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TYPES OF MOMENTUM DEFECT DIFFUSION AT THE BEGINING OF THE WAKE AND WAKE ESTABLISHMENT ZONE FOR THE COUPLED WAKE AND COMPARISON WITH THE ISOLATED WAKE

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ABSTRACT. The article [10] has presented the necessity of the classification into two types: Isolated wakes and coupled wakes. In this report, different types of momentum defect diffusion at the begining of wake are analysed for the two types of coupled wake and isolated wake. According to numerical results obtained, we analyse the existence - long or short - of the wake establishment zone before the wake established zone, and this zone is very different between isolated wake and coupled wake.

1. Introduction

From the established equation about the wake width, we will analyse types of momentum defect diffusion at the beginning of the wake for coupled wake and isolated wake. The difference about the type of momentum deffect diffusion of the two sorts of wakes decides different properties of these two sorts of wakes.

According to obtained numerical results, we analyse the existence of the wake establishment zone before the wake established zone. For coupled wake, the wake establishment zone can be long, short, or negligible - which depends on the boundary layers before the coupled wake and Reynolds number Re_x in wake. But, for isolated wake, the wake establishment zone is considerable, and before the wake establishment zone, according to each case, it exists a zone of form vortexes, in which, the numerical calculation is not converged.

2. General equation about wake width and type of momentum defect gradual diffusion at the beginning of coupled wake & comparison with the type of momentum defect sudden diffusion at the beginning of isolated wake

Existing theories of wake report that the bidimensional wake width is proportional to the square root of the abscissa x [6]:

$$b(x) = k\sqrt{x}, \tag{2.1}$$

where k is a proportional coefficient. While according to the established theory [12], we have:

$$b(x) = \sqrt{\delta_0^2 + k_b \int\limits_0^x \frac{dx}{U^2(x)}} \tag{2.2}$$

where, δ_0 is the boundary layer thickness at the trailing edge, k_b is the expansion coefficient of wake width, and $U(x)$ is the external velocity (with $f_u \approx$ const, see (3.1)).

According to the existing formulas (2.1), we can put a question: is the abscissa $x = 0$ determined at the beginning of wake or at the beginning of boundary layer ?

• If the abscissa $x = 0$ is counted at the beginning of the wake, the formulas (2.1) are not applicable to coupled wake, because the width of coupled wakes is not zero at the beginning.

• If the absccissa $x = 0$ is counted at the beginning of the boundary layer, is the law of development of the boundary layer thickness similar to the law (2.1) of the wake, so that the passageway from the boundary layers to the coupled wake has not leaps about the width.

According to the established equation (2.2), at the beginning of wake: $x = 0$ the wake width is equal to the boundary layer thickness at the trailing edge: $b(x) = \delta_0$. The development of wake width along the flow (abscissa x) depends on x and the external velocity $U(x)$.

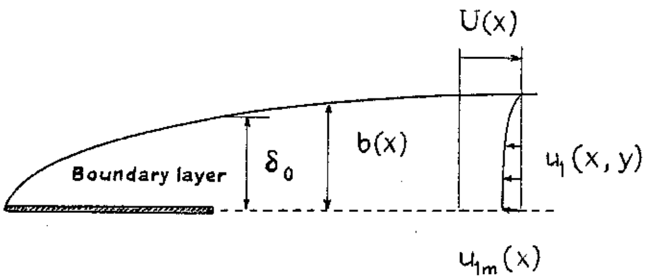

Fig. 1. Coupled wake

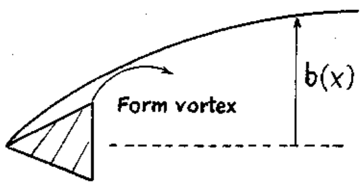

Fig. 2. Isolated wake

We know that the momentum defect in wake is conserved [2]. For the coupled wake - "pur" coupled wake (not mixed wake) [10], the momentum defect is caused by friction losses in boundary layers before the coupled wake. At $x = 0$, from the equation (2), the wake width: $b(x = 0) = \delta_0$. This momentum defect at the beginning of the coupled wake is diffused in all the width δ_0. And we can call this type of momentum defect diffusion type of gradual diffusion.

While, for the isolated wake - "pur" isolated wake (not mixed wake) [10], $\delta_0 = 0$, the momentum defect is caused by form resistance. At $x = 0$, from equation (2.2), the wake width: $b(x) = 0$. Since $x > 0$, the wake width increases according to the law: $b(x) \sim \sqrt{x}$ (with $U(x) = \text{const}$). Thus, the space at the beginning of isolated wake is limited, and the momentum defect is diffused with sudden type. With this type of sudden diffusion of momentum defect, there are form vortexes at the beginning of isolated wake.

