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NUMERICAL SIMULATION OF FLUID MUD LAYER IN ESTUARIES AND COASTAL AREAS

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ABSTRACT. In this paper the formation and development of fluid mud layer happening in estuaries and coastal areas are studied in detail through numerical simulation on the basis of the 2D shallow water equations for tidal flow, the advection-diffusion equation for cohesive sediment transport and the equations for fluid mud transport. Numerical solution of a special case for a part of the Severn estuary is obtained using the finite difference method as an illustration of the applicability of the model in practice. On the basis of the results, initial remarks and evaluation are given.

1. Introduction

In estuaries and coastal regions where there is a high concentration of sediment in suspension, a fluid mud layer is often formed during slack water periods by the process of hindered settling. This amount of sediment comes from the sea or from rivers due to the process of flowing through many areas in a country or around the sides of mountains. Experiments have established the relationship between settling velocity and cohesive sediment concentration (Tsuruya, Murakami and Irie[1]) and it is seen that a peak value of settling velocity occurs at a concentration of approximately $5kg/m^3$. Once the near bed sediment concentration exceeds this value, mud settles towards the bed more quickly than it can dewater and a layer of fluid mud forms. The movement of the fluid mud layer can be described by a restricted form of the shallow water equations. Many complicated physical processes occur at the interface between sediment in suspension and fluid mud and between fluid mud and the rigid bed. These are represented in the model in a parameterised way.

Fluid mud can also be formed by waves which can fluidise a muddy bed. However, in this paper, the study is restricted to the case of calm conditions, with a very high cohesive sediment concentration in suspension. These conditions are typical in the Severn estuary during a spring tide. The study is focused on the formulation of the mathematical model and the numerical simulation of the model on a computer. The functions describing the processes of exchange between suspended sediment, fluid mud and the bed are explained and the results of the application of the model to the Severn estuary are presented. Quantitative measurements of fluid mud in the Severn were not available but the behaviour of the model fits well with the description of fluid mud in the Severn given by Kirby and Parker[2]. It is planned to further develop the model to include the effect of waves.

2. The governing equations and boundary conditions

As well known, the most popular assumption used in the mathematical models up to now is that the effect of the bed changing in respect to time on hydrodynamics process is ignored. This is because this effect is insignificant in comparison with the other factors when the average sediment concentration is not large enough. Therefore the mathematical model describing this phenomenon is divided into two separate models: The model of tidal current and the model of sediment transport. The tidal model is the 2D horizontal shallow water equations without the force of wind(Muir Wood and Fleming [3]),

$$\frac{\partial z}{\partial t} + \frac{\partial (du)}{\partial x} + \frac{\partial (dv)}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial z}{\partial x} + \Omega v + D\Big(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\Big) - gu\frac{\sqrt{u^2+v^2}}{C^2 d} \tag{2.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial z}{\partial y} - \Omega u + D\Big(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\Big) - gv\frac{\sqrt{u^2+v^2}}{C^2 d} \tag{2.3}$$

in which x, y are the coordinates, the x axis goes along the shore and the y axis perpendicular to the shore, u and v are the tidal flow velocity components along the x and y directions, respectively, z is the water level above the chart datum, g-acceleration due to gravity, C-Chezy coefficient, d-total water depth, Ω-Coriolis parameter, D-the eddy viscosity coefficient, and t-time. It should be noted that the terms in the right hand sides of the equations (2.2) and (2.3) present the forces due to the surface slope, rotating of the earth, turbulent diffusion and the friction on the bed in the x and y directions, respectively, the scales of which depend on the situations in question.

The equation describing sediment transport is the advection-diffusion equation based on the sediment mass conservation with the exchange between sediment in suspension and fluid mud or mud on the bed taken into account (Roberts [4], Odd and Cooper [5], Odd and Rodger [6], and Le Hir and Kalikow [7]) as follows,

$$\frac{\partial(dc)}{\partial t}+\frac{\partial(q_x c)}{\partial x}+\frac{\partial(q_y c)}{\partial y}=\frac{dm}{dt}+\frac{\partial}{\partial x}\left(dE_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(dE_y\frac{\partial c}{\partial y}\right) \tag{2.4}$$

in which c is the mass suspended sediment concentration, q_x, q_y components of discharge per unit of width along the x and y directions, respectively, E_x, E_y the diffusion coefficients for sediment along the x and y, z_b the bed level below the chart datum, ρ_m the mud density, and $\frac{dm}{dt}$ the source-sink term.

