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ONE MODIFICATION OF $K - \varepsilon$ TURBULENT MODEL OF TWO-PHASE FLOWS

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ABSTRACT. A modification of k-ε turbulent model is described with application to the numerical investigation of two-phase turbulent jets. In difference of existed two-parameter models, new three-parameter modification have been suggested. In this case an additional equation for transportation of turbulent energy of admixture - k_p is used together with equations of transportation of carrier- phase turbulent energy - k_g and its dissipation - ε. Additional dissipation terms in the above mentioned equations are defined in connection with a new examination of the between-phases interaction forces determining jet flows.

Symbols

$U_{i(i=p,g,o,m)}$ - Axial components of velocity
$V_{i(i=p,g,o,m)}$ - Radial components of velocity
$W_{i(i=p,g,o,m)}$ - Tangential components of velocity
$U'_{i(i=p,g,o,m)}$ - Axial pulsative components of velocity
$V'_{i(i=p,g,o,m)}$ - Radial pulsative components of velocity
$W'_{i(i=p,g,o,m)}$ - Tangential pulsative components of velocity
$\chi_{i(o,m)}$ - Concentrates
x, r, θ - Coordinates of cylindrical coordinates system.
$k_{i(i=p,g)}$ - Kinetic energy of the motion
ε - Dissipative velocity of the motion
p, g, o, m - Symbols of admixture; gas; in the initial section; max. values
Re, Sc - Reynold and Smidth numbers
$\nu_{i(i=p,g)}$ - Turbulent viscosity
R_i - Richarson's number
C_R - Coefficient of aerodynamic resistance
$\sigma_{i(i=k,\varepsilon)}$ - Empirical constant.

1. Introduction

A numerical modeling of two-phase turbulent jets by using $k - \varepsilon$ turbulent model have been done 1^{st} time by S. E. Elghobashi and T. W. Abou-Arab in [1-3] and the method have been developed by L. B. Gavin, V. A. Naumov, V. V. Shor in [4-8]. In these works to close the system equations of motion have been used two equations - for transportation of carrier-phase turbulent energy - k_g and its' dissipation - ε.

In our works at the beginning of 90's [9, 10] have done modification of the model by introducing an additional equation for transportation of turbulent energy of 2^{nd} phase - admixture - k_p with the purpose to explain "two-fluid" character of the flow in turbulent model. The main concept is that the energy, lessened in admixture phase (particle) is transported into carrier (usually gas) phase. The model has not included an additional equation of velocity of turbulent dissipation of admixture because it can be considered by additional component in the equation of velocity of turbulent dissipation of carrier phase, same as performed in [11].

In this work for closing the system equations of motion the authors demonstrated a perfected modification of k-ε model, called k_g-k_p-ε model (or three-parameter model) with formulation of k_p equation. An additional dissipated components are included, taking in consideration components of between-phases interaction forces, which are characterized for a two-phase turbulent jet and the numerical application in investigation of swirling two-phase turbulent jet.

2. k_g-k_p-ε Model equations

Equations for transportation of turbulent (fluctuated) kinetic energy of carrier (gas) phase and of admixture have been received by developing equations in [1, 3] and considering some modification according to the theory of flow's boundary layer.

An equation for transportation of turbulent kinetic energy of carrier (gas) phase for swirling two-phase turbulent jet has a from as below:

$$U_g \frac{\partial k_g}{\partial x} + V_g \frac{\partial k_g}{\partial r} = \frac{\partial}{r \partial r}\left[\frac{r \nu_{tg}}{\sigma_k}\left(\frac{\partial k_g}{\partial r}\right)\right] + \nu_{tg}\left[\frac{\partial U_g}{\partial r}\right]^2 + \nu_{tg}\left[\frac{r\partial}{\partial r}\left(\frac{W_g}{r}\right)\right]^2 + C_R k_g^{3/2} R_i - \varepsilon - \varepsilon_p. \qquad (2.1)$$

The above said equation is represented for stationary axis-symmetrical two-phase flowing jet with unchanged density & temperature of carrier phase ($T_g =$ const & $\rho_g =$ const). In difference with an usual from for single-phase flowing jets k-ε model, this equation included an additional dissipated component ε_p, which describes two-phase character of the flow - dissipation of turbulent energy under influence of carried phase (admixture).

