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NUMERICAL SIMULATION OF FLUID MUD LAYER UNDER CURRENT AND WAVES IN ESTUARIES AND COASTAL AREAS

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ABSTRACT. The formation and development of fluid mud layer under the both factors of current and wave that often happens in estuaries and coastal areas are studied in detail through numerical simulation on the basis of the 2D shallow water equations for tidal flow, the advection-diffusion equation for cohesive sediment transport and the equations for fluid mud transport. At the same time the model for wave propagation is also included. Numerical solution of a special case for a part of the Severn estuary is obtained using the finite difference method as an illustration of the applicability of the model in practice.

1. Introduction

Cohesive sediment transport plays an important role in a wide rage of design, maintenance and management problems in estuaries and coastal regions. Accumulation of sediment in navigation channels and berths often requires very expensive dredging operations. Once a new development appears, it can cause significant changes to the sediment transport patterns, so a good understanding of the likely changes is necessary before any engineering project can proceed. In addition, many pollutants are preferentially adsorbed on to the fine cohesive fraction of the sediment and therefore for environmental reasons it is important to be able to predict the movement of the sediment.

In estuaries and coastal regions where there is a high concentration of sediment in suspension, a fluid mud layer is often formed during slack water periods by the process of hindered settling. This amount of sediment comes from the sea or from rivers due to the process of flowing through many areas in a country or around the sides of mountains. Once the near bed sediment concentration exceeds the peak value, mud settles towards the bed more quickly than it can dewater and a layer of fluid mud forms. The movement of the fluid mud layer can be described by a restricted form of the shallow water equations. Many complicated physical processes occur at the interface between sediment in suspension and fluid mud and between fluid mud and the rigid bed. On the other hand, fluid mud can also be formed by waves which can fluidise a muddy bed, especially, in the case of storm the effect of wave can't be ignored.

Here the study is focused on the numerical simulation of the fluid mud layer development with a very high cohesive sediment concentration in suspension on a computer by using finite difference method. The model include 2D shallow water equations for tidal flow, wave propagation equation, the advection-diffusion equation for cohesive sediment transport and the equations for fluid mud transport. A numerical solution for a concrete case is obtained as an illustration example. On the basis of the results, initial remarks and evaluation are given.

2. The governing equations and boundary conditions

1. Tidal model

As well known, the most popular assumption used in the mathematical models up to now is that the effect of the bed changing in respect to time on hydrodynamics process is ignored. This is because this effect is insignificant in comparison with the other factors when the average sediment concentration is not large enough. Therefore the mathematical model describing this phenomenon is divided into two separate models: The model of tidal current and the model of sediment transport. The tidal model is the $2D$ horizontal shallow water equations without the force of wind (Muir Wood and Fleming [1]),

$$\frac{\partial z}{\partial t}+\frac{\partial(du)}{\partial x}+\frac{\partial(dv)}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial z}{\partial x}+\Omega v+D\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big)-gu\frac{\sqrt{u^2+v^2}}{C^2 d} \tag{2.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial z}{\partial y}-\Omega u+D\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\Big)-gv\frac{\sqrt{u^2+v^2}}{C^2 d} \tag{2.3}$$

in which x, y are the coordinates, the x axis goes along the shore and the y axis perpendicular to the shore, u and v are the tidal flow velocity components along the x and y directions, respectively, z is the water level above the chart datum, g-acceleration due to gravity, C-Chezy coefficient, d-total water depth, Ω-Coriolis parameter, D-the eddy viscosity coefficient, and t-time. It should be noted that the terms in the right hand sides of the equations (2.2) and (2.3) present the forces due to the surface slope, rotating of the earth, turbulent diffusion and the friction on the bed in the x and y directions, respectively, the scales of which depend on the situations in question.

