, q and shear force τ. The upper forces are assumed to be increasingly and depend arbitrarily on a some parameter t (*the loading parameter*)

$$p = p(t), \quad q = q(t), \quad \tau = \tau(t)$$

so that the loading is really performed in a arbitrary process. It is important to determine the critical values $t = t^*$, $p^* = p(t^*)$, $q^* = q(t^*)$, $\tau^* = \tau(t^*)$ at which an instability appears.

An analysis of the stability problem is always made in two stages: the-buckling stage and the post-buckling stage.

1. Pre-buckling stage

At any moment t there exists a plane stress state in the plate

$$\sigma_{11} = -p,\ \sigma_{22} = -q,\ \sigma_{12} = -\tau,\ \sigma_{13} = \sigma_{23} = \sigma_{33} = 0 \tag{2.1}$$

so that

$$\begin{aligned} \sigma &= \frac{\sigma_{11} + \sigma_{22}}{3} = -\frac{p+q}{3}, \\ \sigma_u &= \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 + 3\sigma_{12}^2} = \sqrt{p^2 - pq + q^2 + 3\tau^2} \end{aligned} \tag{2.2}$$

The components of deformation velocity are determined according to the theory of elastoplastic process [1]. In case of process with average curvature, they are of the forms

$$\begin{aligned} \dot{\varepsilon}_{11} &= \frac{1}{A}\left(-\dot{p} + \frac{1}{2}\dot{q}\right) - \left(\frac{1}{P} - \frac{1}{A}\right)\frac{p\dot{p} + q\dot{q} + 3\tau\dot{\tau} - \frac{1}{2}p\dot{q} - \frac{1}{2}q\dot{p}}{p^2 - pq + q^2 + 3\tau^2}\left(p - \frac{1}{2}q\right) \\ \dot{\varepsilon}_{22} &= \frac{1}{A}\left(-\dot{q} + \frac{1}{2}\dot{p}\right) - \left(\frac{1}{P} - \frac{1}{A}\right)\frac{p\dot{p} + q\dot{q} + 3\tau\dot{\tau} - \frac{1}{2}p\dot{q} - \frac{1}{2}q\dot{p}}{p^2 - pq + q^2 + 3\tau^2}\left(q - \frac{1}{2}p\right) \\ \dot{\varepsilon}_{12} &= -\frac{3\dot{\tau}}{A} - \frac{3}{2}\left(\frac{1}{P} - \frac{1}{A}\right)\frac{p\dot{p} + q\dot{q} + 3\tau\dot{\tau} - \frac{1}{2}p\dot{q} - \frac{1}{2}q\dot{p}}{p^2 - qp + q^2 + 3\tau^2}\tau \end{aligned} \tag{2.3}$$

where

$$A = \frac{\sigma_u}{s},\ P = \Phi'(s),\ s = s_0 + \frac{2}{\sqrt{3}}\int_0^t \left(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{22} + \dot{\varepsilon}_{12}^2\right)^{1/2} dt \tag{2.4}$$

$\Phi(s)$ - a known function concerned with the material used, s - the arc-length of the strain trajectory.

Suppose the loaded process of the plate starts at the natural state. At $t = 0$

$$p = q = \tau = 0, \quad \varepsilon_{11} = \varepsilon_{22} = \varepsilon_{12} = 0, \quad s = 0. \tag{2.5}$$

With equations (2.1) ÷ (2.5) we can find the corresponding deformation process of the plate in the pre-buckling stage.

In the inverse problem, the desired strain process is assumed to be given

$$\varepsilon_{11} = \varepsilon_{11}(t), \quad \varepsilon_{22} = \varepsilon_{22}(t), \quad \varepsilon_{12} = \varepsilon_{12}(t) \tag{2.6}$$

then the external forces acting on the plate must perform in the process that can be determined from the following equations

