

ON THE INFLUENCE OF SECOND ORDER NON-LINEARITY ON RANDOM VIBRATIONS

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1. Introduction

For many years the stochastic averaging method (SAM) has been a very useful tool for investigating non-linear random vibration systems, see e.g. [1-5]. However, the effect of some non-linear terms cannot be investigated by using the classical first order SAM. The procedure for obtaining higher approximate solutions for the Fokker-Planck (FP) equation was developed in [6, 7] and then applied to Van der Pol oscillator under white noise excitation [8]. In the paper this procedure is further developed to non-linear systems of Duffing and Van der Pol types taking into account some second order non-linear terms. It is shown that the effect of this non-linearity can be detected by the procedure proposed while it cannot be investigated by using the classical first order stochastic averaging method (SAM).

2. SAM of coefficients in FP equation

Consider a single - degree - of freedom system whose motion equation takes the form

$$\ddot{x} + \omega^2 x = \varepsilon f_1(x, \dot{x}) + \varepsilon^2 f_2(x, \dot{x}) + \sqrt{\varepsilon} \sigma \dot{\xi}(t) \quad (2.1)$$

where ω , σ are positive constants, ε is a positive small parameter, f_1 and f_2 are functions in x and \dot{x} . The random excitation $\dot{\xi}(t)$ is a Gaussian white noise process with unit intensity. Using the following change of variables

$$\begin{cases} x = a \cos \varphi \\ \dot{x} = -\omega a \sin \varphi, \quad \varphi = \omega t + \theta(t) \end{cases} \quad (2.2)$$

one gets the Ito differential equation system [4]

$$\begin{cases} da = (\varepsilon k_1 + \varepsilon^2 r_1) dt - \frac{\sqrt{\varepsilon}}{\omega a} \sigma \sin \varphi d\xi(t) \\ d\varphi = (\omega + \varepsilon k_2 + \varepsilon^2 r_2) dt - \frac{\sqrt{\varepsilon}}{\omega a} \sigma \cos \varphi d\xi(t) \end{cases} \quad (2.3)$$

where

$$\begin{cases} k_1 = -\frac{\sigma^2}{2\omega^2 a} \cos^2 \varphi - \frac{f_1(a, \varphi)}{\omega} \sin \varphi, & r_1 = -\frac{f_2(a, \varphi)}{\omega} \sin \varphi \\ k_2 = -\frac{\sigma^2}{2\omega^2 a^2} \sin 2\varphi - \frac{f_1(a, \varphi)}{\omega a} \cos \varphi, & r_2 = -\frac{f_2(a, \varphi)}{\omega a} \cos \varphi \end{cases} \quad (2.4)$$

$$f_i(a, \varphi) = f_i(x, \dot{x}) \Big|_{x=a \cos \varphi, \dot{x}=-\omega a \sin \varphi} \quad (i = 1, 2).$$

The FP equation for the stationary probability density function $W(a, \varphi)$ corresponding to the system (2.3) takes the form [6, 7];

$$\omega \frac{\partial \phi}{\partial \varphi} = -\varepsilon [k_i, k_{ij}] \ell[\phi] - \varepsilon^2 \left(\frac{\partial r_1}{\partial a} + \frac{\partial r_2}{\partial \varphi} + r_1 \frac{\partial \phi}{\partial a} + r_2 \frac{\partial \phi}{\partial \varphi} \right) \quad (2.5)$$

where

$$\phi(a, \varphi) = \ln W(a, \varphi) \quad (2.6)$$

$$\begin{aligned} [k_i, k_{ij}] \ell[\phi] = & \left(\frac{\partial k_1}{\partial a} + \frac{\partial k_2}{\partial \varphi} - \frac{1}{2} \frac{\partial^2 k_{11}}{\partial a^2} - \frac{\partial^2 k_{12}}{\partial a \partial \varphi} - \frac{1}{2} \frac{\partial^2 k_{22}}{\partial \varphi^2} \right) \\ & + \left(k_1 - \frac{\partial k_{11}}{\partial a} - \frac{\partial k_{12}}{\partial \varphi} \right) \frac{\partial \phi}{\partial a} + \left(k_2 - \frac{\partial k_{12}}{\partial a} - \frac{\partial k_{22}}{\partial \varphi} \right) \frac{\partial \phi}{\partial \varphi} \\ & - \frac{1}{2} \left\{ k_{11} \left[\frac{\partial^2 \phi}{\partial a^2} + \left(\frac{\partial \phi}{\partial a} \right)^2 \right] + 2k_{12} \left(\frac{\partial^2 \phi}{\partial a \partial \varphi} + \frac{\partial \phi}{\partial a} \cdot \frac{\partial \phi}{\partial \varphi} \right) \right. \\ & \left. + k_{22} \left[\frac{\partial^2 \phi}{\partial \varphi^2} + \left(\frac{\partial \phi}{\partial \varphi} \right)^2 \right] \right\} \end{aligned} \quad (2.7)$$