2. Wave model

The governing equation used by the model can be derived from the time-independent form of the Mild-Slope equation:

$$\frac{\partial}{\partial x}\left(cc_g\frac{\partial \eta}{\partial x}\right)+\frac{\partial}{\partial y}\left(cc_g\frac{\partial \eta}{\partial y}\right)+\frac{\omega^2 c_g}{c}\eta=0 \tag{2.4}$$

in which η is the complex water surface elevation, c-the wave celerity, c_g-the wave group velocity, ω - the angular wave frequency. By representing $\eta = Ae^{iS}$ (A is the wave amplitude and S is the wave phase) and neglecting diffraction effects, the ray tracing model is obtained. In the case when wave height curvature in the main propagation direction (y direction) is much less than in the lateral direction (x direction), the governing equations are the following:

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x} \tag{2.5}$$

$$\frac{\partial Q}{\partial y}=\frac{1}{Q}\Big(-P\frac{\partial Q}{\partial x}+k\frac{\partial k}{\partial y}+\frac{1}{2}\frac{\partial}{\partial y}\Big(\frac{1}{H}\frac{\partial^2 H}{\partial x^2}\Big)\Big) \tag{2.6}$$

$$\frac{\partial}{\partial x}(H^2MP)+\frac{\partial}{\partial y}(H^2MQ)=0 \tag{2.7}$$

in which P and Q are components of Wave number vector $\mathbf{K}$, H is wave height, k is the wave separation factor. Wave number is represented in the vector form and

$$\mathbf{K}=\nabla S=(P,Q) \quad \text{and} \quad M=\frac{c_g}{|\mathbf{K}|} \tag{2.8}$$

3. Suspended sediment model

The equation describing sediment transport is the advection-diffusion equation based on the sediment mass conservation with the exchange between sediment in suspension and fluid mud or mud on the bed taken into account (Roberts [2], Odd and Cooper [3], Odd and Rodger [4], and Le Hir and Kalikow [5]) as follows

$$\frac{\partial(dc)}{\partial t}+\frac{\partial(q_x c)}{\partial x}+\frac{\partial(q_y c)}{\partial y}=\frac{dm}{dt}+\frac{\partial}{\partial x}\left(dE_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(dE_y\frac{\partial c}{\partial y}\right) \tag{2.9}$$

in which c is the mass suspended sediment concentration, q_x, q_y components of discharge per unit of width along the x and y directions, respectively, E_x, E_y the diffusion coefficients for sediment along the x and y, z_b the bed level below the chart datum, ρ_m the mud density, and $\frac{dm}{dt}$ the source-sink term.

4. Fluid mud model

The model for fluid mud layer includes the equation of fluid mud mass conservation and the equations of momentum conservation that are the restricted form of the $2D$ horizontal shallow water equations (Roberts [4]),

$$\frac{\partial}{\partial t}(c_m d_m) + \frac{\partial}{\partial x}(c_m d_m u_m) + \frac{\partial}{\partial y}(c_m d_m v_m) = \frac{dm}{dt} \tag{2.10}$$

$$\frac{\partial u_m}{\partial t} + \frac{1}{d_m \rho_m}(\tau_0 - \tau_i)_x - \Omega v_m + \frac{\rho_w}{\rho_m} g \frac{\partial z}{\partial x} + g\frac{\Delta\rho}{\rho_m}\frac{\partial z_m}{\partial x} + g\frac{d_m}{2\rho_m}\frac{\partial \Delta\rho}{\partial x} = 0 \tag{2.11}$$

$$\frac{\partial v_m}{\partial t} + \frac{1}{d_m \rho_m}(\tau_0 - \tau_i)_y + \Omega u_m + \frac{\rho_w}{\rho_m} g \frac{\partial z}{\partial y} + g\frac{\Delta\rho}{\rho_m}\frac{\partial z_m}{\partial y} + g\frac{d_m}{2\rho_m}\frac{\partial \Delta\rho}{\partial y} = 0 \tag{2.12}$$

in which c_m is the mass concentration of fluid mud, in general a function of time and space, d_m is the fluid mud depth, u_m and v_m the fluid mud velocity components in the x and y directions, respectively, z_m the elevation of the interface between fluid mud and sediment in suspension, τ_0 and τ_i the shear stress vectors on the bed and on the interface, respectively, ρ_w the water density, and ρ_m the fluid mud density, the relationship of which with the water density is the following,

$$\rho_m = \rho_w + \Delta\rho, \quad \Delta\rho = 0.62 c_m. \tag{2.13}$$

From equations (2.11)–(2.12) it should be noted that the external forces making fluid mud move in turn are the shear stress on the bed and on the mud-water interface, the Coriolis force, the slope of water surface, the slope of mud-water interface, and gradient of density that is ignored in the study.