With the similar procedure for receiving equation for transportation turbulent energy (fluctuation) of carrier-phase,the authors come to the equation for transportation turbulent energy of 2^{nd} phase - admixture [9], taking into account some modifications: the first is: exclusion matrix of internal tensions - admitting that a dissipation of frictional tension is missed (this error can be compensated by frictional components in equation (2.1)). The second is concerning to an additional dissipated component ε_p^*, this component has a plus sign (+) because ε_p^* led to generating turbulent energy of admixture from the above of carrier-phase. So, the equation for transportation turbulent energy of admixtures in swirling two-phase turbulent jet is:

$$U_p\frac{\partial k_p}{\partial x}+V_p\frac{\partial k_p}{\partial r}=\frac{\partial}{r\partial r}\left[\frac{r\nu_{tp}}{\sigma_k}\left(\frac{\partial k_p}{\partial r}\right)\right]+\nu_{tp}\left[\frac{\partial U_p}{\partial r}\right]^2$$
$$+\nu_{tp}\frac{r\partial}{\partial r}\left[\left(\frac{W_p}{r}\right)\right]^2+C_R k_p^{3/2}R_i+\varepsilon_p^* \qquad (2.2)$$

The components of the right side of equation (2.2) are: the 1^{st} is described for diffusive transportation of pulsated energy; the 2^{nd} determines turbulent energy's generating; the third is an additional dissipated component, through it can be considered influence of carrier-phase (gas) on the turbulent energy of admixture.

The same as in "two-parameter" model of turbulence, applied for two-phase flows [1-8], we used an equation for velocity of dissipation of turbulent pulsative energy. The from of that equation for swirling isothermal stationary axis-symmetrical two-phase flowing jets is as below:

$$U_g\frac{\partial \varepsilon}{\partial x}+V_g\frac{\partial \varepsilon}{\partial r}=\frac{\partial}{r\partial r}\left[\frac{r\nu_t}{\sigma_\varepsilon}\left(\frac{\partial \varepsilon}{\partial r}\right)\right]-\Phi_p+C_{\varepsilon_1}\nu_{tg}\frac{\varepsilon}{k_g}\left[\frac{\partial U_g}{\partial r}\right]^2$$
$$+C_{\varepsilon_1}\frac{\varepsilon}{k_g}\left[\frac{r\partial}{\partial r}\left(\frac{W}{r}\right)\right]^2-\frac{\varepsilon^2}{k_g}(C_{\varepsilon_2}-C_{\varepsilon_3}\chi_m) \qquad (2.3)$$

The 2^{nd} on the right side of the equation (2.3) is an additional dissipated component Φ_p which described influence of between-phase interaction forces on the equation of turbulent energy's dissipation.

According to Kolmogorov [1], between turbulent friction ν_t and turbulent (pulsative) kinetic energy exists the relation:

$$\nu_t=C_v k^{0.5}L$$

Applied for two-phase turbulent flows, taking into consideration of the two equations for turbulent energy of carrier-phase k_g and of admixture k_p the equations for turbulent friction of two phases can be written as follows:

$$\nu_{tg}=C_v k_g^{0.5}L; \qquad \nu_{tp}=C_v k_p^{0.5}L. \qquad (2.4)$$

A dissipation, occurred in the result of friction influences has the from:

$$\varepsilon = C_D k_g^{0.5} L. \tag{2.5}$$

In the equations (2.4), (2.5) C_v and C_D are empirical constants, L is a turbulent micro-ratio, proportional to radial dimension - of the flow, for example the relation between L and difference of radial coordinates, where the carrier-phase velocities are 0.9 and 0.1 of its' max. values respectively in the same profile is:

$$L = C_\lambda (r_{0.1} - r_{0.9}). \tag{2.6}$$

The empirical constants in the model are: $C_v = 0.09$; $C_D = 1$; $C_\lambda = 0.625$ and others are given in the table 1:

Table 1

C_v	C_{ε_1}	C_{ε_2}	C_{ε_3}	σ_k	σ_ε
0.009	1.44	1.92	0.8	1.0	1.3

An additional dissipated component ε_p is determined by the method described in [8] as follows:

$$\varepsilon_p = \frac{1}{\rho_g} \sum \overline{F_i' V_{gi}'}. \tag{2.7}$$

This component ε_p considers an interaction of admixture on turbulent energy of carrier-phase, while taking into account and pulsation of between-phase interaction forces f_i, which are: resistive force f_A; Magnus's force f_M, Saffman's force f_s for two-phase turbulent flows and in addition the forces of rotation f_w and f_{pw} for swirling two-phase turbulent jet. So, the additional dissipated component will be as below:

$$\varepsilon_p = \varepsilon_{pA} + \varepsilon_{pM} + \varepsilon_{pS} \text{ for Two-phase Turbulent Flows and} \tag{2.8}$$
$$\varepsilon_p = \varepsilon_{pA} + \varepsilon_{pM} + \varepsilon_{pS} + \varepsilon_{pW} + \varepsilon_{ppW} \text{ for Swirling Two-phase Turbulent Flows}$$

where, an additional dissipated component ε_{pA} from resistive force f_A has a form:

$$\varepsilon_{pA} = \frac{1}{\rho_g} \sum \overline{F_{Ai}' V_{gi}'} = \frac{2\rho_p \beta k_g}{\rho_g} \left[1 - \exp\left(\frac{-B}{\beta \tau_t}\right)\right] \tag{2.9}$$

$\tau_t = (B_g/\varepsilon)^{0.5}$ - timing micro-ratio, B - empirical constant ($B = 0.3$), $k_g = 0.5 V_g^2$. An additional dissipated component from Magnus's force is determined from the equation:

$$\varepsilon_{pM} = \frac{1}{\rho_g} \sum \overline{F_{Mi}' V_{gi}'} = \frac{2\rho_p k_g \lambda_w}{\rho_g} \left[1 - \exp\left(\frac{-B}{\beta \tau_t}\right)\right] \left[\frac{W_p}{r} + 0.5 \frac{\partial U_g}{\partial r}\right] \tag{2.10}$$

An additional dissipated component from Saffman's force after re-working out becomes:

$$\varepsilon_{pS} = \frac{1}{\rho_g}\sum \overline{F'_{Si}V'_{gi}} = 2k_g K_{s1}\left[1-\exp\left(\frac{-B}{\beta\tau_t}\right)\right]\sqrt{\frac{\partial U_g}{\partial r}} \tag{2.11}$$

For swirling two-phase turbulent jet it is necessary to add an additional dissipated components ε_{pW} and ε_{ppW}, which are respectively [9]:

$$\begin{aligned}
\varepsilon_{pW} &= \frac{1}{\rho_g}\sum \overline{F'_{Wi}V'_{gi}} = \frac{2\sqrt{2}\rho_p k_W k_p\sqrt{k_g}}{3\rho_g}\\
\varepsilon_{pWW} &= \frac{1}{\rho_g}\sum \overline{F'_{Wi}V'_{gi}} = \frac{2\sqrt{2}k_{pW}k_g^{1.5}}{3}\\
k_p &= 0.5\sum \overline{V'^{\,2}_{pi}}; \quad k_g = 0.5\sum \overline{V'^{\,2}_{gi}}.
\end{aligned} \tag{2.12}$$