The model for fluid mud layer includes the equation of fluid mud mass conservation and the equations of momentum conservation that are the restricted form of the 2D horizontal shallow water equations (Roberts [4]),

$$\frac{\partial}{\partial t}(c_m d_m)+\frac{\partial}{\partial x}(c_m d_m u_m)+\frac{\partial}{\partial y}(c_m d_m v_m)=\frac{dm}{dt} \tag{2.5}$$

$$\frac{\partial u_m}{\partial t}+\frac{1}{d_m\rho_m}\left(\tau_0-\tau_i\right)_x-\Omega v_m+\frac{\rho_w}{\rho_m}g\frac{\partial z}{\partial x}+g\frac{\Delta\rho}{\rho_m}\frac{\partial z_m}{\partial x}+\frac{gd_m}{2\rho_m}\frac{\partial\Delta\rho}{\partial x}=0 \tag{2.6}$$

$$\frac{\partial v_m}{\partial t}+\frac{1}{d_m\rho_m}\left(\tau_0-\tau_i\right)_y+\Omega u_m+\frac{\rho_w}{\rho_m}g\frac{\partial z}{\partial y}+g\frac{\Delta\rho}{\rho_m}\frac{\partial z_m}{\partial y}+\frac{gd_m}{2\rho_m}\frac{\partial\Delta\rho}{\partial y}=0 \tag{2.7}$$

in which c_m is the mass concentration of fluid mud, in general a function of time and space, d_m is the fluid mud depth, u_m and v_m are the fluid mud velocity components in the x and y directions, respectively, z_m-the elevation of the interface between fluid mud and sediment in suspension, τ_0- and τ_i-the shear stress vectors on the bed and on the interface, respectively, ρ_w-the water density, and ρ_m-the fluid mud density, the relationship of which with the water density is the following,

$$\rho_m=\rho_w+\Delta\rho, \quad \Delta\rho=0.62c_m. \tag{2.8}$$

From equations (2.6)-(2.7) it should be noted that the external forces making fluid mud move in turn are the shear stress on the bed and on the mud-water interface, the Coriolis force, the slope of water surface, the slope of mud-water interface, and gradient of density that is ignored in the study.

About the source-sink term presenting the exchange at the bed or at the mud-water interface in the equations (2.4)-(2.5), the following processes are introduced:

* Erosion:

$$\frac{dm}{dt}=m_e\left(\frac{\tau}{\tau_e}-1\right)H(\tau-\tau_e), \quad H(x)=\begin{cases}1, & x>0\\ 0, & x\le 0\end{cases} \tag{2.9}$$

in which $m_e(Kg/N/s)$ is the erosion rate parameter, $\tau(N/m^2)$ is the actual shear stress at the fluid mud-water interface or at the bed-water interface in the absence of fluid mud, τ_e-the critical bed shear stress for erosion, and $H(x)$-the usual Heaviside step function.

* Settling of mud from suspension:

$$\frac{dm}{dt} = v_s(c)c\Big(1 - \frac{\tau}{\tau_d}\Big)H(\tau_d - \tau), \quad v_s(c) = \begin{cases} v_{\min}, & c < \dfrac{v_{\min}}{R_0} \\ cR_0, & c \geq \dfrac{v_{\min}}{R_0} \end{cases} \tag{2.10}$$

where τ_d is the critical bed shear stress for deposition, $v_s(m/s)$-the settling velocity, $v_{\min}$-the minimum settling velocity, and $R_0(m^4/kg/s)$ are given from experiments. This process only occurs on the mud-water interface or on the bed in the case without fluid mud.

* Entrainment:

$$\begin{aligned} &\frac{dm}{dt} = v_e c_m H(10 - R_i), \quad v_e = \frac{0.1\Delta U}{(1 + 63R_i^2)^{3/4}}, \\ &R_i = \frac{\Delta\rho g d_m}{\rho_w \Delta U^2}, \quad \Delta U^2 = (u - u_m)^2 + (v - v_m)^2 \end{aligned} \tag{2.11}$$

where v_e is the entrainment velocity (m/s), and R_i-the bulk Richardson number representing the degree of the flow stratification. Therefore, the entrainment only happens when the stratification of the flow is not strong enough on the water-mud interface.