$$k_{11} = \frac{\sigma^2}{\omega^2} \sin^2 \varphi, \quad k_{12} = \frac{\sigma^2}{2\omega^2 a} \sin 2\varphi, \quad k_{22} = \frac{\sigma^2}{\omega^2 a^2} \cos^2 \varphi. \quad (2.8)$$

According to [6, 7] a solution of Eq. (2.5) can be found in the form:

$$\phi(a, \varphi) = \phi_0(a, \varphi) + \varepsilon \phi_1(a, \varphi) + \varepsilon^2 \phi_2(a, \varphi) + \dots \quad (2.9)$$

Substituting (2.9) into (2.5) and comparing the coefficients of like powers of ε one obtains a system of separable equations for the unknown functions $\phi_n(a, \varphi)$ ($n = 0, 1, 2, \dots$) as follows:

$$\varepsilon^0: \omega \frac{\partial \phi_0}{\partial \varphi} = 0 \quad (2.10)$$

$$\varepsilon^1: \omega \frac{\partial \phi_1}{\partial \varphi} = -[k_i, k_{ij}] \ell[\phi_0] \quad (2.11)$$

$$\varepsilon^2: \omega \frac{\partial \phi_2}{\partial \varphi} = [k_i, k_{ij}] \ell[\phi_0, \phi_1] \quad (2.12)$$

$$\varepsilon^3: \omega \frac{\partial \phi_3}{\partial \varphi} = [k_i, k_{ij}] \ell[\phi_0, \phi_1, \phi_2] \quad (2.13)$$

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where the operators $[k_i, k_{ij}] \ell[\phi_0, \phi_1]$, $[k_i, k_{ij}] \ell[\phi_0, \phi_1, \phi_2]$ are defined as

$$\begin{aligned}
[k_i, k_{ij}] \ell[\phi_0, \phi_1] = & - \left(k_1 - \frac{\partial k_{11}}{\partial a} - \frac{\partial k_{12}}{\partial \varphi} \right) \frac{\partial \phi_1}{\partial a} - k_2 \left(k_2 - \frac{\partial k_{12}}{\partial a} - \frac{\partial k_{22}}{\partial \varphi} \right) \frac{\partial \phi_1}{\partial \varphi} \\
& + \frac{1}{2} k_{11} \left(\frac{\partial^2 \phi_1}{\partial a^2} + 2 \frac{\partial \phi_0}{\partial a} \frac{\partial \phi_1}{\partial a} \right) + k_{12} \left(\frac{\partial^2 \phi_1}{\partial a \partial \varphi} + \frac{\partial \phi_0}{\partial a} \frac{\partial \phi_1}{\partial \varphi} + \frac{\partial \phi_0}{\partial \varphi} \frac{\partial \phi_1}{\partial a} \right) \\
& + k_{22} \left(\frac{\partial^2 \phi_1}{\partial \varphi^2} + 2 \frac{\partial \phi_0}{\partial \varphi} \frac{\partial \phi_1}{\partial \varphi} \right) - \left(\frac{\partial r_1}{\partial a} + \frac{\partial r_2}{\partial \varphi} + r_1 \frac{\partial \phi_0}{\partial a} + r_2 \frac{\partial \phi_0}{\partial \varphi} \right), \quad (2.14)
\end{aligned}$$