3. Wake establishment zone for the coupled wake

Flow laws in the boundary layers and the coupled wake are different. Thus, there is always a transition zone from the boundary layers to the coupled wake - from the flow with wall to the free flow (without wall). If neglecting form losses behind the profile ("pur" coupled wake), characteristics of the transition zone depend on characteristics of the boundary layers at the trailing edge [7, 8] and the Reynolds number (Re_x). The momentum defect is gradually diffused, and then we can call this zone wake establishment zone (it is conventional that the wake establishment zone is a zone, in which, flow parameters are fluctuated and there are not singularities).

Using the established equation about the wake width (2.2) in a numerical method [3, 9], we calculated some cases (one case was compared with an experimental case of Andropoulos and Bradshaw [3]), and numerical results in figures 3, 4 show the wake establishment zone for coupled wakes (Case a: coupled wake without pressure gradient with the external velocity $U(x) = 0.5\,\mathrm{m/s}$; Case b: coupled wake without pressure gradient with $U(x) = 10\,\mathrm{m/s}$; $v = 10^{-6}\,\mathrm{m^2s^{-1}}$). The figure 3 represents converged results of 4 longitudinal iterances for the integral f_u, and the figure 4 converged results of 4 longitudinal iterances for the integral f_y, where:

$$f_u = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{b}\right), \tag{3.1}$$

$$f_y = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{y_c}\right), \tag{3.2}$$

with, $u_1 = u_1(x, y)$ is the velocity defect distribution in the wake,

$u_{1m} = u_{1m}(x)$ is centerline velocity defect,

$y_c = y_c(x)$ is the distance from the axis to the point, at which, the velocity defect is a half of the centerline velocity defect: $u_1(x, y_c) = 0.5u_{1m}(x)$.

For the integral f_u, Schlichting semi-empirically found its constancy for isolated wakes without pressure gradient [1]:

$$f_u = c_u = \text{const} \tag{3.3}$$

Our numerical results also demonstrate the constancy of the integral f_u for wakes without pressure gradient [14]. That is, the integral f_u has special properties.

For the integral f_y, our numerical results demonstrate its constancy for all cases: wakes without pressure gradients and accelerating, decelerating wakes [16, 17]:

$$f_y = c_y = \text{const} \quad \text{[universal integral]}. \tag{3.4}$$

This is equivalent to the experimental conclusion of Schlichting and Reichard (1929-1932) about the demensionless velocity defect [18]:

$$\frac{u_1}{u_{1m}} = f\left(\frac{y}{y_c}\right) \quad \text{[univeral relation]}. \tag{3.5}$$

That is, the integral f_y also is a special integral, which we call universal integral [14].

Beside special properties of these two integrals f_u and f_y, numerical results show that these integrals are very sensitive with errors. Thus, we represent in following figures numerical results of only the two special parameters above: f_u and f_y.

• For the case a, numerical results in figures 3.a, 4.a show that at the beginning of wake, exists a bad converged zone, which lengthens about 0.15 m. We can consider this zone wake establishment zone. Next, it is the wake established zone. However, the numerical calculation is converged only on the length of 25 m (zone C). After the zone C, the numerical calculation is diverged (zone D). This is explained by factor that: at the wake establishment zone, calculation knots bring certain errors. In the next calculation process, these errors add to accumulated numerical errors in the zone C. When the total error is too big, the calculation is diverged. We can find that, the longer the wake establishment zone, the shorter the numerical converged zone (zone C), and vice verca.

• For the case *b*, numerical results in figures 3.b, 4.b show that the wake establishment zone is so small that not visible in the figures. Thus, the numerical converged zone C is very long, till 32 m, there are not yet numerical unstabilities. Why is the wake establishment zone in this case more moderate? This can be that when the external velocity $U(x)$ is big, according to the equation (2.2), the expansion of wake width $b(x)$ upstream of wake is small. Thus, the passageway from the boundary layers to the coupled wake at the trailing edge is gradual. That also shows inconveniences of existing formulas (2.1) about the wake width, when in these formulas, these is not the presence of the external velocity $U(x)$. And numerical results calculated for asymmetric coupled wakes [11, 15] show that, parallelly with the axis displacement of the asymmetric, the wake establishment zone of the asymmetric wakes is more important than one of symmetric wakes with same flow conditions.