* Dewatering

$$\frac{dm}{dt} = v_0 c_m H(\tau_d - \tau) \tag{2.12}$$

where v_0 is the dewatering velocity (m/s). This phenomenon only occurs on the bed when fluid mud layer exists and the bed shear stress is less than the critical value τ_d.

As mentioned above, the entrainment is a process easily causing the numerical instability, so it requires to treat carefully.

To close the problem mathematically the initial and boundary conditions for the situation under consideration are required. They are as follows,

* The initial conditions:

Due to the area of interest is not large enough, the initial water surface can be horizontal. The condition for concentration of suspended sediment is subjunctively given so that it should be suitable to the case of mud flow.

$$\begin{aligned} &u(x,y,0) = 0, \quad v(x,y,0) = 0, \quad z(x,y,0) = 12.4 \\ &c(x,y,0) = 5, \quad d_m(x,y,0) = 0, \quad u_m(x,y,0) = 0, \quad v_m(x,y,0) = 0 \end{aligned} \tag{2.13}$$

* The boundary conditions:

The 4 kinds of boundaries that are necessary to consider here in turn are the river boundary ($x = L$), the open sea boundary ($x = 0$), the offshore boundary ($y = 0$) and the land boundary ($(x, y) \in Ln$):

$$\begin{aligned}
&q_{tx}(x,y,t)\big|_{x=L} = f_1(t), \ q_{ty}(x,y,t)\big|_{x=L} = 0, \\
&c(x,y,t)\big|_{x=L} = 5, \quad d_m(x,y,t)\big|_{x=L} = 0, \\
&z(x,y,t)\big|_{x=0} = f_2(t), \quad v(x,y,t)\big|_{x=0} = 0, \\
&\frac{\partial u}{\partial x}\Big|_{x=0} = 0, \quad \frac{\partial c}{\partial x}\Big|_{x=0} = 0, \quad \mathbf{v}\cdot\mathbf{n} = 0, \ (x,y) \in Ln
\end{aligned} \tag{2.14}$$

in which q_{tx}, q_{ty} are the components of the total water discharge vector, $f_i(t)$ ($i = 1, 2$) the given functions at the river boundary, $\mathbf{v}$ the flow velocity vector, and $\mathbf{n}$ the normal vector unit on the land boundary.

3. Numerical solutions and a test application

Owing to the feature of the model, the equations (2.1)-(2.3) together with given boundary and initial conditions that present the tidal flow have been solved numerically firstly. The ADI method was used, with staggered grid for the derivative in respect to space and Leap-frog scheme for time. The corresponding difference equations are given in previous paper (Chung and Roberts [8]). The finite difference method is also used to solve the equations (2.4)-(2.7) together with the initial and boundary conditions (2.13)-(2.14) for the sediment transport model. Specially, QUICKEST (see Leonard [9]) is used for the advection-diffusion equation (2.4) and QUICK [9] for the equation (2.5) to get more accuracy. The difference equations corresponding to the equations (2.4)-(2.7) are the following,

$$\begin{aligned}
&(dc)_{ij}^{n+1} = \\
&(dc)_{ij}^{n} + \frac{\Delta t}{\Delta x}\Big[u_{i-1j}^{n}c_{i-1}^{*} - u_{ij}^{n}c_{i}^{*} - (dE_x)_{i-1j}\Big(\frac{\partial c}{\partial x}\Big)_{i-1}^{*} + (dE_x)_{ij}\Big(\frac{\partial c}{\partial x}\Big)_{i}^{*}\Big] \\
&+ \frac{\Delta t}{\Delta y}\Big[v_{ij}^{n}c_{j}^{*} - v_{ij-1}^{n} - (dE_y)_{ij}\Big(\frac{\partial c}{\partial y}\Big)_{j}^{*} + (dE_y)_{ij-1}\Big(\frac{\partial c}{\partial y}\Big)_{j-1}^{*}\Big] + \Delta t\Big(\frac{dm}{dt}\Big)_{ij},
\end{aligned}$$