$$\begin{aligned}
[k_i, k_{ij}] \ell[\phi_0, \phi_1, \phi_2] = & - \left(k_1 - \frac{\partial k_{11}}{\partial a} - \frac{\partial k_{12}}{\partial \varphi} \right) \frac{\partial \phi_2}{\partial a} - \left(k_2 - \frac{\partial k_{12}}{\partial a} - \frac{\partial k_{22}}{\partial \varphi} \right) \frac{\partial \phi_2}{\partial \varphi} \\
& + \frac{1}{2} K_{11} \left[\frac{\partial^2 \phi_2}{\partial a^2} + 2 \frac{\partial \phi_0}{\partial a} \frac{\partial \phi_2}{\partial a} + \left(\frac{\partial \phi_1}{\partial a} \right)^2 \right] \\
& + k_{12} \left(\frac{\partial^2 \phi_2}{\partial a \partial \varphi} + \frac{\partial \phi_0}{\partial \varphi} \frac{\partial \phi_2}{\partial \varphi} + \frac{\partial \phi_1}{\partial a} \frac{\partial \phi_1}{\partial \varphi} + \frac{\partial \phi_2}{\partial a} \frac{\partial \phi_0}{\partial \varphi} \right) \quad (2.15) \\
& + \frac{1}{2} k_{22} \left[\frac{\partial^2 \phi_2}{\partial \varphi^2} + 2 \frac{\partial \phi_0}{\partial \varphi} \frac{\partial \phi_2}{\partial \varphi} + \left(\frac{\partial \phi_1}{\partial \varphi} \right)^2 \right] - \left(r_1 \frac{\partial \phi_1}{\partial a} + r_2 \frac{\partial \phi_1}{\partial \varphi} \right).
\end{aligned}$$

From (2.10), (2.11) and (2.12) one gets:

$$\phi_0 = \phi_0(a) \quad (2.16)$$

$$\phi_1(a, \varphi) = -\frac{1}{\omega} \int [k_i, k_{ij}] \ell[\phi_0] d\varphi + \phi_{10}(a) = \phi_{11}(a, \varphi) + \phi_{10}(a) \quad (2.17)$$

$$\phi_2(a, \varphi) = \frac{1}{\omega} \int [k_i, k_{ij}] \ell[\phi_0, \phi_1] d\varphi + \phi_{20}(a) = \phi_{22}(a, \varphi) + \phi_{20}(a). \quad (2.18)$$

The arbitrary integration function $\phi_{n0}(a)$ must be chosen from the condition for the function $\phi_{n+1}(a, \varphi)$ to be periodic in φ . In particular, for $n = 0$, one gets:

$$\langle [k_i, k_{ij}] \ell[\phi_0] \rangle = 0 \quad (2.19)$$

where $\langle \cdot \rangle$ denotes the averaging operator with respect to phase φ :

$$\langle F(\varphi) \rangle = \frac{1}{2\pi} \int_0^{2\pi} F(\varphi) d\varphi. \quad (2.20)$$

From (2.19) one gets:

$$\phi_0(a) = \int \frac{2\langle k_1 \rangle}{\langle k_{11} \rangle} da, \quad W_0(a) = \exp \{ \phi_0(a) \}. \quad (2.21)$$

Following the proposed procedure one has the third approximate solution to the FP equation (2.5) in the form:

$$W(a, \varphi) = \exp \left\{ \phi_0(a) + \varepsilon [\phi_{10}(a), \phi_{10}(a, \varphi)] + \varepsilon^2 [\phi_{20}(a) + \phi_{22}(a, \varphi)] \right\}. \quad (2.22)$$

From (2.22) one obtains the approximate mean square response in the form:

$$E[x^2] = \int_0^{2\pi} \int_0^{\infty} a^2 \cos^2 \varphi \exp \left\{ \phi_0(a) + \varepsilon [\phi_{10}(a) + \phi_{11}(a, \varphi)] + \varepsilon^2 [\phi_{20}(a) + \phi_{22}(a, \varphi)] \right\} da d\varphi. \quad (2.23)$$

3. Application

In order to illustrate this procedure we consider two following oscillators with non-linear stiffness and damping.

3.1. Modified Duffing oscillator

Consider the following Duffing equation with second order non-linear term:

$$\ddot{x} + \omega^2 x = \varepsilon(-2h\dot{x} - \beta x^3) - \varepsilon^2 \gamma x^2 \dot{x} + \sqrt{\varepsilon} \sigma \dot{\xi}(t) \quad (3.1)$$

where β , γ and h are constants and $h, \beta > 0$. In this case one gets:

$$f_1(x, \dot{x}) = -2h\dot{x} - \beta x^3, \quad f_2(x, \dot{x}) = -\gamma x^2 \dot{x}. \quad (3.2)$$