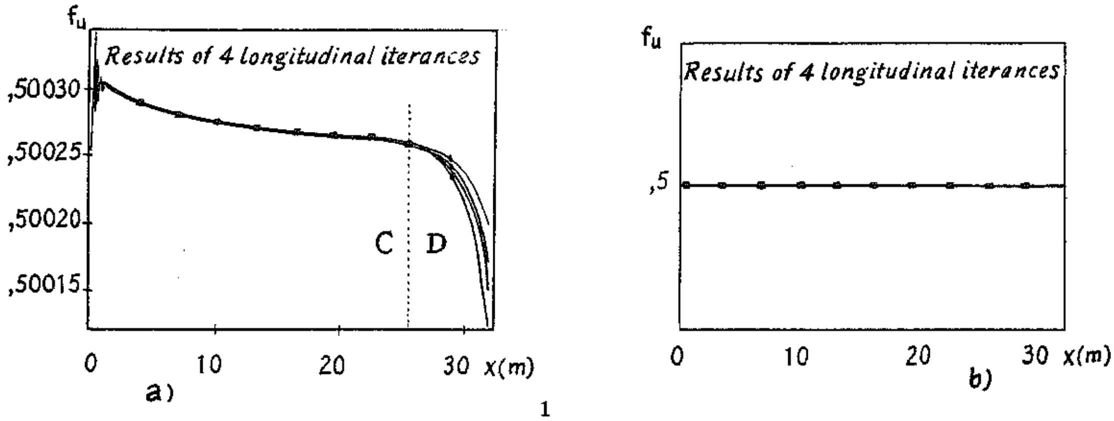

Fig. 3. Integral $f_u = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{b}\right)$ - coupled wakes

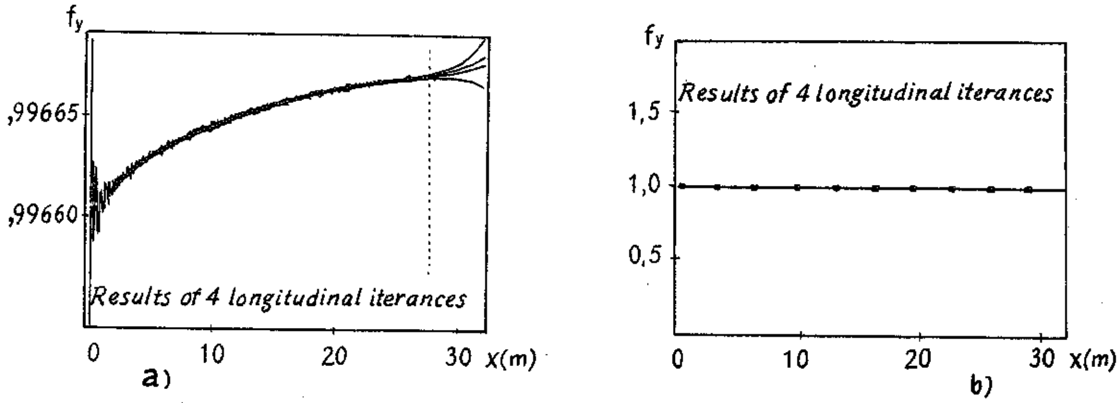

Fig. 4. Universal integral $f_y = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{y_c}\right)$ - coupled wakes

Note that in the zone D after the zone C, although the numerical calculation not converged (this is caused by too large numerical errors), but physically, the ›ne D belongs to the wake established zone (which always follows flow laws of ie zone C in cases of constant external velocity).

Also using the same method as for the calculation of coupled wake [13], we ılculated some cases of isolated wakes. The numerical calculation must then ımpt a certain distance (x_{up}) from the beginning point of the wake. In figures and 6, represent numerical results on the integral f_u and the integral f_y for vo cases without pressure gradient with different drag coefficients c_x, different kternal velocities $U(x)$ and similar cross dimensions. For the case c: $c_x = 1.22$:ylinder body), $x_{up} \geq 0.21$; for the case d: $c_x = 1.55$ (triangle body), $x_{up} \geq 0.28$. erhaps, this can be explained by the factor that: for isolate wakes, before the 'ake establishment, there is a zone of form vortexes, where Prandtl's equation ystem is not verified, and thus, the numerical calculation can not converged at eginning knots. We can see that for the two cases of coupled wakes above, the ad converged zone upstream of the wake lengthens about 0.15 m in the case a, nd this zone is negligible in the case b. But here, for the two cases of isolated rakes, the bad converged zone is much more than one of the cases of coupled rakes: about 1 m for the case c and about 2.5 m for the case d. We find that flow .pstream of coupled wakes is much more tender than the flow upsream of isolated rakes.