$$\begin{aligned}
&\Big[1 + \frac{\Delta t}{d_{mij}^{n}\rho_m}(\tau_0' + \tau_{ix}')\Big]u_{mij}^{n+1} = u_{mij}^{n} + \frac{\Delta t}{d_{mij}^{n}\rho_m}\tau_{ix}'u_{ij}^{n+1} + \Delta t\Omega v_{mij}^{n} \qquad (3.1)\\
&- \frac{\Delta t}{\Delta x}\frac{g\rho_w}{\rho_m}(z_{i+1j}^{n} - z_{ij}^{n}) - \frac{\Delta t}{\Delta x}\frac{g\Delta\rho}{\rho_m}(d_{mi+1j}^{n} - d_{mij}^{n} + udep_{ij} - udep_{i+1j}),\\
&\Big[1 + \frac{\Delta t}{d_{mij}^{n}\rho_m}(\tau_0' + \tau_{iy}')\Big]v_{mij}^{n+1} = v_{mij}^{n} + \frac{\Delta t}{d_{mij}^{n}\rho_m}\tau_{iy}'v_{ij}^{n+1} - \Delta t\Omega u_{mij}^{n} \qquad (3.2)\\
&- \frac{\Delta t}{\Delta y}\frac{g\rho_w}{\rho_m}(z_{ij}^{n} - z_{ij+1}^{n}) - \frac{\Delta t}{\Delta y}\frac{g\Delta\rho}{\rho_m}(d_{mij}^{n} - d_{mij+1}^{n} + vdep_{ij+1} - vdep_{ij}),
\end{aligned}$$

$$
\begin{aligned}
c_m d_{mij}^{n+1} &= c_m d_{mij}^{n} + \frac{\Delta t}{\Delta x} c_m \left(u_{mi-1j}^{n} d_{mi-1}^{*} - u_{mij}^{n} d_{mi}^{*}\right) \\
&+ \frac{\Delta t}{\Delta y} c_m \left(v_{mij}^{n} d_{mj}^{*} - v_{mij-1}^{n} d_{mj-1}^{*}\right) + \Delta t \left(\frac{dm}{dt}\right)_{ij}, \\
& i = 2, I-1, \quad j = 2, J-1
\end{aligned}
\tag{3.3}
$$

ı which,

$$
c_1^{*} = \frac{1}{2}\left(c_{ij}^{n} + c_{i+1j}^{n}\right) - \frac{\Delta x}{2}\sigma_i GRAD_i + \frac{\Delta x^2}{2}\left[\alpha_i - \frac{1}{3}\left(1 - \sigma_i^2\right)\right] CURV_i
$$

$$
\sigma_i = u_{ij}^{n}\frac{\Delta t}{\Delta x}, \quad \alpha_i = \left[\frac{1}{2}\left(z_{ij}^{n} + z_{i+1j}^{n}\right) + udep_{ij}\right] E_x \frac{\Delta t}{\Delta x^2},
$$

$$
GRAD_i = \frac{c_{i+1j}^{n} - c_{ij}^{n}}{\Delta x}, \quad CURV_i = \begin{cases} \dfrac{1}{\Delta x^2}\left(c_{i+1j}^{n} - 2c_{ij}^{n} + c_{i-1j}^{n}\right), & u_{ij}^{n} \geq 0 \\ \dfrac{1}{\Delta x^2}\left(c_{i+2j}^{n} - 2c_{i+1j}^{n} + c_{ij}^{n}\right), & u_{ij}^{n} < 0 \end{cases}
$$

$$
d_{mi}^{*} = \frac{1}{2}\left(d_{ij}^{n} + d_{i+1j}^{n}\right) - \frac{\Delta x^2}{8} CURV_{di},
$$

$$
CURV_{di} = \begin{cases} \dfrac{1}{\Delta x^2}\left(d_{mi+1j}^{n} - 2d_{mij}^{n} + d_{mi-1j}^{n}\right), & u_{mij}^{n} \geq 0 \\ \dfrac{1}{\Delta x^2}\left(d_{mi+2j}^{n} - 2d_{mi+1j}^{n} + d_{mij}^{n}\right), & u_{mij}^{n} < 0 \end{cases}
$$

