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STABILITY OF VISCOELASTIC PLATES IN SHEAR

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1. Introduction

In [1] the stability of viscoelastic plates in compression is investigated

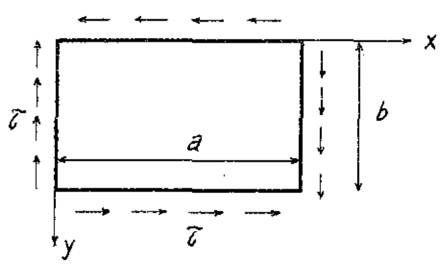
In this paper we consider the stability of viscoelastic plates in shear. The problem is solved by theory of pseudo-bifurcation points and "elastic analogy" method [2] in two cases: isotropic viscoelastic and orthotropic viscoelastic plates.

2. Stability of viscoelastic isotropic plates in shear

A. Elastic stability of plates in shear

Let us consider elastic stability of plates ($a \times b$) with simply supported edges.

From [3] we have the elastic stability equation of plates in shear as follows:



$$D\Big(\frac{\partial^4 \Delta W}{\partial x^4} + 2\frac{\partial^4 \Delta W}{\partial x^2 \partial y^2} + \frac{\partial^4 \Delta W}{\partial y^4}\Big) + 2N_{xy}\frac{\partial^2 \Delta W}{\partial x \partial y} = 0 \tag{2.1}$$

where $N_{xy} = \tau_{xy} \cdot \delta$: shearing force

δ : thickness of plates

ΔW : "stimulus" displacement.

Modulus

$$D = \frac{E\delta^3}{12(1-\vartheta^2)} = \frac{G\delta^3}{6(1-\vartheta)} \cdot \tag{2.2}$$

The critical stress of elastic plates depends on the rate a/b:

$$\tau_{cr} = K\frac{\pi^2 D}{b^2\delta} = K\frac{\pi^2\delta^2}{6b^2(1-\vartheta)} \cdot G \tag{2.3}$$

a/b	1.0	1.1	1.2	1.3	1.4	1.6	1.8	2.0	3.0	5.0	∞
K	9.34	8.47	7.97	7.57	7.3	6.9	6.64	6.47	6.04	5.71	5.34

B. Viscoelastic stability of plates in shear

Let us consider the equation of state [4]:

$$\gamma(t) = \frac{\tau(t)}{G(t)} - 2\big[1+\vartheta(t)\big]\int_0^t \tau(t_1)K(t,t_1)dt_1 \tag{2.4}$$

where: $\tau(t)$, $\gamma(t)$ - shearing stress and shearing strain and

$$G(t) = \frac{E(t)}{2\big[1+\vartheta(t)\big]} \tag{2.5}$$

It can be assumed that $E(t) = E = \text{const}$, $G(t) = G = \text{const}$, $\vartheta(t) = \vartheta = \text{const}$, from (2.5) we get :

$$G = \frac{E}{2(1+\vartheta)} \tag{2.6}$$

We denote $\gamma_0(t)$, $\tau_0(t)$ - strain and stress in basic state, $\gamma(t)$, $\tau(t)$ - in the adjacent state such that

$$\Delta\tau = \tau - \tau_0 \ll \tau_0, \quad \Delta\gamma = \gamma - \gamma_0 \ll \gamma_0, \quad \Delta\tau,\ \Delta\gamma\ - \text{called "stimulus"}.$$

Equation (2.4) can be written for $\Delta\tau$, $\Delta\gamma$ in the form:

$$\Delta\gamma(t) = \frac{\Delta\tau(t)}{G} - 2(1+\vartheta)\int_0^t \Delta\tau(t_1)K(t,t_1)dt_1.$$

With the help of (2.6) we have:

$$\Delta\gamma = \frac{1}{G}\Big[\Delta\tau - E\int\limits_0^t \Delta\tau(t_1)K(t,t_1)dt_1\Big] \tag{2.7}$$

Using the definition "pseudo - bifurcation of N-degree (PBN) [2] ($\overset{(N)}{\Delta}\tau \neq 0$, $\overset{(N)}{\Delta}\gamma \neq 0$, $\overset{(K)}{\Delta}\tau = 0$, $\overset{(K)}{\Delta}\gamma = 0$, $K = 0, 1, \ldots, M$; $K \neq N$ and $N < M$) and expanding the function $\Delta\tau(t_1)$ into a series we can get "elastic analogy" of N-degree.

