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ANALYSIS OF DYNAMICS OF AUTOMOBILE CRANES WITH THE ACCOUNT OF THE ROPE ELASTICITY

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ABSTRACT. Up to now, there is still a little of attention paid on the dynamical analysis of automobile crane system and only simple dynamical models are used for the related studies. In this paper, a relatively comprehensive model describing the dynamics of automobile crane with only the bodylifting mechanism taken into account, is presented. The numerical solution has been derived, illustrated by graphics and assessed.

1. Introduction

Automobile cranes are very popular loading-discharging tools. The study on dynamics of their working mechanism, however, is still limited and many related problems have not been solved adequately. In the literature, the models describing dynamics of automobile cranes are still simple [1-3]. The interaction between bodylifting and rodlifting mechanism are not taken into account and the role of the rope elasticity in the working mechanism of the system is not specified in these models. Thus, from the practical point of view, many problems have not been solved, such as how to keep the lifting body from shaking?, how the rod is shaking during the operation of the bodylifting mechanism?, how the movement pattern of the lifting body is when only the rodlifting mechanism works and what effects will be caused due to the elasticity of the rope?,...

In this paper, a more comprehensive model for description of the dynamics of automobile crane in the case when only the bodylifting mechanism works and the numerical method for solving its basic equations are presented.

The obtained results allow to assess the impact of elasticity of the rope on the system, especially the rod vibration even when the rodlifting mechanism does not work, the changes in movement pattern of the body and of the tension of the rope.

2. Model

The bodylifting mechanism is considered as a drum (disk). The model is developed based on the following assumptions:

- The rod is rigid body.
- The lifting body is a particle of the mass m.
- The rope is considered as a massless spring.
- The vibration of lifting body is plane.
- The mass and internal friction of pulleys in polyspacts are neglected.

The real model and its operation scheme of the automobile crane is described in the Fig. 1.

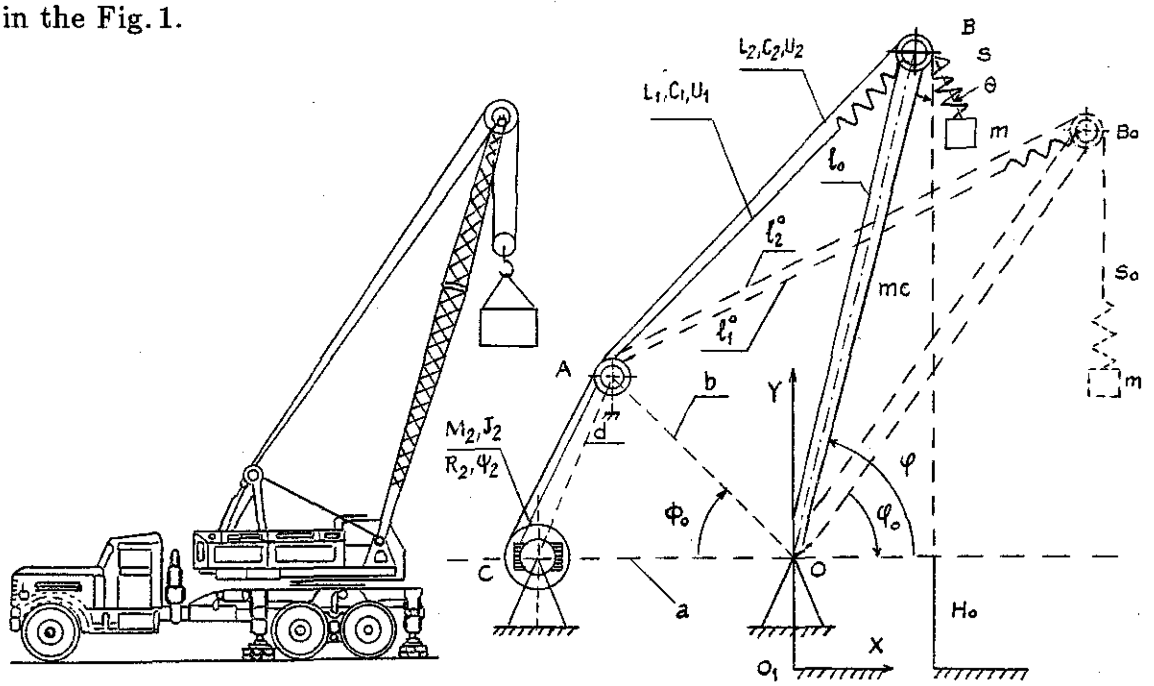

Fig. 1. Real model and operation scheme of the automobile crane

The system has 4 degrees of freedom including:

φ - the shaking angle of the rod from the horizontal plane,

θ - the shaking angle of the lifting body from the vertical,

S - the distance between the lifting body and the rod top, which describes the movement of the lifting along the rope,

ψ_2 - the rotation angle of the rope winding drum.

The dimension parameters of the system are as follows (Fig. 1):

a - the distance between the center of bodylifting drum and the rod foot,

b - the distance between the center of direction changing pulley and the rod foot,

d - the distance between the center of body lifting drum and the direction changing pulley,

R_2 - the radius of the bodylifting drum,

r - the radius of the direction changing pulley.