$$\begin{aligned}
\frac{dp}{dt} &= -\frac{4}{3}A\Big(\dot\varepsilon_{11} + \frac{\dot\varepsilon_{22}}{2}\Big) + (A-P)\frac{p\dot\varepsilon_{11} + q\dot\varepsilon_{22} + 2\tau\dot\varepsilon_{12}}{p^2 - pq + p^2 + 3\tau^2}p \\
\frac{dq}{dt} &= -\frac{4}{3}A\Big(\dot\varepsilon_{22} + \frac{\dot\varepsilon_{11}}{2}\Big) + (A-P)\frac{p\dot\varepsilon_{11} + q\dot\varepsilon_{22} + 2\tau\dot\varepsilon_{12}}{p^2 - pq + p^2 + 3\tau^2}q \\
\frac{d\tau}{dt} &= -\frac{2}{3}A\dot\varepsilon_{12} + (A-P)\frac{p\dot\varepsilon_{11} + q\dot\varepsilon_{22} + 2\tau\dot\varepsilon_{12}}{p^2 - pq + p^2 + 3\tau^2}\tau.
\end{aligned} \tag{2.7}$$

2. Post-buckling stage

Let t increase until it reaches the value t^* at which a bifurcation of equilibrium states appears. It means: with an infinitesimal small increment of the external forces there are possible increments of deformation (including the bending deformation) in the plate. According to the assumption of straight normal

$$\delta\varepsilon_{ij} = \delta\varepsilon^*_{ij} - z \cdot \delta\chi_{ij} \tag{2.8}$$

where

$$\delta\varepsilon^*_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big), \quad \delta\chi_{ij} = \frac{\partial^2 \delta w}{\partial x_i \partial x_j} \tag{2.9}$$

$\delta\varepsilon^*_{ij}$ - increments of deformation components of the middle surface,

δu_i, δw - increments of in-plane displacement and deflection of the middle surface.

$\delta\chi_{ij}$ - increments of curvature and torsion associated with instability.

The corresponding stress increment can be determined according to the theory of the elastoplastic process

$$\begin{aligned}
\delta\sigma_{ij} &= \frac{2}{3}A\big(\delta\varepsilon_{ij} + \delta_{ij}\cdot\delta\varepsilon_{kk}\big) + (P-A)\frac{\sigma_{k\ell}\delta\varepsilon_{k\ell}}{\sigma_u^2}\cdot\sigma_{ij} \\
\delta s &= \frac{2}{\sqrt{3}}\big(\delta\varepsilon_{11}^2 + \delta\varepsilon_{22}^2 + \delta\varepsilon_{11}\delta\varepsilon_{22} + \delta\varepsilon_{12}^2\big)^{1/2} \\
&\qquad (i,j,k,\ell = 1,2)
\end{aligned} \tag{2.10}$$

The increments of membrane forces and bending moments are determined with using (2.10)

$$\delta N_{ij} = \int\limits_{-h/2}^{h/2} \delta\sigma_{ij} dz = \frac{2}{3}A_1\left(\delta\varepsilon_{ij}^* + \delta_{ij}\delta\varepsilon_{kk}^*\right) - \frac{2}{3}A_2\left(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}\right) + $$
$$+ \overline{\sigma}_{ij}\left[(P_1 - A_1)\varepsilon - (P_2 - A_2)\chi\right], \qquad (2.11)$$

$$\delta M_{ij} = \int\limits_{-h/2}^{h/2} \delta\sigma_{ij} z dz = \frac{2}{3}A_2\left(\delta\varepsilon_{ij}^* + \delta_{ij}\delta\varepsilon_{kk}^*\right) - \frac{2}{3}A_3\left(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}\right) + $$
$$+ \overline{\sigma}_{ij}\left[(P_2 - A_2)\varepsilon - (P_3 - A_3)\chi\right] \qquad (2.12)$$

where

$$A_m = \int\limits_{-h/2}^{h/2} A \cdot z^{m-1} dz, \quad P_m = \int\limits_{-h/2}^{h/2} P \cdot z^{m-1} dz, \quad (m = 1, 2, 3) \qquad (2.13)$$

$$\overline{\sigma}_{ij} = \frac{\sigma_{ij}}{\sigma_u}, \quad \varepsilon = \frac{\sigma_{ij}}{\sigma_u}\delta\varepsilon_{ij}^* = \overline{\sigma}_{ij}\delta\varepsilon_{ij}^*, \quad \chi = \frac{\sigma_{ij}}{\sigma_u}\delta\chi_{ij} = \overline{\sigma}_{ij}\delta\chi_{ij} \qquad (2.14)$$