Same as for the carrier-phase, the additional dissipated components in equations for turbulent energy of admixture (particles) will be as flows [9]:

$$\begin{aligned}
\varepsilon^*_{pA} &= \frac{1}{\rho_g}\sum \overline{F'_{Ai}V'_{pi}} = 2\beta k_g\left[\exp\left(\frac{-B}{\beta\tau_t}\right)-\frac{k_p}{k_g}\right]\\
\varepsilon^*_{pM} &= \frac{1}{\rho_g}\sum \overline{F'_{Mi}V'_{pi}} = \frac{2\rho_p k_g\lambda_w}{\rho_g}\left[\exp\left(\frac{-B}{\beta\tau_t}\right)-\frac{k_p}{k_g}\right]\left[\frac{W_p}{r}+0.5\frac{\partial U_g}{\partial r}\right]\\
\varepsilon^*_{pW} &= \frac{1}{\rho_g}\sum \overline{F'_{Wi}V'_{pi}} = \frac{2\sqrt{2}k_{pW}k_p^{1.5}}{3}\ .
\end{aligned} \tag{2.13}$$

General from of additional dissipated components ε^*_p in the equation for turbulent energy of admixture will have a from:

$$\varepsilon^*_p = \varepsilon^*_{pA} + \varepsilon^*_{pM} + \varepsilon^*_{pW}. \tag{2.14}$$

An additional dissipated component Φ_p in the equation for velocity of turbulent energy's dissipation is determined by the function [8]

$$\Phi_p = 2\frac{\nu}{\rho_g}\sum_i\sum_k \overline{\left(\frac{\partial F'_i}{\partial x_i}\right)\left(\frac{\partial V'_{gi}}{\partial x_k}\right)} \tag{2.15}$$

The same as an additional dissipated component in the equation for turbulent energy ε^*_p, the values of Φ_p and its' components Φ_{pA}, Φ_{pM}, Φ_{pS}, Φ_{pW}, ϕ_{ppW} can be determined by following equation [8]:

$$\Phi_p = \Phi_{pA} + \Phi_{pM} + \Phi_{pS} + \Phi_{pW} + \Phi_{ppW}$$
$$\Phi_{pM} = 2\beta \frac{\rho_p}{\rho_g} \frac{C_{t1}\varepsilon}{C_{t1} + \beta T_E} + \frac{2\beta\nu C_{i1}}{\rho_g(C_{t1} + \beta T_E)} \sum_j \frac{\partial \rho_p}{\partial x_j} \frac{\partial k_p}{\partial x_j}$$
$$\Phi_{pA} = 2\beta \frac{\rho_p}{\rho_g} \frac{C_{t1}\varepsilon}{C_{t1} + \beta T_E} \tag{2.16}$$
$$\Phi_{pS} = 2K_{S1}\sqrt{\frac{\partial U_g}{\partial r}} \frac{C_{t1}\varepsilon}{C_{t1} + \beta T_E}$$
$$\Phi_{pW} = \frac{4\sqrt{2}}{3} \frac{\rho_p}{\rho_g} \frac{K_W \sqrt{k_p}\, \beta\varepsilon T_E}{C_{t1} + \beta T_E}$$

where, the constants are defined as: $T_E = \lambda_E / U_g$ - integral-ratio on the time, $\lambda_E = C_{t2} L$ - integral-ratio on distance, $C_{t2} = 0.575$, $C_{t1} = 1.48 \div 1.6$