Using (3.2) for (2.4) one gets:

$$\begin{cases} k_1 = \frac{\sigma^2}{4\omega^2 a} - ha + \left(ha + \frac{\sigma^2}{4\omega^2 a} \right) \cos 2\varphi + \frac{\beta}{4\omega} a^3 \sin 2\varphi + \frac{\beta}{8\omega} a^3 \sin 4\varphi \\ k_2 = \frac{3\beta}{8\omega} a^2 - \left(h + \frac{\sigma^2}{2\omega^2 a} \right) \sin 2\varphi + \frac{\beta}{2\omega} a^2 \cos 2\varphi + \frac{\beta}{8\omega} a^2 \cos 4\varphi \\ r_1 = -\gamma a^3 \cos^2 \varphi \sin^2 \varphi, \quad r_2 = -\gamma a^2 \cos^3 \varphi \sin \varphi. \end{cases} \quad (3.3)$$

Using (2.10), (2.13), (2.14) and (2.16), after some calculations one obtains the second approximate probability density function in the form:

$$W(a, \varphi) = W_0(a) \exp \left\{ \varepsilon [\phi_{10}(a) + \phi_{11}(a, \varphi)] \right\} \quad (3.4)$$

where:

$$W_0(a) = Ca \exp \left\{ -\frac{2h\omega^2}{\sigma^2} a^2 \right\} \quad (3.5)$$

C is a normalization constant

$$\begin{cases} \phi_{10}(a) = -\left(\frac{\gamma\omega^2}{8\sigma^2} + \frac{3\beta h}{8\sigma^2}\right) a^4 \\ \phi_{11}(a) = -\frac{\beta h}{8\sigma^2} a^4 (4 \cos 2\varphi + \cos 4\varphi). \end{cases} \quad (3.6)$$

The probability density function $W(a, \varphi)$ must satisfy the condition:

$$\lim_{a \rightarrow \infty} W(a, \varphi) = 0. \quad (3.7)$$

Thus, the parameter γ must be chosen such that the coefficient of a^4 in the expression of $W(a, \varphi)$ is negative. Hence, from (3.6) one gets:

$$\gamma > -\frac{8\beta h}{\omega^2}. \quad (3.8)$$

On the other hand, using the classical SAM [5] one obtain the probability density function of amplitude as follows:

$$W(a) = \exp \left\{ \int \frac{2\langle k_1 + r_1 \rangle}{\langle k_{11} \rangle} da \right\} = Ca \exp \left\{ -\frac{2h\omega^2}{\sigma^2} a^2 - \varepsilon \frac{\gamma\omega^2}{8\sigma^2} a^4 \right\}. \quad (3.9)$$

It can be seen that the results in (3.4), (3.5), (3.6) and in (3.9) are different from each other and the effect of the non-linear term βx^3 is lost in (3.9).

Using (3.4), (3.5) and (3.6) one gets the approximate mean square response:

$$E[x^2] = \frac{\sigma^2}{4h\omega^2} - \varepsilon \frac{(6\beta h + \gamma\omega^2)\sigma^4}{32h^3\omega^6} + \varepsilon^2 \dots \quad (3.10)$$

We consider the case $\omega = \beta = \sigma = 1$, $h = 0.25$. So, the condition for γ in (3.8) becomes

$$\gamma > -2. \quad (3.11)$$

The mean square responses corresponding to some different values of the coefficient γ are given in Table 1. One can see the effect of second order non-linear term $\varepsilon^2 \gamma x^2 \dot{x}$: If $\gamma > 0$ the non-linear term $\varepsilon^2 \gamma x^2 \dot{x}$ reduces the mean square response $E[x^2]$ and increases it if:

$$-2 < \gamma < 0 \quad (3.12)$$

Table 1. Mean-square response to Duffing system
(Effect of second order non-linear coefficient γ)

N	γ	$E[x^2]$
1	-1	$1 - \varepsilon + \varepsilon^2 \dots$
2	-0.5	$1 - 2\varepsilon + \varepsilon^2 \dots$
3	0	$1 - 3\varepsilon + \varepsilon^2 \dots$
4	0.5	$1 - 4\varepsilon + \varepsilon^2 \dots$
5	1	$1 - 5\varepsilon + \varepsilon^2 \dots$