About the source-sink term presenting the exchange at the bed or at the mud–water interface in the equations (29)–(2.10), the following processes are introduced:

* Erosion:

$$\frac{dm}{dt} = m_e\Big(\frac{\tau}{\tau_e} - 1\Big)H(\tau - \tau_e), \quad H(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{2.14}$$

in which $m_e(Kg/N/s)$ is the erosion rate parameter, $\tau(N/m^2)$ is the actual shear stress at the fluid mud–water interface or at the bed–water interface in the absence of fluid mud, τ_e the critical bed shear stress for erosion, and $H(x)$ the usual Heaviside step function.

* Settling of mud from suspension:

$$\frac{dm}{dt} = v_s(c)c\Big(1 - \frac{\tau}{\tau_d}\Big)H(\tau_d - \tau), \quad v_s(c) = \begin{cases} v_{\min}, & c < \dfrac{v_{\min}}{R_0} \\ cR_0, & c \geq \dfrac{v_{\min}}{R_0} \end{cases} \tag{2.15}$$

where τ_d is the critical bed shear stress for deposition, $v_s(m/s)$ the settling velocity, $v_{\min}$ the minimum settling velocity, and $R_0(m^4/kg/s)$ are given from experiments. This process only occurs on the mud–water interface or on the bed in the case without fluid mud.

* Entrainment:

$$\begin{aligned} &\frac{dm}{dt} = v_e c_m H(10 - R_i), \quad v_e = \frac{0.1\Delta U}{(1 + 63R_i^2)^{3/4}}, \quad R_i = \frac{\Delta\rho g d_m}{\rho_w \Delta U^2}, \\ &\Delta U^2 = (u - u_m)^2 + (v - v_m)^2, \end{aligned} \tag{2.16}$$

where v_e is the entrainment velocity (m/s), and R_i-the bulk Richardson number representing the degree of the flow stratification. Therefore, the entrainment only happens when the stratification of the flow is not strong enough on the water–mud interface.

* Dewatering

$$\frac{dm}{dt} = v_0 c_m H(\tau_d - \tau) \tag{2.17}$$

where v_0 is the dewatering velocity (m/s). This phenomenon only occurs on the bed when fluid mud layer exists and the bed shear stress is less than the critical value τ_d.

As mentioned above, the entrainment is a process easily causing the numerical instability, so it requires to treat carefully.

To close the problem mathematically the initial and boundary conditions for the situation under consideration are required. They are as follows,

* The initial conditions:

$$\begin{aligned} &u(x,y,0) = 0, \quad v(x,y,z) = 0, \quad z(x,y,0) = 12.4, \\ &c(x,y,0) = 5, \quad d_m(x,y,0) = 0, \quad u_m(x,y,0) = 0, \quad v_m(x,y,0) = 0. \end{aligned} \tag{2.18}$$

* The boundary conditions:

The 4 kinds of boundaries that are necessary to consider here in turn are the river boundary $(x = L)$, the open sea boundary $(x = 0)$, the offshore boundary $(y = 0)$ and the land boundary $((x,y) \in Ln)$:

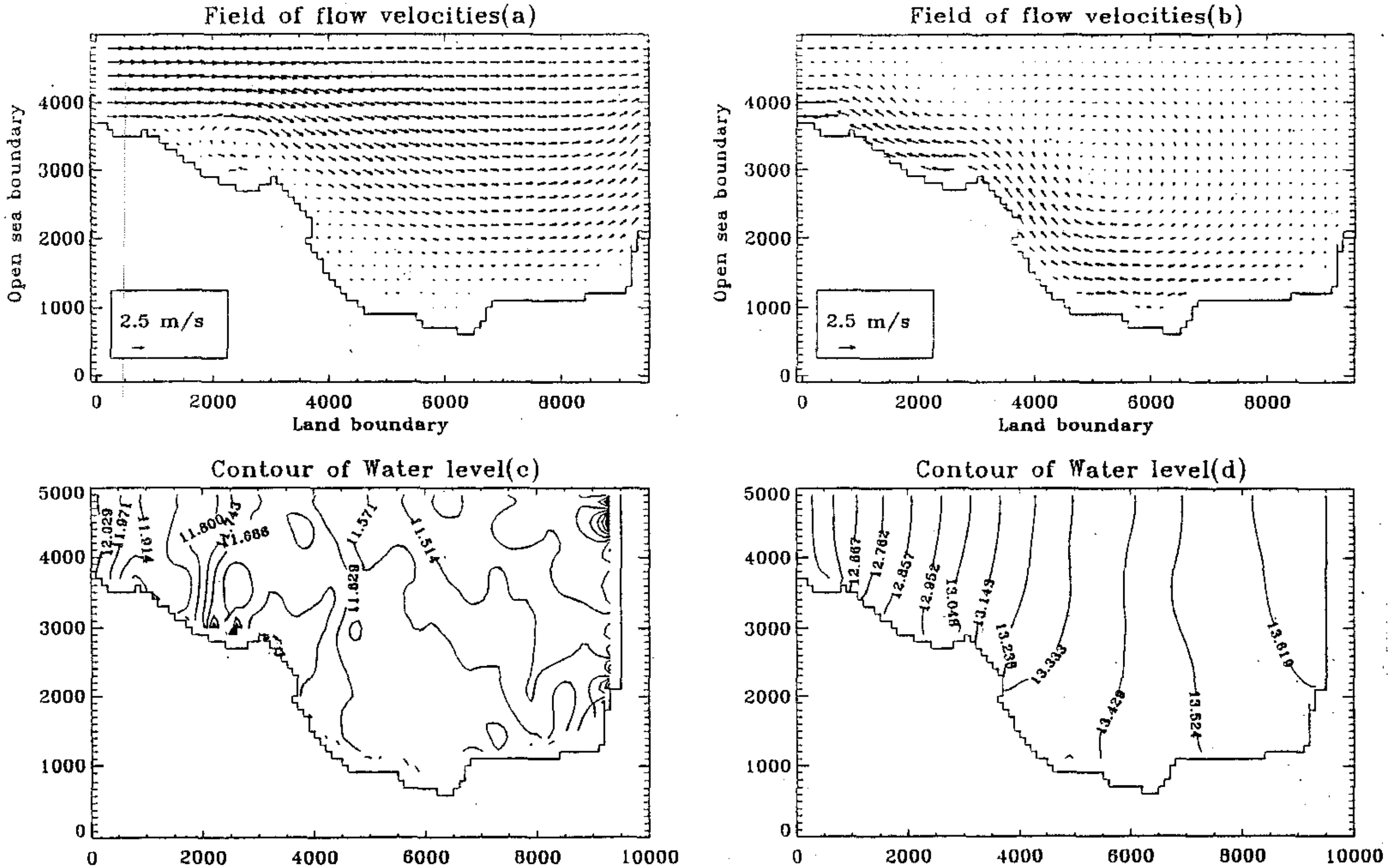

Fig. 1. Results of computation for tidal model t=38400s (left), t=44400s (right)

$$
\begin{aligned}
&q_{tx}(x,y,t)\big|_{x=L} = f_1(t), \quad q_{ty}(x,y,t)\big|_{x=L} = 0,\\
&c(x,y,t)\big|_{x=L} = 5, \quad d_m(x,y,t)\big|_{x=L} = 0,\\
&z(x,y,t)\big|_{x=0} = f_2(t), \quad v(x,y,t)\big|_{x=0} = 0,\\
&\frac{\partial u}{\partial x}\Big|_{x=0} = 0, \quad \frac{\partial c}{\partial x}\Big|_{x=0} = 0, \quad \mathbf{v.n} = 0, \quad (x,y) \in Ln,
\end{aligned}
\tag{2.19}
$$

in which q_{tx}, q_{ty} are the components of the total water discharge vector, $f_i(t)$ $(i = 1,2)$ the given functions at the river boundary, $\mathbf{v}$-the flow velocity vector, and $\mathbf{n}$-the normal vector unit on the land boundary.