3. System equations & numerical results of swirling two-phase turbulent jet

The k_g-k_p-ε model is used by authors for closing system equation, describing the motion of the swirling axis-symmetrical two-phase turbulent jet on the basis of two-fluid scheme, which in dimensionless form are as below:

$$\frac{\partial(\bar{r}\,\overline{U}_g)}{\partial \bar{x}} + \frac{\partial(\bar{r}\,\overline{V}_g)}{\partial \bar{r}} = 0 \tag{3.1}$$
$$\frac{\partial(\bar{r}\,\overline{U}_p)}{\partial \bar{x}} + \frac{\partial(\bar{r}\,\overline{V}_p)}{\partial \bar{r}} = 0 \tag{3.2}$$

$$\frac{\partial \chi}{\partial \bar{x}}(\bar{r}\,\overline{U}_p) + \frac{\partial \chi}{\partial \bar{r}}(\bar{r}\,\overline{V}_p) = \frac{1}{\rho_g} \frac{\partial}{\partial \bar{r}}\Big(\rho_g \bar{r} \frac{\overline{\nu}_{tp}}{S_c} \frac{\partial \chi}{\partial \bar{r}}\Big) \tag{3.3}$$
$$\frac{\partial \overline{U}_g}{\partial \bar{x}}(\bar{r}\,\overline{U}_g) + \frac{\partial \overline{U}_g}{\partial \bar{r}}(\bar{r}\,\overline{V}_g) + \frac{\partial \overline{P}}{\partial \bar{x}} = \frac{1}{\rho_g} \frac{\partial}{\partial \bar{r}}\Big(\rho_g \bar{r} \frac{\overline{\nu}_{tg}\,\partial \overline{U}_g}{\partial \bar{r}}\Big) - \overline{F}_x \bar{r} \tag{3.4}$$
$$\frac{\partial \overline{U}_P}{\partial \bar{x}}(\bar{r}\,\overline{U}_p) + \frac{\partial \overline{U}_p}{\partial \bar{r}}\Big(\bar{r}\,\overline{V}_p \frac{\overline{\nu}_{tp}\partial \chi}{S_c\,\partial \bar{r}}\Big) = \frac{1}{\rho_p} \frac{\partial}{\partial \bar{r}}\Big(\rho_p \bar{r} \frac{\overline{\nu}_{tp}\,\partial \overline{U}_p}{\partial \bar{r}}\Big) + \overline{F}_x\,\bar{r} \tag{3.5}$$
$$\frac{\partial \overline{W}_g}{\partial \bar{x}}(\bar{r}^2\,\overline{U}_g) + \frac{\partial \overline{W}_g \bar{r}}{\partial \bar{r}}(\bar{r}\,\overline{V}_g) = \frac{1}{\rho_g} \frac{\partial}{\partial \bar{r}}\Big(\rho_g \bar{r}^2 \overline{\nu}_{tg}\big(\frac{\partial \overline{W}_g}{\partial \bar{r}} - \frac{\overline{W}_g}{\bar{r}}\big)\Big) \tag{3.6}$$
$$\frac{\partial \overline{W}_p}{\partial \bar{x}}(\bar{r}^2\,\overline{U}_p) + \frac{\partial \overline{W}_p \bar{r}}{\partial \bar{r}}\Big(\bar{r}\,\overline{V}_p \frac{\overline{\nu}_{tp}}{S_c} \frac{\partial \chi}{\partial \bar{r}} \bar{r}\Big) =$$
$$= \frac{1}{\rho_p} \frac{\partial}{\partial \bar{r}}\Big(\rho_p \bar{r}^2 \overline{\nu}_{tp}\big(\frac{\partial \overline{W}_p}{\partial \bar{r}} - \frac{\overline{W}_p}{\bar{r}}\big)\Big) \tag{3.7}$$

$$\frac{\partial \overline{P}}{\partial \overline{x}} = \rho_g \frac{\overline{W}_g^{\,2}}{\overline{r}} + F_y, \tag{3.8}$$

$$\frac{\partial \overline{k}_g}{\partial \overline{x}}(\overline{r}\,\overline{U}_g) + \frac{\partial \overline{k}_g}{\partial \overline{r}}(\overline{r}\,\overline{V}_g) = \frac{1}{\rho_g}\frac{\partial}{\partial \overline{r}}\Big(\rho_g \overline{r}\frac{\overline{\nu}_{tg}}{\sigma_k}\frac{\partial \overline{k}_g}{\partial \overline{r}}\Big) +$$