3.2. Modified Van der Pol oscillator

Now we consider the nonlinear system described by the equation

$$\ddot{x} + \omega^2 x = \varepsilon(1 - \gamma x^2)\dot{x} - \varepsilon^2 \beta x^3 + \sqrt{\varepsilon} \sigma \dot{\xi}(t) \quad (3.13)$$

where $\gamma, \beta > 0$. In this cases one gets:

$$f_1(x, \dot{x}) = (1 - \gamma x^2)\dot{x}, \quad f_2(x, \dot{x}) = -\beta x^3 \quad (3.14)$$

$$\begin{cases} k_1 = (1 - \gamma a^2 \cos^2 \varphi) a \sin^2 \varphi + \frac{\sigma^2}{2\omega^2 a} \cos^2 \varphi \\ k_2 = (1 - \gamma a^2 \cos^2 \varphi) \sin \varphi \cos \varphi - \frac{\sigma^2}{\omega^2 a^2} \sin \varphi \cos \varphi \\ r_1 = \frac{\beta a^3 \cos^3 \varphi \sin \varphi}{\omega}, \quad r_2 = \frac{\beta a^2 \cos^4 \varphi}{\omega} \end{cases} \quad (3.15)$$

Similarly, one gets the third approximate density function as follows:

$$W(a, \varphi) = W_0(a) \exp \left\{ \varepsilon [\phi_{10}(a) + \phi_{11}(a, \varphi)] + \varepsilon^2 [\phi_{20}(a) + \phi_{22}(a, \varphi)] \right\} \quad (3.16)$$

where

$$W_0(a) = C a \exp \left\{ \frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma \omega^2}{8\sigma^2} a^4 \right\} \quad (3.17)$$

C is a normalization constant.

$$\begin{cases} \phi_{10}(a) = 0 \\ \phi_{11}(a, \varphi) = \frac{\gamma}{32\omega\sigma^2} (12\sigma^2 a^2 + 4\omega^2 a^2 - \gamma\omega^6 a^6) \sin 2\varphi \\ \quad + \frac{\gamma\omega}{64\sigma^2} (\gamma a^2 - 4a^4) \sin 4\varphi \end{cases} \quad (3.18)$$

$$\begin{aligned} \phi_{20}(a) = & -\frac{9\gamma}{2^4\omega\sigma^2}a^2 + \left(\frac{15\gamma^2}{2^7\omega^2} - \frac{9\gamma}{2^6\sigma^2}\right)a^4 + \left(\frac{5\gamma^2}{3 \cdot 2^5\sigma^2} - \frac{\beta\gamma}{2^4\sigma^2}\right)a^6 + \\ & + \left(\frac{3\beta\gamma\omega^2}{2^8\sigma^4} - \frac{61\gamma^3}{2^{13}\sigma^2}\right)a^8 - \frac{3\beta^2\gamma\omega^2}{5 \cdot 2^8\sigma^4}a^{10} \end{aligned} \quad (3.19)$$

$$\phi_{22}(a, \varphi) = C_1(a) \cos 2\varphi + C_2(a) \cos 4\varphi + C_3(a) \cos 6\varphi + C_4(a) \cos 8\varphi \quad (3.20)$$

$$\left\{ \begin{aligned} C_1(a) &= -\left(\frac{9\gamma}{2^4\omega^2} + \frac{\beta}{2\omega^2}\right)a^2 + \left(\frac{13\gamma^2}{2^6\omega^2} - \frac{3\gamma}{2^4\sigma^2} - \frac{\beta}{2\sigma^2}\right)a^4 \\ &\quad + \left(\frac{23\gamma^2}{2^8\sigma^2} + \frac{\beta\gamma}{2^3\sigma^2}\right)a^6 - \frac{5\gamma^3}{2^9\sigma^2}a^8 \\ C_2(a) &= -\frac{\beta}{2^3\omega^2}a^2 + \left(\frac{3\gamma}{2^6\sigma^2} - \frac{13\gamma^2}{2^8\omega^2} - \frac{\beta}{2\sigma^2}\right)a^4 \\ &\quad + \left(\frac{\beta\gamma}{2^6\sigma^2} - \frac{11\gamma^2}{2^8\sigma^2}\right)a^6 + \frac{9\gamma^3}{2^{10}\sigma^2}a^8 \\ C_3(a) &= \frac{7\gamma^2}{3 \cdot 2^8\sigma^2}a^6 - \frac{5\gamma^3}{3 \cdot 2^9\sigma^2}a^8 \\ C_4(a) &= \frac{\gamma^3}{2^{12}\sigma^2}a^8. \end{aligned} \right. \quad (3.21)$$

Using the classical SAM one gets the approximate probability density function:

$$W(a) = W_0(a) = Ca \exp \left\{ \frac{\omega^2}{\sigma^2}a^2 - \frac{\gamma\omega^2}{8\sigma^2}a^4 \right\}. \quad (3.22)$$