3. Numerical solutions and a test application

The equations (2.1)-(2.3) together with given boundary and initial conditions that present the tidal flow have been solved numerically firstly. The ADI method was used, with staggered grid for the derivative in respect to space and Leap-frog scheme for time (Chung and Roberts [6]). For the wave model the Predictor-Corrector method is used, in which the discretisation of eqs. (2.5)-(2.7) is carried out by the row for P, Q and H, respectively. It should be noted that an average oprerator is proposed to improve the solution, that is

$$
Q_{ij} = \frac{1}{r_s}\Big(r_1 Q_{i-1j-1} + r_2 Q_{ij-1} + r_1 Q_{i+1j-1} + Q_{ij}\Big), \quad r_s = 2r_1 + r_2 + r \tag{3.1}
$$

and fully similar to (3.1) for P_{ij} and H_{ij}.

The finite difference method is also used to solve the equations (2.9)-(2.12) together with the initial and boundary conditions (2.18)-(2.19) for the sediment transport model. Specially, QUICKEST (see Leonard [7]) is used for the advection–diffusion equation (2.9) and QUICK [7] for the equation (2.10) to get more accuracy.

In order to get an overview for the short term prediction the above difference equation systems are solved over a tidal period, some results of which are illustrated in the figures 1-2. For tidal flow, an important feature of the model is worth to pay attention, that is the representation of the flooding and drying of the intertidal flats and numerical treatment to ensure stability and accuracy because of the very large tidal range in the Severn. At the same time, the results obtained from tidal model have been compared with the field measurements [8] and discussed in more detail (Chung and Roberts [6]). In this work the results of computation at two time points $t = 38400s$ and $t = 44400s$ corresponding to before and after high water, respectively (high water at $t = 42000s$), are displayed as the characteristics evaluations over a tidal period. It shows that in both the case the suspended sediment concentrations are quite high in general. This is completely reasonable, because wave factor actually hinders settling of sediment in suspension and remains concentration at higher value. However, this is only the evaluation for a short time of a tidal period without big change of wave strength. For the case, in which wave strength is considerably variable, such as before and