First of all, we use the definition of "pseudo - bifurcation" of 0-degree (PBO) and expand $\Delta\tau(t_1)$ in (2.7) into a series, as a result of simple transformations we obtain "elastic analogy"

$$\Delta\tau = \widetilde{G}_0\Delta\gamma,$$

where

$$\widetilde{G}_0 = G\Big[1 - E\int\limits_0^t K(t,t_1)dt_1\Big]^{-1}. \tag{2.8}$$

Using PB1 similarly, we get:

$$\Delta\dot{\tau} = \widetilde{G}_1 \cdot \dot{\gamma},$$

where

$$\widetilde{G}_1 = G\Big[1 - E\int\limits_0^t (t_1 - t)\dot{K}(t,t_1)dt_1\Big]^{-1}. \tag{2.9}$$

For PB2, similarly, as a result we get:

$$\Delta\ddot{\tau} = \widetilde{G}_2 \cdot \Delta\ddot{\gamma},$$

where

$$\widetilde{G}_2 = G\Big[1 - \frac{E}{2}\int\limits_0^t (t_1 - t)^2\ddot{K}(t,t_1)dt_1\Big]^{-1}. \tag{2.10}$$

On the other hand, from [2] it is known that PB2 point is the limit of the stability region (criterion of the creep stability), we can use PB2 to determine critical time.

In this case, in the formula (2.3) we replace G by $\widetilde{G}_2$ from (2.10), we have:

$$\tau^* = K\frac{\pi^2\delta^2}{6b^2(1-\vartheta)}\, G\Big[1 - \frac{E}{2}\int\limits_0^t (t_1 - t)^2 K(t,t_1)dt_1\Big]^{-1}.$$

If we denote $\dfrac{\tau^*}{\tau_{cr}} = \omega$, where τ^* - stress acting really, τ_{cr} - critical stress by Euler.

We get:

$$\omega = \frac{\tau^*}{\tau_{cr}} = \Big[1 - \frac{E}{2}\int\limits_0^t (t_1 - t)^2 \tilde{K}(t, t_1) dt_1\Big]^{-1}. \tag{2.11}$$

To determine critical time, now we choose creep kernel (in practice for concrete) in the form [4].

$$K(t, t_1) = \frac{\partial}{\partial t_1}\left[\varphi(t_1)(1 - e^{-\gamma(t - t_1)})\right], \tag{2.12}$$

where $\varphi(t_1)$ - function of oldness with the condition $\lim\limits_{t_1 \to \infty} \varphi(t_1) = C_0$ in [4]:

$$\varphi(t_1) = C_0 + A_0 e^{-\beta t_1} \tag{2.13}$$

C_0, A_0, β - material constants.

Substituting (2.12), (2.13) in (2.11), we obtain:

$$\begin{aligned} \omega = \Big\{ 1 + EC_0\Big[1 - e^{-\gamma t}(1 + \gamma t + \frac{\gamma^2 t^2}{2})\Big] \\ + \frac{EA_0\gamma^2}{(\gamma - \beta)^2}\Big[e^{-\beta t} - e^{-\gamma t}\big[1 + (\gamma - \beta)t + \frac{(\gamma - \beta)^2 \cdot t^2}{2}\big]\Big]\Big\}^{-1} \end{aligned} \tag{2.14}$$

For concrete [5] we have:

$$E = 2 \cdot 10^4 Mpa, \quad \gamma = 0.026\frac{1}{\text{day}}, \quad A_0 = 0.482 \cdot 10^{-3}\frac{\text{day}}{Mpa},$$
$$\beta = 0.03\frac{1}{\text{day}}, \quad C_0 = 0.09 \cdot 10^{-3}\frac{1}{Mpa}$$

Using these material constants and formula (2.14) we can represent ω in terms of t.

We now construct the diagram $\omega \sim t$

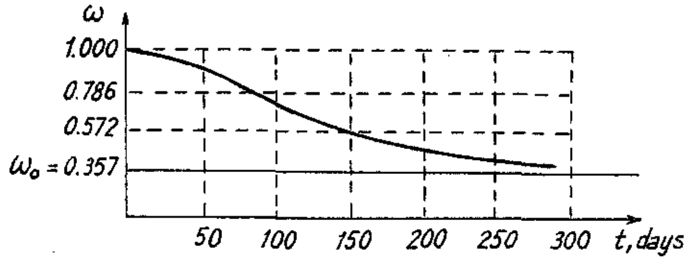


Fig. 1

From this diagram, we arrive at the conclusion:

- If $\omega < \omega_0$ - equilibrium of plate is always stable for any value of t.

- If $\omega > \omega_0$ - the loss of stability is in determined time.