Eliminating the value $\frac{2}{3}\left(\delta\varepsilon_{ij} + \delta_{ij}\delta\varepsilon_{kk}\right)$ from the expressions of δN_{ij}, δM_{ij} gives us

$$\delta M_{ij} = \frac{A_2}{A_1}\delta N_{ij} + \frac{2}{3}\left(\frac{A_2^2}{A_1} - A_3\right)(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}) + \qquad (2.15)$$
$$+ \overline{\sigma}_{ij}\left[\left(A_3 - \frac{A_2^2}{A_1} + \frac{P_2^2}{P_1} - P_3\right)\chi + \frac{3}{2}\left(\frac{P_2}{P_1} - \frac{A_2}{A_1}\right)\overline{S}_{ij}\delta N_{ij}\right].$$

The increments of displacements, internal forces and moments satisfy the following equations

$$\frac{\partial \delta N_{ij}}{\partial x_j} = 0$$
$$\frac{\partial^2 \delta M_{ij}}{\partial x_i \partial x_j} + N_{ij}\delta\chi_{ij} = 0 \qquad (2.16)$$
$$\frac{\partial^2 \delta\varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \delta\varepsilon_{22}}{\partial x_1^2} = 2\frac{\partial^2 \delta\varepsilon_{12}}{\partial x_1 \partial x_2}$$

and the concrete boundary condition.

In general, when an instability occurs, there may exist two regions of active and passive deformations in the structure. The expression for the increment of deformation work is

$$\begin{aligned}\delta W &= \sigma_{ij}\delta\varepsilon_{ij} = \sigma_{ij}(\delta\varepsilon_{ij} - z\delta\chi_{ij}) \\ &= \sigma_u\left(\frac{\sigma_{ij}}{\sigma_u}\delta\varepsilon^*_{ij} - z\frac{\sigma_{ij}}{\sigma_u}\delta\chi_{ij}\right) = \sigma_u(\varepsilon - z\chi).\end{aligned} \tag{2.17}$$

In the region of active deformation $\delta W > 0$, so that the stress-strain relationship is taken by (2.10). In the region of passive deformation $\delta W < 0$, so that the Hookian relationship is applied. Dividing boundary z_0 of the two regions is determined by

$$\varepsilon - z_0\chi = 0 \tag{2.18}$$

Using equations (2.11) we can get the following equation to find out z_0

$$P_1 z_0 - P_2 = \frac{3}{2}\frac{\overline{S}_{ij}\delta N_{ij}}{\chi} \tag{2.19}$$

Accepting Ilyushin's approximate statement which says $\delta N_{ij} = 0$ we obtain

$$\delta\varepsilon^*_{ij} = 0 \tag{2.20}$$

$$\delta s = \frac{2}{\sqrt{3}}z\left[\left(\frac{\partial^2\delta w}{\partial x_1^2}\right)^2 + \left(\frac{\partial^2\delta w}{\partial x_2^2}\right)^2 + \left(\frac{\partial^2\delta w}{\partial x_1^2}\right)\left(\frac{\partial^2\delta w}{\partial x_2^2}\right) + \left(\frac{\partial^2\delta w}{\partial x_1\partial x_2}\right)^2\right]^{1/2} \tag{2.21}$$