So, one can see that the effect of the nonlinear term $\varepsilon^2\beta x^3$ cannot be shown in (3.22) by using the classical SAM, but it can be detected in the higher order approximate probability density function (3.16) - (3.21). The approximate mean square response in this case takes the form:

$$E[x^2] = \int_0^{2\pi} \int_0^\infty a^2 \cos^2 \varphi W(a, \varphi) da d\varphi = E[x^2]_0 + \varepsilon^2 E[x^2]_2 + \dots \quad (3.23)$$

where

$$E[x^2]_0 = \frac{\int_0^\infty a^3 \exp \left\{ \frac{\omega^2}{\sigma^2}a^2 - \frac{\gamma\omega^2}{8\sigma^2}a^4 \right\} da}{2 \int_0^\infty a \exp \left\{ \frac{\omega^2}{\sigma^2}a^2 - \frac{\gamma\omega^2}{8\sigma^2}a^4 \right\} da} \quad (3.24)$$

$$\begin{aligned}
E[x^2]_2 = & \left\{ 2 \int_0^{\infty} a \exp\left(\frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma\omega^2}{8\sigma^2} a^4\right) da \int_0^{\infty} a^3 \exp\left(\frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma\omega^2}{8\sigma^2} a^4\right) \left[\phi_{20}(a) \right. \right. \\
& + \left. \left. \frac{3\langle\phi_{11}^2(a, \varphi)\rangle}{4} + \frac{C_1(a)}{2} \right] da \right. \\
& - \int_0^{\infty} a^3 \exp\left(\frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma\omega^2}{8\sigma^2} a^4\right) da \int_0^{\infty} a \exp\left(\frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma\omega^2}{8\sigma^2} a^4\right) \left[\phi_{20}(a) \right. \\
& + \left. \left. \frac{\langle\phi_{11}^2(a, \varphi)\rangle}{2} \right] da \right\} \frac{1}{4 \left[\int_0^{\infty} a \exp\left(\frac{\omega^2}{\sigma^2} a^2 - \frac{\gamma\omega^2}{8\sigma^2} a^4\right) da \right]^2}. \quad (3.25)
\end{aligned}$$

We consider the case where $\omega = \gamma = 1$, $\varepsilon = 0.2$. The mean square responses corresponding to the different values of the coefficient β and the intensity of white noise σ^2 are given in Table 2. It is seen that the mean square response for $\beta = 0$ is greater than that for values $\beta > 0$. Hence, the non-linear terms $\varepsilon^2 \beta x^3$ reduces the mean square response.

Table 2. Mean - square response to Van der Pol system
(Effect of second order non-linear coefficient β)

N	σ^2	$E[x^2]$			
		$\beta = 0$	$\beta = 5$	$\beta = 10$	$\beta = 15$
1	0.1	0.2091	0.2038	0.2023	0.2012
2	1.0	0.3754	0.3652	0.3568	0.3479
3	5.0	0.7243	0.7135	0.6974	0.6678
4	10.0	0.9634	0.9476	0.9256	0.9074
5	20.0	1.2954	1.2015	1.1845	1.1626

4. Conclusion

For many years the stochastic averaging method has been a very useful tool for investigating non-linear random vibration systems. However, the effect of some non-linear terms cannot be investigated by using the classical first order SAM. In

the paper, in the examples of two nonlinear systems this difficulty may be overcome by considering higher approximate solutions of SAM and we can see the influence of second order non-linear terms on the system response.

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ẢNH HƯỞNG CỦA CÁC SỐ HẠNG PHI TUYẾN BẬC HAI

Phương pháp trung bình ngẫu nhiên bậc nhất kinh điển đã được áp dụng

rộng rãi đối với các hệ cơ học phi tuyến, tuy nhiên phương pháp này không thể hiện được nhiều hiệu ứng phi tuyến của hệ đang xét. Trong bài báo này các tác giả trình bày phương pháp xác định nghiệm xấp xỉ bậc cao của phương trình Fokker-Planck. Kết quả áp dụng cho các hệ dao động dạng Duffing và Van der Pol cho thấy rõ hiệu ứng của các số hạng phi tuyến bậc hai trong xấp xỉ bậc cao của hàm mật độ xác suất cũng như trong đáp ứng bình phương trung bình.