$\omega = \omega_0$ - the limit of long time stability, $\omega_0 = \dfrac{1}{1+EC_0}$.

On the basis of the diagram or the formula (2.14) we can determine critical time t_{cr} for given stress ω or τ^*.

Example. For a square plate with simply supported edges: $a = b = 3\text{m}$, $\nu = 0.25$, $\delta = 6\,\text{cm}$, we can obtain

$$\tau_{cr} = 9.34\frac{\pi^2 D}{b^2 \cdot \delta} = 65.47 Mpa.$$

Using this τ_{cr} and (2.14) we get the relation $\tau^* \sim t_{cr}$

τ^*(MPa)	65.5	64.4	58.6	49.8	41.6	35.5	31.4	28.6	26.8	25.6	24.8	24.3	23.9	23.4
t_{cr}(days)	0	25	50	75	100	125	150	175	200	225	250	275	300	∞

Now we choose creep kernel in the form :

$$K(t-t_1) = \frac{-A}{(t-t_1)^\alpha}, \tag{2.15}$$

where $0 < \alpha < 1$, $A > 0$; A, α - material constants

Substituting (2.15) into (2.11) we have:

$$t_{cr} = \left[\frac{2(1-\alpha)\left(\frac{1}{\omega}-1\right)}{AE\alpha(\alpha+1)}\right]^{\frac{1}{1-\alpha}}. \tag{2.16}$$

Using the following constants (for polymer)

$$A = 2\cdot 10^{-4}\frac{\text{cm}^2}{KN\cdot \text{day}^{1/4}}, \quad \alpha = 0.75, \quad E = 0.267\cdot 10^4\frac{KN}{\text{cm}^2}.$$

We can find the critical time for plate acted by a given shearing force.

3. Stability of viscoelastic orthotropic plate in shear

We denote e^0_{ij}, σ^0_{ij} strain and stress in the basic state, e_{ij}, σ_{ij} strain and stress in the adjacent state, such that

$$\Delta e_{ij} = e^0_{ij} - e_{ij} \ll e^0_{ij}, \qquad \Delta\sigma_{ij} = \sigma^0_{ij} - \sigma_{ij} \ll \sigma^0_{ij}.$$

The viscoelastic orthotropic law for elements in plane stress can be written

$$\begin{aligned}
\Delta\sigma_{11}(t) &= b_{11}\Delta e_{11}(t) - b_{11}\int_0^t \Delta e_{11}(t_1)R_{11}(t,t_1)dt_1 + \\
&\quad + b_{12}\Delta e_{22}(t) - b_{12}\int_0^t \Delta e_{22}(t_1)R_{12}(t,t_1)dt_1, \\
\Delta\sigma_{22}(t) &= b_{21}\Delta e_{11}(t) - b_{21}\int_0^t \Delta e_{11}(t_1)R_{21}(t,t_1)dt_1 + \\
&\quad + b_{22}\Delta e_{22}(t) - b_{22}\int_0^t \Delta e_{22}(t_1)R_{22}(t,t_1)dt_1, \\
\Delta\sigma_{12}(t) &= 2b^*\Delta e_{12}(t) - 2b^*\int_0^t \Delta e_{12}(t_1)R_{12}(t,t_1)dt_1
\end{aligned} \tag{3.1}$$

or in more general form:

$$\Delta\sigma_{ij}(t) = b_{ijmn}\Delta e_{mn}(t) - b_{ijmn}\int_0^t \Delta e_{mn}(t_1)R_{ijmn}(t,t_1)dt_1 \tag{3.2}$$

$$(i = 1,2; \quad j = 1,2)$$

where

$$\begin{aligned}
b_{11} &= \frac{E_1}{1-\vartheta_1\vartheta_2}; \quad b_{12} = b_{21} = \frac{\vartheta_1 E_2}{1-\vartheta_1\vartheta_2} = \frac{\vartheta_2 E_1}{1-\vartheta_1\vartheta_2}; \\
b_{22} &= \frac{E_2}{1-\vartheta_1\vartheta_2}; \quad 2b^* = G_{12} = \frac{E_{45^\circ}}{2(1+\vartheta_{45^\circ})}
\end{aligned} \tag{3.3}$$

$R_{ijmn}(t,t_1)$ - relaxation kernel assumed isotropic.