Now we can calculate

$$\begin{aligned}A_m &= \int_{-h/2}^{h/2} A\cdot z^{m-1}dz = \int_{-h/2}^{z_0} 3G\cdot z^{m-1}dz + \int_{z_0}^{h/2} A\cdot z^{m-1}dx = \\ &= \frac{1}{m}\left\{3G\left[z_0^m - \left(-\frac{h}{2}\right)^m\right] + A\left[\left(\frac{h}{2}\right)^m - z_0^m\right]\right\} \\ P_m &= \int_{-h/2}^{h/2} P.\cdot z^{m-1}dz = \int_{-h/2}^{z_0} 3Gz^{m-1}dz + \int_{z_0}^{h/2} Pz^{m-1}dz = \\ &= \frac{1}{m}\left\{3G\left[z_0^m - \left(-\frac{h}{2}\right)^m\right] + P\left[\left(\frac{h}{2}\right)^m - z_0^m\right]\right\}\end{aligned}$$

so

$$A_3 - \frac{A_2^2}{A_1} = \frac{Gh^3}{4}\Psi_A, \quad P_3 = P_2\frac{h\bar{z}_0}{2} = \frac{Gh^3}{4}\Psi_P$$

where

$$\Psi_A = \frac{1}{2}\Big[1 + \varphi_A + (1-\varphi_A)\overline{z}_0^3 - \frac{3}{4}\,\frac{(1-\varphi_A)^2\big(1-\overline{z}_0^2\big)^2}{1+\varphi_A+(1-\varphi_A)\overline{z}_0}\Big]$$

$$\Psi_P = \frac{1}{2}\Big[1 + \varphi_P + \frac{3}{2}(1-\varphi_P)\overline{z}_0 - \frac{1}{2}(1-\varphi_P)\overline{z}_0^3\Big]$$

$$\varphi_A = \frac{A}{3G}\,, \quad \varphi_P = \frac{P}{3G}\,.$$

Equation (2.19) determining the bound of the active and passive deformation regions reduces to

$$\overline{z}_0^2(3G-P) + 2\overline{z}_0(3G+P) + (3G-P) = 0, \quad \overline{z}_0 = \frac{2z_0}{h}$$

and solution to which is $\overline{z}_0 = -\dfrac{\sqrt{3G}-P}{\sqrt{3G}+P}$.

The expressions of moments now have the forms

$$\delta M_{ij} = \frac{Gh^3}{4}\Big[-\frac{2}{3}\Psi_A\big(\delta\chi_{ij} + \delta_{ij}\delta\chi_{kk}\big) + (\Psi_A - \Psi_P)\frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_u^2}\delta\chi_{k\ell}\Big] \tag{2.22}$$

The stability equation (2.16) in this case becomes

$$\alpha_1\frac{\partial^4\delta w}{\partial x^2} + \alpha_2\frac{\partial^4\delta w}{\partial x^3\partial y} + \alpha_3\frac{\partial^4\delta w}{\partial x^2\partial y^2} + \alpha_4\frac{\partial^4\delta w}{\partial x\partial y^3} + \alpha_1\frac{\partial^4\delta w}{\partial y^4} + \\ + \frac{3}{G\Psi_A h^2}\Big(p\frac{\partial^2\delta w}{\partial x^2} + 2\tau\frac{\partial^2\delta w}{\partial x\partial y} + q\frac{\partial^2\delta w}{\partial y^2}\Big) = 0 \tag{2.23}$$

where

$$\alpha_1 = 1 - \frac{3}{4}\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{p^2}{\sigma_u^2}\,, \quad \alpha_2 = -3\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{p\tau}{\sigma_u^2}$$

$$\alpha_3 = 2 - 3\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{\tau^2}{\sigma_u^2} - \frac{3}{2}\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{pq}{\sigma_u^2}$$

$$\alpha_4 = -3\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{q\tau}{\sigma_u^2}\,, \quad \alpha_5 = 1 - \frac{3}{4}\Big(1-\frac{\Psi_P}{\Psi_A}\Big)\frac{q^2}{\sigma_u^2}$$

Now we return to the two cases of boundary condition.

If four edges of the plate are clamped stiffly then the boundary conditions are

$$\begin{aligned} \delta w = 0, \quad \frac{\partial\delta w}{\partial x} = 0 \quad \text{at } x = 0,\ x = a \\ \delta w = 0. \quad \frac{\partial\delta w}{\partial y} = 0 \quad \text{at } y = 0,\ y = b \end{aligned} \tag{2.24}$$

The expression of δw chosen for this case is the following series

$$\delta w = \sum_{m=1}^{N}\sum_{n=1}^{N} D_{mn}\Big(1-\cos\frac{2m\pi x}{a}\Big)\Big(1-\cos\frac{2n\pi y}{b}\Big) \tag{2.25}$$

which satisfies the boundary conditions (2.24).