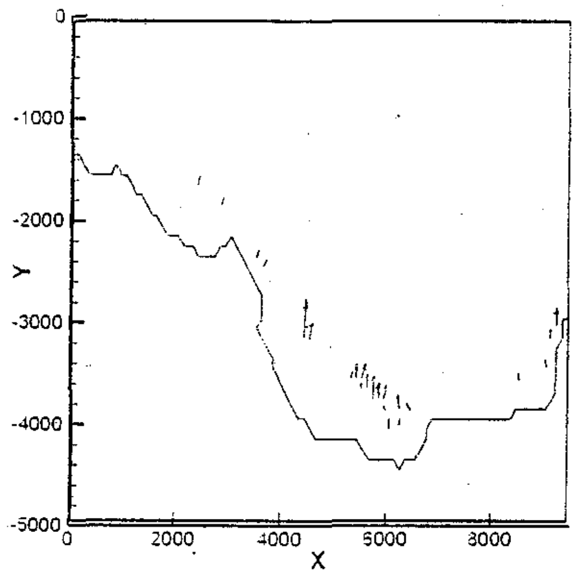

Fig. 2a

Field of fluid mud velocities at t=38400s

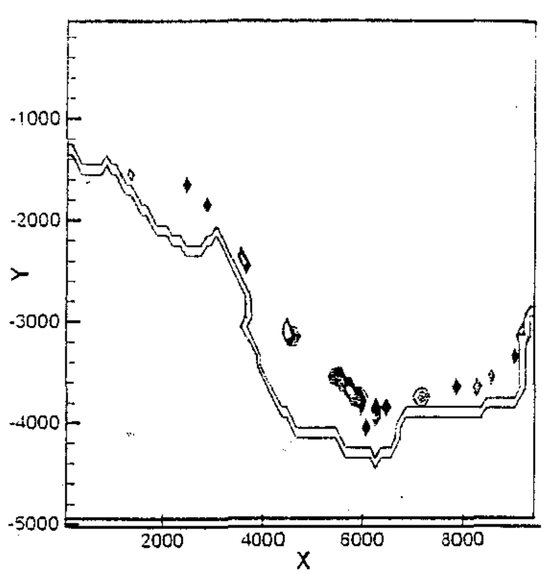

Fig. 2b

Contours of mud depth at t=38400s

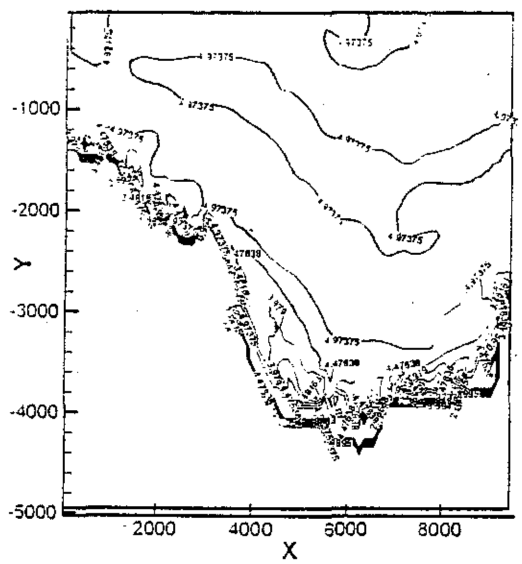

Fig. 2c

Contours of concentration at t=38400s

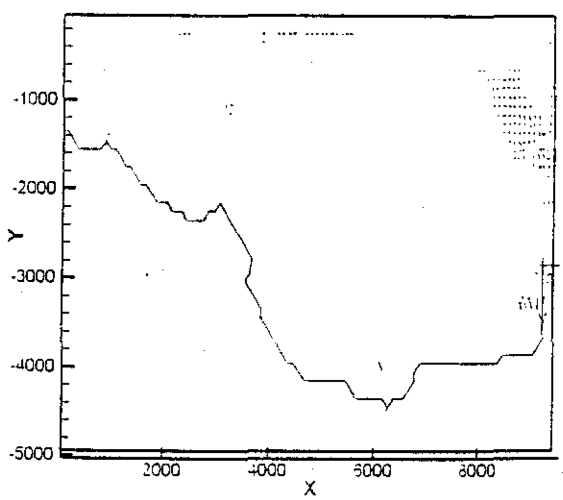

Fig. 2d

Field of fluid mud velocities at t=44400s

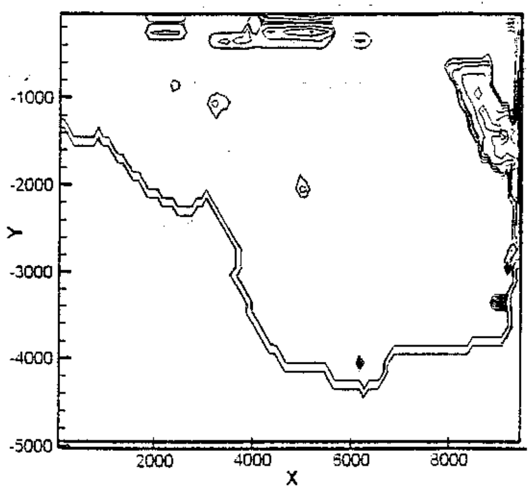

Fig. 2e

Contours of depth at t=44400s

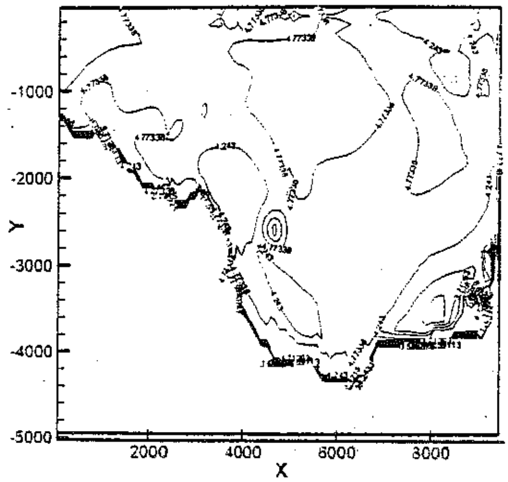

Fig. 2f

Contours of concentration at t=44400s

after a storm, the fluid mud layer by wave will really become a problem for navigational channels and berths. Also from the Fig. 2 it shows that mud concentration in suspension in the area with fluid mud is smaller than the initial value (4.5kg/m^3 in the deep area and 3-4kg/m^3 in the shallow area with low currents), while it is higher in the narrow band. As soon as the fluid is formed, it starts to move with very slow velocity as illustrated in Fig. 2, causing serious accumulation in the deep channel at slack water. For the case under consideration the maximum mud layer depth is about 0.7m that is really bigger in comparison with the case of no wave (Chung [9]). The peak value of fluid mud layer appears when the water level achieves the minimum value that is explained obviously, because at this time flow velocity is enough small and makes sediment in suspension be able to settle downwards near bed and therefore, fluid mud layer depth increases rapidly. The interrelation between suspended sediment concentration and fluid mud depth is analysed and shows suitability on their sensitivity by considering the results of computation at the two positions corresponding to very shallow and deep areas. In both the cases it shows that when suspended sediment concentration goes down to a minimum value, the mud depth achieves the maximum. And as soon as this drying position becomes flooding again it has the behaviour similar with the other position mentioned above and a similar situation happens somewhere else as shown in Fig. 2.