Expanding $\Delta e_{mn}(t_1)$ in (3.2) into a series and using definition of pseudo-bifurcation of N-degree we obtain "elastic analogy" as follows:

$$\Delta \overset{(N)}{\sigma}_{ijmn} = \widetilde{b}_{ijmn}\Delta \overset{(N)}{e}_{mn}$$

where

$$\widetilde{b}_{ijmn} = b_{ijmn}\Big[1 - \frac{1}{N!}\int_0^t (t_1 - t)^N \overset{(N)}{R}_{ijmn} dt_1\Big]. \tag{3.4}$$

In [6] the critical shear stress of elastic orthotropic square plate $(a \times a)$ with fixed edges in the condition:

$D_{11} = 10D_{22};\ \ D_{12} + 2D_{66} = 1.67D_{22}$ is determined:

$$\tau_{cr} = 503.6 \cdot \frac{D_{22}}{a^2} = 42\frac{\delta^3}{a^2}b_{22} \tag{3.5}$$

where

$$D_{22} = \frac{\delta^3}{12} \cdot b_{22}.$$

According to the criterion of creep stability $(N = 2)$ from (3.4) we have

$$\widetilde{b}_{22} = b_{22}\Big[1 - \frac{1}{2}\int_0^t (t_1 - t)^2 \ddot{R}(t, t_1) dt_1\Big]. \tag{3.6}$$

Substituting $\widetilde{b}_{22}$ into b_{22} of (3.5) we can get

$$\omega = \frac{\tau^*}{\tau_{cr}} = 1 - \frac{1}{2}\int_0^t (t_1 - t)^2 \ddot{R}(t, t_1) dt_1. \tag{3.7}$$

If we express the curve of stress relaxation in the form

$$\frac{\sigma}{\sigma_0} = 1 - \gamma(1 - e^{-\alpha t}) \tag{3.8}$$

where γ, α - material constants, σ_0 - initial stress,

then relaxation kernel will be:

$$R(t, t_1) = \gamma\alpha e^{-\alpha t} \qquad (0 < \gamma < 1, \quad 0 < \alpha < 1). \tag{3.9}$$

Substituting $R(t, t_1)$ from (3.9) into (3.7) we get

$$\omega = \frac{\tau^*}{\tau_{cr}} = 1 - \gamma\Big[1 - e^{-\alpha t}(1 + \alpha t + \frac{\alpha^2 t^2}{2})\Big]. \tag{3.10}$$

Let us consider a plate made of polymer with $\gamma = 0.5$, $\alpha = 0.3$. According to the formula (3.10) we can plot the curve $\omega \sim t$, (Fig. 2).

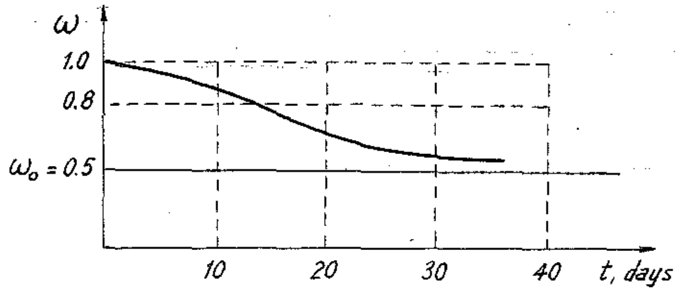


Fig. 2

Consequently, if $\omega < \omega_0$ the plate is stable for all the time, if $\omega_0 < \omega$ the loss of stability is in determined time. $\omega = \omega_0$ - the limit of long time stability, $\omega_0 = 1 - \gamma$.

4. Conclusion

Using the theory of pseudo-bifurcation points and new criterion of creep stability the authors have solved the problem of stability of viscoelastic (isotropic and orthotropic) plate in shear and obtained the analytic formula of critical time.

In choosing different kernels the similar type of relation $\omega \sim t_{cr}$ is determined.

The difference is only in the formula of critical force by Euler.

In relation $\omega \sim t_{cr}$ let $t \to \infty$, we can find the limit of long time stability. If stress is smaller than this limit the equilibrium of plate is always stable.

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ỔN ĐỊNH CỦA TẤM ĐÀN NHỚT CHỊU CẮT

Trong bài báo các tác giả đã sử dụng lý thuyết về các điểm phân nhánh giả và phương pháp "tương tự đàn hồi" để giải bài toán ổn định của tấm đàn nhớt (đẳng hướng và trực hướng) chịu cắt. Với cách làm trên đã lập được quan hệ có dạng giống định luật Húc nhưng với mô đun đàn hồi giả tạo, sử dụng kết quả đã tìm được ở trạng thái đàn hồi (theo Euler) để đi tìm thời gian tới hạn ứng với tải trọng tác dụng xác định. Có xét đến các nhân từ biến khác nhau.