$$
d_{mj}^{*} = \frac{1}{2}\left(d_{ij}^{n} + d_{ij+1}^{n}\right) - \frac{\Delta y^2}{8} CURV_{dj},
$$

$$
CURV_{dj} = \begin{cases} \dfrac{1}{\Delta y^2}\left(d_{mij}^{n} - 2d_{mij+1}^{n} + d_{mij+2}^{n}\right), & v_{mij}^{n} \geq 0 \\ \dfrac{1}{\Delta y^2}\left(d_{mij-1}^{n} - 2d_{mij}^{n} + d_{mij+1}^{n}\right), & v_{mij}^{n} < 0 \end{cases}
$$

$$
\tau_0' = \rho_m f_m \sqrt{\left(u_{mij}^{n}\right)^2 + \left(v_{mij}^{n}\right)^2}, \quad \Delta u_{ij}^{n} = u_{ij}^{n} - u_{mij}^{n}, \quad \Delta v_{ij}^{n} = v_{ij}^{n} - v_{mij}^{n},
$$

$$
\tau_{ix}' = \frac{f_x \rho_w}{8}\sqrt{\left(\Delta u_{ij}^{n}\right)^2 + \left(\Delta v_{in}^{n}\right)^2}, \quad \tau_{iy}' = \frac{f_y \rho_w}{8}\sqrt{\left(\Delta u_{ij}^{n}\right)^2 + \left(\Delta v_{ij}^{n}\right)^2}, \quad f = \frac{g}{C^2},
$$

where $udep_{ij}$ and $vdep_{ij}$ are the bed elevations below the chart datum corresponding to the positions of u_{ij} and v_{ij}, respectively, and f_m-the friction factor on the bed for mud flow.

The above difference equation systems are solved over a tidal period. The results of computation at two time points $t = 38400s$ and $t = 44400s$ corresponding to before and after high water, respectively (high water at $t = 42000s$), are displayed on Figures (1-4) as the characteristics evaluations over a tidal period. They present the behaviour of the mud velocity field, distributions of fluid mud depths and suspended mud concentration. From here it can be seen that at $t = 38400s$ mud concentration in suspension remains at a level higher than the initial level ($5 kg/m^3$) nearly everywhere due to the effect of high current, so the fluid mud layer only appears in the area near the land boundary. When $t = 44400s$ current