If two opposite edges of the plate (e.g., edges belong to $y = 0$ and $y = b$) are clamped stiffly while the two others are simply supported then the boundary conditions are

$$\begin{aligned} \delta w &= 0, \quad \frac{\partial^2 \delta w}{\partial x^2} = 0 \quad \text{at } x = 0,\ x = a,\\ \delta w &= 0, \quad \frac{\partial \delta w}{\partial y} = 0 \quad \text{at } \ y = 0,\ y = b. \end{aligned} \tag{2.26}$$

The corresponding expression of δw can be chosen as

$$\delta w = \sum_{m=1}^{N}\sum_{n=1}^{N} D_{mn}\sin\frac{m\pi x}{a}\Big(1-\cos\frac{2n\pi y}{b}\Big) \tag{2.27}$$

which satisfies the boundary conditions (2.26).

Series (2.25) and series (2.27) contains $N_0 = N^2$ terms, each of them corresponds with only one couple (m, n). By numbering the terms of series (2.25) and series (2.27) from 1 to N_0, we can rewrite these series as follows

$$\delta w = \sum_{k=1}^{N_0} B_k \delta w_k \tag{2.28}$$

where $B_1 = D_{11},\ B_2 = D_{21}, \ldots, B_N = D_{N1}$,

$B_{N+1} = D_{12},\ B_{N+2} = D_{22}, \ldots, B_{N_0} = D_{NN}$.

The index k in δw_k is determined in the same way

$$\delta w_1 = \Big(1-\cos\frac{2\pi x}{a}\Big)\cdot\Big(1-\cos\frac{2\pi y}{b} \qquad \text{or } \delta w_1 = \sin\frac{1\pi x}{a}\cdot\Big(1-\cos\frac{2\pi y}{b}\Big)$$

$$\delta w_2 = \Big(1-\cos\frac{4\pi x}{a}\Big)\cdot\Big(1-\cos\frac{2\pi y}{b}\Big) \qquad \text{or } \delta w_2 = \sin\frac{2\pi x}{a}\cdot\Big(1-\cos\frac{2\pi y}{b}\Big)$$

....

$$\delta w_N = \Big(1-\cos\frac{2N\pi x}{a}\Big)\cdot\Big(1-\cos\frac{2\pi y}{b}\Big) \quad \text{or } \delta w_N = \sin\frac{N\pi x}{a}\cdot\Big(1-\cos\frac{2\pi y}{b}\Big)$$

$$\delta w_{N+1} = \Big(1-\cos\frac{2\pi x}{a}\Big)\cdot\Big(1-\cos\frac{4\pi y}{b}\Big) \quad \text{or } \delta w_{N+1} = \sin\frac{1\pi x}{a}\cdot\Big(1-\cos\frac{4\pi y}{b}\Big)$$

....

$$\delta w_{N_0} = \Big(1-\cos\frac{2N\pi x}{a}\Big)\Big(1-\cos\frac{2N\pi y}{b}\Big) \quad \text{or } \delta w_{N_0} = \sin\frac{N\pi x}{a}\Big(1-\cos\frac{2N\pi y}{b}\Big).$$

To determine the critical forces, in here the Bubnov-Galerkin method is used. According to this method, we put series (2.28) into the stability equation (2.23) and use the notation $\Omega(\delta w)$ for the left side of the received equation. Afterward, multiplying $\Omega(\delta w)$ by δw_i $(i = 1, 2, \ldots, N_0)$ and integrating all over the volume of the plate gives us

$$\int_0^a \int_0^b \Omega(\delta w) \delta w_i dx dy = 0 \quad (i = 1, 2, \ldots, N_0) \tag{2.29}$$

From the structure of δw, if let i be sequently assigned with values $1, 2, \ldots, N_0$ then from equation (2.29) we can get a system of N_0 linear-algebraic equations of the unknowns B_k $(k = 1, 2, \ldots, N_0)$. This system has the form

$$\left(C_{ik}\right) \cdot \left\{B_k\right\} = 0 \tag{2.30}$$

where

$\left\{B_k\right\}^T = (B_1, B_2, \ldots, B_{N_0})$

$\left(C_{ik}\right)$ - matrix of N_0 columns and N_0 rows, $C_{ik} = \int_0^a \int_0^b \delta w_i \cdot \Omega(\delta w_k) dx dy$

$\Omega(\delta w_k)$ - the left side of the equation which is received from putting δw_k in equation (2.23) instead of putting δw.