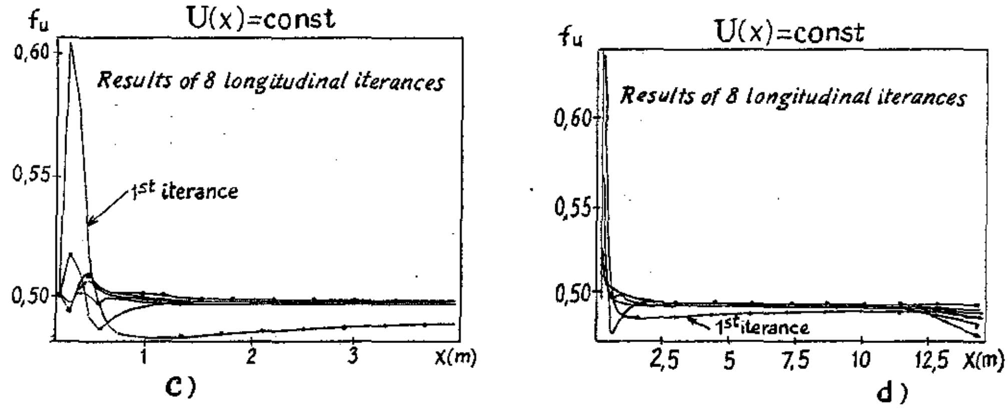

Fig. 5. Integral $f_u = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{b}\right)$ - isolated wakes

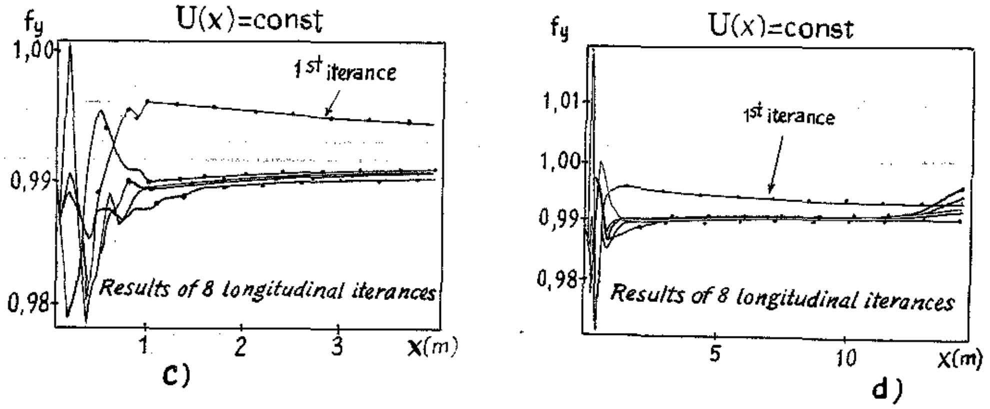

Fig. 6. Universal integral $f_y = \int_0^1 \frac{u_1}{u_{1m}} d\left(\frac{y}{y_c}\right)$ - isolated wakes

4. Conclusion

The presented analysis about the different types of momentum defect diffusion at the beginning of the wake for coupled wake for and isolated wake, and the studies about the wake establishment zone for these two sorts of wakes show that it is very necessary to classify the wakes into two types: isolated wake and coupled wakes. And thus, it is necessary to have two respective study domains for the wakes: the study domain of isolated wakes and the study domain of coupled wakes.

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CÁC KIỂU KHUYẾCH TÁN ĐỘNG LƯỢNG THIẾU HỤT Ở ĐẦU VẾT VÀ VÙNG VẾT ĐANG THIẾT LẬP CỦA VẾT LIÊN HỢP VÀ SO SÁNH VỚI VẾT ĐƠN

Trong bài viết [10], đã trình bày sự cần thiết phải phân loại vết thành hai loại: vết đơn và vết liên hợp. Bài viết này đề cập tới các kiểu khuyếch tán động lượng thiếu hụt ở đầu vết, đối với hai loại vết đơn và vết liên hợp. Với hai loại vết này, các kiểu khuyếch tán động lượng thiếu hụt ở đầu vết rất khác nhau, và chính điểm khác nhau này chi phối những đặc điểm khác nhau cơ bản của hai loại vết nói trên. Theo kết quả số nhận được, sự tồn tại của vùng vết đang thiết lập - dài hoặc ngắn - ở trước vùng vết thiết lập được phân tích trong bài này. Vùng vết đang thiết lập này rất khác nhau giữa vết đơn và vết liên hợp.