$$+ \overline{\nu}_{tg}\Big(\frac{\partial \overline{U}_g}{\partial \overline{r}}\Big) + C_R \overline{k}_g^{3/2} R_i - \overline{\varepsilon} - \overline{\varepsilon}_p^*, \tag{3.9}$$

$$\frac{\partial \overline{k}_p}{\partial \overline{x}}(\overline{r}\,\overline{U}_p) + \frac{\partial \overline{k}_p}{\partial \overline{r}}\Big(\overline{r}\,\overline{V}_p \frac{\overline{\nu}_{tp}}{S_c}\frac{\partial \overline{k}_p}{\partial \overline{r}}\overline{r}\Big) = \frac{1}{\rho_p}\frac{\partial}{\partial \overline{r}}\Big(\rho_p \overline{r}\frac{\overline{\nu}_{tp}}{\sigma_k}\frac{\partial \overline{k}_p}{\partial \overline{r}}\Big) +$$

$$+ \overline{\nu}_{tp}\Big(\frac{\partial \overline{U}_p}{\partial r}\Big) + C_R \overline{k}_p^{3/2} R_i + \overline{\varepsilon}_p^*, \tag{3.10}$$

$$\frac{\partial \overline{\varepsilon}}{\partial \overline{x}}(\overline{r}\,\overline{U}_g) + \frac{\partial \overline{\varepsilon}}{\partial \overline{r}}(\overline{r}\,\overline{V}_g) = \frac{1}{\overline{\rho}_g}\frac{\partial}{\partial r}\Big(\rho_g \overline{r}\frac{\overline{\nu}_{tg}}{\sigma_\varepsilon}\frac{\partial \overline{\varepsilon}}{\partial \overline{r}}\Big) + C_{\varepsilon 1}\overline{\nu}_{tg}\frac{\overline{\varepsilon}}{\overline{k}_g}\Big(\frac{\partial \overline{U}_g}{\partial \overline{r}}\Big)^2$$

$$+ C_{\varepsilon_1}\frac{\overline{\varepsilon}}{\overline{k}_g}\Big[\overline{r}\frac{\partial}{\partial \overline{r}}\Big(\frac{\overline{W}_g}{\overline{r}}\Big)\Big]^2 - C_{\varepsilon 2}\frac{\overline{\varepsilon}^2}{\overline{k}_g} - \overline{\Phi}_p, \tag{3.11}$$

where: $\overline{x} = x/r_0$; $\overline{U}_i = U_i/U_0$; $\overline{V}_i = V_i/V_0$; $\overline{k}_g = k_g/U_0^2$; $\overline{k}_p = k_p/U_0^2$, $\overline{P} = P/(\rho_g U_0^2)$; $\overline{W}_i = W_i/W_0$; $\overline{\nu}_{ti} = \nu_{ti}/(U_0 r_0)$; $\overline{\varepsilon} = \varepsilon r_0/U_0^3$; $\overline{\Phi}_p = \Phi_p r_0^2/U_0^4$; $\overline{F} = F r_0/(\rho_g U_0^2)$

Boundary Conditions:

- In the axis of the flow ($r = 0$)

$$\frac{\partial \overline{U}_g}{\partial \overline{r}} = \frac{\partial \overline{U}_p}{\partial \overline{r}} = 0, \quad \frac{\partial \chi}{\partial \overline{r}} = 0; \quad \frac{\partial \overline{k}_p}{\partial r} = \frac{\partial \overline{k}_g}{\partial \overline{r}} = \frac{\partial \overline{\varepsilon}}{\partial \overline{r}} = 0$$

$\overline{W}_p = \overline{W}_g = 0$; $\overline{V}_p = \overline{V}_g = 0$

- On the boundary layer ($U_i = 0$)

$\overline{W}_g = \overline{W}_p = 0$; $\overline{U}_p = \overline{U}_g = 0$; $\overline{k}_p = \overline{k}_g = 0$, $\overline{\varepsilon} = \overline{P} = 0$.