Notice that the coefficients in stability equation (2.23) depend on p, q, τ so C_{ik} in system (2.30) depend on p, q, τ as well.

Because $B_k \neq 0$ so according to the condition of existence of non-trivial solution we can get the relation

$$\det\left(C_{ik}\right) = 0 \tag{2.31}$$

In the process of solving differential equations (2.3) (or differential equations (2.7) according to concrete expressions of $p(t)$, $q(t)$, $\tau(t)$ (or of $\varepsilon_{11}(t)$, $\varepsilon_{22}(t)$, $\varepsilon_{12}(t)$), relation (2.31) is used to determine the critical values t^*, p^*, q^*, τ^*.

3. The way of solving and concrete results

In case of the direct problem we have known the expressions $p(t)$, $q(t)$, $\tau(t)$, $\Phi(s)$. In the inverse problem the known expressions are $\varepsilon_{11}(t)$, $\varepsilon_{22}(t)$, $\varepsilon_{12}(t)$, $\Phi(s)$.

Let t increase from $t = 0$ with a some step Δt. Solving differential equations (2.3) or (2.7) by Runge-Kutta we will receive the corresponding process of deformation or loading, respectively. The calculation process is continued until condition (2.31) is satisfied. The values of t, p, q, τ received at the end of the calculation process are respectively accepted as the critical values t^*, p^*, q^*, τ^*.

In the following part, the author would give out some concrete results received from solving the direct problem and the inverse problem on stability of rectangular plates in the two upper cases of boundary condition. The material of the considered plates is the steel marked 30ХГСА which has $G = 0.8667 \cdot 10^6$ kG/cm^2 and function $\Phi(s)$ presented in [1, 4]. The programming to solve the problem is implemented in Turbo Pascal language.

1. *The first set of results* (Direct problem - First case of boundary condition)

$$a/h = 32, \quad b/h = 35$$

$$p(t) = 2400t, \quad q(t) = 1200t^2 + 2000t, \quad \tau = 800(e^t - 1)$$

$$t^* = 0.974; \; p^* = 2337.6, \; q^* = 3086.4, \; \tau^* = 1310.8, \; \sigma_u = 3604.6 \text{ (kG/cm}^2)$$

2. *The second set of results* (Direct problem - First case of boundary condition)

$$a/h = 38, \quad b/h = 32$$

$$p(t) = 1700t, \quad q(t) = \begin{cases} 1500 & \text{if } t < 0.25 \\ 1500(t + 0.75) & \text{if } t \geq 0.25 \end{cases}, \quad \tau = 1800\sqrt{t}$$

$$t^* = 1.552; \; p^* = 2638, \; q^* = 3453, \; \tau^* = 2242, \; \sigma_u^* = 4986 \text{ (kG/cm}^2)$$

3. *The third set of results* (Direct problem - Second case of boundary condition)

$$a/h = 38, \quad b/h = 35$$

$$p(t) = 1700t, \quad q(t) = \begin{cases} 1500 & \text{if } t < 0.25 \\ 1500(t + 0.75) & \text{if } t \geq 0.25 \end{cases}, \quad \tau = 1800\sqrt{t}$$

$$t^* = 1.334; \; p^* = 2267, \; q^* = 3126, \; \tau^* = 2079, \; \sigma_u^* = 4560 \text{ (kG/cm}^2)$$