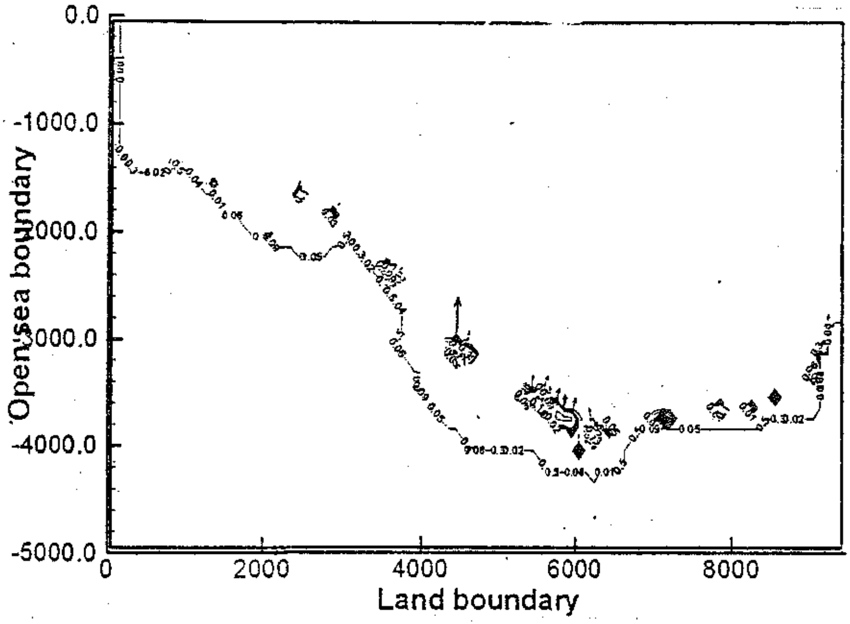

Fig. 1. Contour of fluid mud depth and vector field at $t = 38400$s

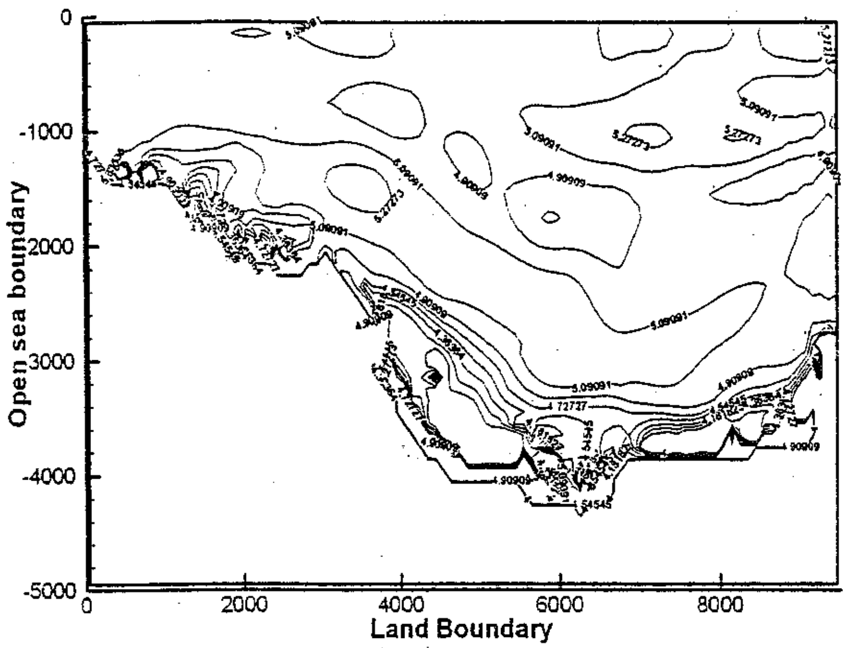

Fig. 2. Contour of sus. sed. concentration at $t = 38400$s

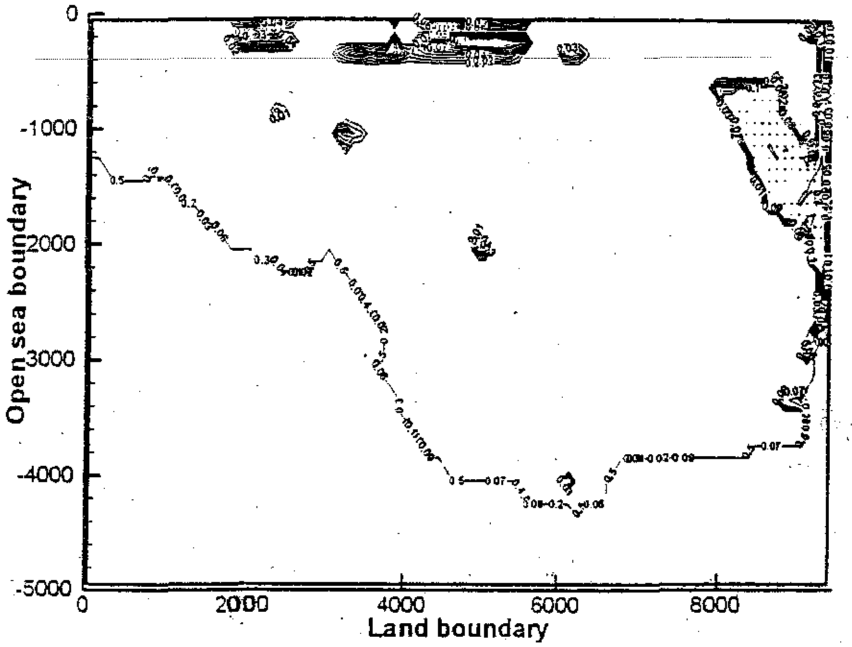

Fig. 3. Contour of fluid mud depth and vector field at $t = 44400$s

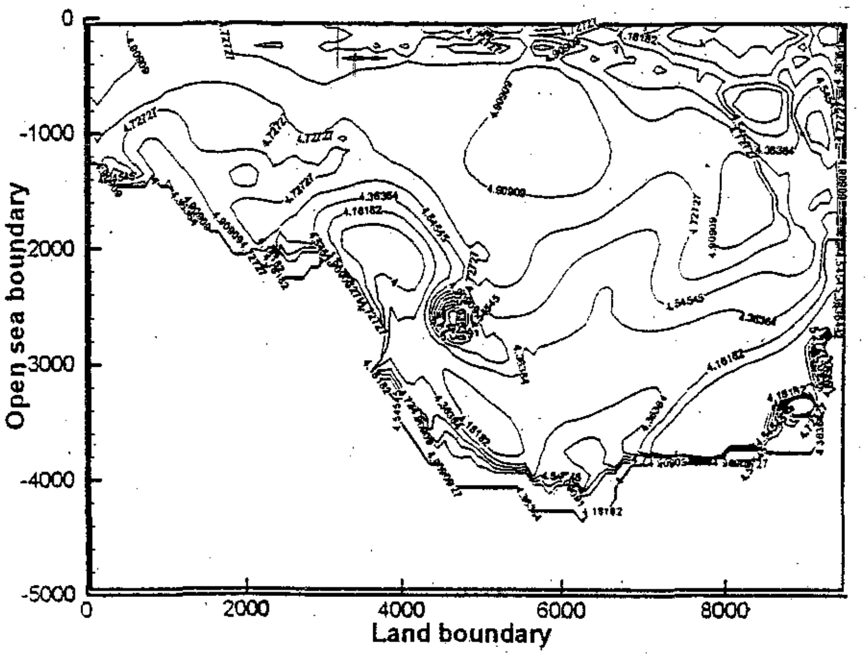

Fig. 4. Contour of sus. sed. concentration at $t = 44400$s

direction changes first in shallow water, because it has less momentum, while the flow velocity in the middle becomes smaller, so the formation of fluid mud appears and settling flux is large mainly in the area far from the shore. It starts to move with very slow velocity as seen from the results, tending to accumulate in the deep channel at slack water. For the case under consideration the maximum mud layer depth is about 0.2m. The peak value of fluid mud layer appears when the water level achieves the minimum value that is explained obviously because of enough small flow velocity. In general the results show a sensible behaviour, compared with the description of mud in the Severn given by Kirby and Parker [5] and show to be applicable in practice.

Conclusion

A numerical simulation for the formation and moving of fluid mud in estuaries and coastal areas is implemented and applied to the Severn estuary as an illustration example. The finite difference method is used, in which ADI method with staggered grid for the derivative in respect to space and Leap-frog scheme for time