Initial Values:

$U_{i0} = U_0$; $\chi_0 = \rho_p/\rho_g$; $k_{i0} = (0.1U_{i0})^2$; $V_{i0} = 0$; $\varepsilon_0 = C_D k_{g0}^{1.5}/L$; $W_0 = S_0 r_0$;

where S_0 is initial swirling factor.

The system of 11 equations with 11 unknowns $(\overline{U}_i, \overline{V}_i, \overline{W}_i, \overline{k}_{i(i=p,g)}; \overline{\varepsilon}; \chi, \overline{P})$ is numerically solved with finite difference method using Duifort-Frankel differential scheme [9].

On Fig. 1 showing the numerical results of U_{gm} and U_{pm}, receiving by solving before said system equation for the case: $S_0 = 0$, $\chi_0 = 0.5\&1.0$; $D_p = 50\,\mu m$; $U_0 = 19\&21$ m/s, comparison with experiments of Shriber [8]. The prediction and experiments show good agreement, which illustrates the capability of the model in predicting complex flows.

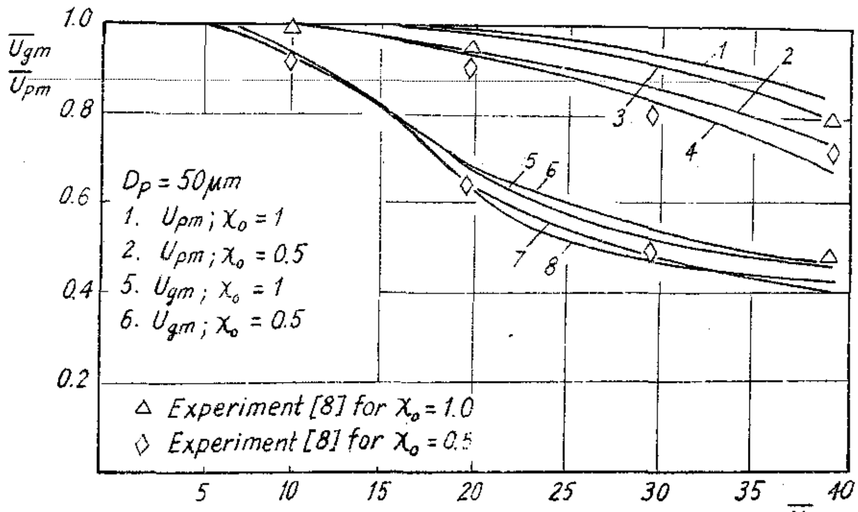

Fig. 1

Conclusion

The k_g-$\dot{k}_p$-ε model proposed in this paper shows that from physical point of view the 2^{nd} phase (admixture) in two-phase flows can be considered as independent phase with own characters and the relation between two phases is existed through between-phases interaction forces. The numerical computation can be used for predicting complex two-phase flows.

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VỀ MỘT SỰ BIẾN DẠNG CỦA MÔ HÌNH K-ε
ĐỐI VỚI DÒNG CHẢY RỐI HAI PHA

Sự biến dạng mô hình K-ε của sự rối, được xem xét, ứng dụng trong việc nghiên cứu dòng phun rối hai pha. Sự khác nhau đối mô hình 2 thông số đang tồn tại là ở đây được đưa vào sự biến dạng với mô hình 3 thông số mới. Trong đó ngoài 2 phương trình về năng lượng mạch động của pha kéo theo K_g và sự tản mạn của nó ε, được đưa vào phương trình bổ sung về năng lượng rối của pha tạp chất - K_p. Những thành phần tản mạn bổ sung ở các phương trình trên được xác định trên cơ sở về một cách nhìn nhận mới đối với việc xác định các lực trong dòng phun của sự tương tác giữa các pha với nhau.