4. *The fourth set of results* (Direct problem - Second case of boundary condition)

$$a/h = 35, \quad b/h = 40$$

$$p(t) = 1200t^2 + 200t, \quad q(t) = \begin{cases} 0 & \text{if } t < 0.25 \\ 2000\sqrt{t - 0.25} & \text{if } t \geq 0.25 \end{cases}, \quad \tau = 800(e^t - 1)$$

$$t^* = 0.764; \; p^* = 2228.4, \; q^* = 1434, \; \tau^* = 917, \; \sigma_u^* = 2520 \text{ (kG/cm}^2)$$

5. *The fifth set of results* (Inverse problem - First case of boundary condition)

$$a/h = 32, \quad b/h = 35$$

$$\varepsilon_{11} = -0.0215\sqrt{t}, \quad \varepsilon_{22} = -0.0255(e^t - 1), \quad \varepsilon_{12} = -0.065t$$

$t^* = 0.0628;\ p^* = 4746.1,\ q^* = 3312.3\ \tau^* = 1547,\ \sigma_u = 4995$ (kG/cm^2)

6. *The sixth set of results* (Inverse problem - First case of boundary condition)

$a/h = 35,\ b/h = 27$

$$\varepsilon_{11} = \begin{cases} -0.015 & \text{if } t < 0.25 \\ -0.0115(t+0.75) & \text{if } t > 0.25 \end{cases},\ \varepsilon_{22} = -0.0255t^2,\ \varepsilon_{12} = -0.065t$$

$t^* = 0.0622;\ p^* = 2411.3,\ q^* = 4822.6,\ \tau^* = 1519.3,\ \sigma_u^* = 4935.5$ (kG/cm^2)

7. *The seventh set of results* (Inverse problem - Second case of boundary condition)

$a/h = 35,\ b/h = 27$

$$\varepsilon_{11} = \begin{cases} -0.015 & \text{if } t < 0.25 \\ -0.0115(t+0.75) & \text{if } t \geq 0.25 \end{cases},\ \varepsilon_{22} = -0.0255t^2,\ \varepsilon_{12} = -0.065t$$

$t^* = 0.0164;\ p^* = 2387,\ q^* = 4774,\ \tau^* = 781,\ \sigma_u^* = 4348$ (kG/cm^2)

8. *The eighth set of results* (Inverse problem - Second case of boundary condition)

$a/h = 25,\ b/h = 40$

$\varepsilon_{11} = -0.0135t,\ \varepsilon_{22} = -0.0345(e' - 1),\ \varepsilon_{12} = -0.0165\sqrt{t}$

$t^* = 0.081;\ p^* = 2421,\ q^* = 3283.6,\ \tau^* 2258.6,\ \sigma_u^* = 4898.1$ (kG/cm^2)

4. Conclusion

The problem on the elastoplastic stability of thin rectangular plates under complex loading has been solved in the following aspects

- Deriving the governing equations,

- Making the algorithm and programs to solve,

- Giving out some concrete results belonging to both the direct problem and inverse problem in two cases of boundary condition.

If assigning different expressions to $p(t)$, $q(t)$, $\tau(t)$ or $\varepsilon_{11}(t)$, $\varepsilon_{22}(t)$, $\varepsilon_{12}(t)$ and running the corresponding programs then we can investigate the influence of complexity of loading process or deformation process on stability of the plate. Moreover, if changing the value of the ratios a/h and b/h then we can also consider the influence of the geometrical relations on stability of the structure.

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GIẢI BÀI TOÁN ỔN ĐỊNH ĐÀN DẺO CỦA TẤM MỎNG HÌNH CHỮ NHẬT TRONG HAI TRƯỜNG HỢP KHÁC NHAU CỦA ĐIỀU KIỆN BIÊN

Trong công trình của mình, tác giả áp dụng lý thuyết quá trình đàn dẻo để thiết lập các phương trình giải của bài toán ổn định đàn dẻo của tấm mỏng hình chữ nhật chịu tải trọng phức tạp. Bài toán được giải trong nhiều trường hợp khác nhau của điều kiện biên, mỗi trường hợp đó lại được giải theo hai chiều thuận và ngược. Giá trị tới hạn của các tải trọng ngoài được xác định theo phương pháp Bubnov-Galerkin.