is used for tidal flow model. Especially, QUICK and QUICKEST schemes with very high accuracy are applied to the equation of fluid mud mass conservation and the equation of advection-diffusion, respectively in the mud transport model. With the grid size of $100m \times 100m$, the computational area consists of 95×50 cells, which is not too coarse and is acceptable in simulation and prediction purposes. For fluid mud model, although no data measurements are available, but the obtained results show a sensible behaviour, compared with the description of mud in the Severn given by Kirby and Parker [2]. Therefore the model shows to be applicable in practice, and that the numerical simulation can be fully carried out on PC.

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MÔ PHỎNG SỐ LỚP BÙN LỎNG VÙNG CỬA SÔNG VÀ VEN BIỂN

Trong bài báo này phương pháp mô phỏng số được sử dụng để nghiên cứu sự hình thành và phát triển của lớp bùn lỏng ở vùng cửa sông và ven biển một cách chi tiết. Các mô hình toán học được sử dụng, đó là các phương trình sóng nước nông 2 chiều ngang đối với dòng triều, phương trình khuyếch tán đối với nồng độ bùn cát lơ lửng và hệ phương trình động lực mô tả quá trình hình thành và phát triển của bùn lỏng. Lời giải số cho một trường hợp cụ thể ở khu vực vùng cửa sông Severn đã được thu nhận nhờ sử dụng phương pháp sai phân hữu hạn, được xem như một minh họa cho tính khả thi của mô hình vào thực tế. Dựa vào những kết quả nhận được một vài nhận xét đánh giá ban đầu được nêu ra.