Conclusion

The formation and development of fluid mud layer in estuaries and coastal areas are numerically simulated and successfully applied to the Severn estuary as an illustration example. Using the finite difference method, numerical solutions for the mathematical models are obtained, in which ADI method with staggered grid for the derivative in respect to space and Leap-frog scheme for time is used for tidal flow model. For wave model the predictor-corrector method shows an effect. Especially, QUICK and QUICKEST schemes with very high accuracy are applied to the equation of fluid mud mass conservation and the equation of advection-diffusion, respectively in the mud transport model. With the grid size of $100m \times 100m$, the computational area consists of 95×50 cells, which is not too coarse and is acceptable in simulation and prediction purposes. Computed results are quite sensible and show the feasibility of the models.

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MÔ PHỎNG SỐ LỚP BÙN LỎNG VÙNG CỬA SÔNG
VÀ VÊN BIỂN DƯỚI TÁC ĐỘNG CỦA DÒNG VÀ SÓNG

Sự hình thành và phát triển lớp bùn lỏng ở vùng cửa sông và ven biển dưới sự tác động của cả hai yếu tố dòng và sóng đã được nghiên cứu chi tiết thông qua phương pháp mô phỏng số. Mô hình toán học mô tả quá trình vật lý phức tạp này bao gồm hệ phương trình nước nông 2 chiều ngang đối với dòng triều, phương trình khuyếch tán xác định nồng độ bùn cát lơ lửng và hệ các phương trình mô tả sự phát triển của lớp bùn lỏng. Đồng thời hệ phương trình đối với khúc xạ sóng vùng nước nông cũng được sử dụng. Lời giải số cho 1 trường hợp cụ thể ở vùng cửa sông Severn bằng phương pháp sai phân hữu hạn đã thu nhận xem như một minh họa cho việc áp dụng mô hình trong việc giải quyết các bài toán trong thực tế.