3. Lagrangian functions

For the model given in the section 2, the Lagrangian function $L = T - \Pi$ can be calculated as follows:

Kinetic energy (K.E.)

$$T = T_m + T_c + T_2,$$

where T_m - K.E. of the body, T_c - K.E. of the rod, T_2 - K.E. of the bodylifting drum, and

$$T_m = \frac{m}{2}\left[\dot{S}^2 + S^2\dot{\theta}^2 + \ell_0^2\dot{\varphi}^2 - 2\ell_0\dot{S}\dot{\varphi}\cos(\theta-\varphi) + 2\ell_0 S\dot{\varphi}\dot{\theta}\sin(\theta-\varphi)\right], \tag{3.1}$$

$$T_c = \frac{1}{2}\left(m_c\frac{\ell_0^2}{4} + J_c\right)\dot{\varphi}^2 \Rightarrow \frac{1}{2}J_c^*\dot{\varphi}^2, \tag{3.2}$$

$$T_2 = \frac{1}{2}J_2\dot{\psi}_2^2. \tag{3.3}$$

Potential energy (P.E.) $\qquad \Pi = \Pi_m + \Pi_c + \Pi_{c1}^* + \Pi_{c2},$

where Π_m - P. E. of the lifting body, Π_c - P.E. of the rod, Π_{c1}^* - P.E. of the rodlifting rope, Π_{c2} - P.E. of the bodylifting rope, and

$$\Pi_m = mgy_m \Rightarrow mg(\ell_0\sin\varphi - S\cos\theta + H_0), \tag{3.4}$$

$$\Pi_c = m_c g y_c \Rightarrow m_c g\left(\frac{1}{2}\ell_0\sin\varphi + H_0\right), \tag{3.5}$$

$$\Pi_{c1}^* = \frac{1}{2}c_1(\Delta L_1^*)^2 \Rightarrow c_1(\Delta L_{1d}^* + \Delta L_{1t})^2, \tag{3.6}$$

$$\Pi_{c2} = \frac{1}{2}c_2(\Delta L_2)^2 \Rightarrow \frac{1}{2}c_2(\Delta L_{2d} + \Delta L_{2t})^2, \tag{3.7}$$

in which

$$\Delta L_{1d}^* = u_1(\ell_{(\varphi)} - \ell_{(\varphi_0)}), \tag{3.8}$$

$$\Delta L_{2d} = u_2(S - S_0) + \ell_{(\varphi)} - \ell_{(\varphi_0)} + R_2(\psi_2 - \psi_2^0), \tag{3.9}$$

$$\ell_{(\varphi_0)} = \sqrt{b^2 + \ell_0^2 + 2b\ell_0\cos(\phi_0 + \varphi_0)}\ , \tag{3.10}$$

$$\ell_{(\varphi)} = \sqrt{b^2 + \ell_0^2 + 2b\ell_0\cos(\phi_0 + \varphi)}\ , \tag{3.11}$$

$$\Delta L_1^* = \Delta L_{1t} + \Delta L_{1d}^* = \Delta_1^0 + u_1[\ell_{(\varphi)} - \ell_{(\varphi_0)}]$$

(Deformation of the rodlifting rope),

$$\Delta L_2 = \Delta L_{2t} + \Delta L_{2d} = \Delta_2^0 + u_2(S - S_0) + \ell_{(\varphi)} - \ell_{(\varphi_0)} + R_2(\psi_2 - \psi_2^0)$$

(Deformation of the bodylifting rope). (3.13)

4. Equations of motion

By applying the Lagrangian equation to the model described above, the following system of differential equations describing the dynamics of automobile crane system when only the rodlifting mechanism works is obtained:

$$
\begin{aligned}
J_c^*\ddot{\varphi} &= -\frac{1}{2}m_c g\ell_0 \cos\varphi - c_2 u_2 \ell_0 \Delta L_2 \cos(\theta-\varphi) \\
&\quad + (c_1 u_1 \Delta L_1^* + c_2 \Delta L_2)\frac{b\ell_0 \sin(\phi_0+\varphi)}{\ell_{(\varphi)}}, \\
S\ddot{\theta} + 2\dot{S}\dot{\theta} + g\sin\theta &= \ell_0\cos(\theta-\varphi)\dot{\varphi}^2 - \ell_0\sin(\theta-\varphi)\ddot{\varphi}, \\
\ddot{S} + \frac{c_2 u_2 \Delta L_2}{m} - S\dot{\theta}^2 - g\cos\theta &= \ell_0\cos(\theta-\varphi)\ddot{\varphi} + \ell_0\sin(\theta-\varphi)\dot{\varphi}^2, \\
J_2\ddot{\psi}_2 + c_2 R_2 \Delta L_2 &= M_2,
\end{aligned}
\tag{4.1}
$$

where

J_c^* - moment of inertia of the rod with respect to the rotation axis of the rod,
J_2 - moment of inertia of the body lifting mechanism with respect to the rotation axis of the drum,
M_2 - rotation moment of body lifting drum,

$$M_2 = M_1 + (M_{kd}^{\max} - M_1)\Big(1 - \frac{t^2}{t_{kd}^2}\Big), \quad \text{if} \quad t \le t_{kd}, \tag{4.2}$$

$$M_2 = \frac{mgR_2}{u_2}, \quad \text{if} \quad t > t_{kd}, \tag{4.3}$$

in which, t_{kd} is the time needed for starting the work of the mechanism,
c_1, c_2 - stiffness of rod lifting and body lifting ropes, respectively,
u_1, u_2 - number of the rope branches of polyspats.

5. Equilibrium equations

Equilibrium equations of the bodylifting mechanism are derived from the equations of motion by vanishing the derivatives of generalized coordinates. Equilibrium positions can be determined by solving the equilibrium equations (algebraic equations). Assuming that M_2^0 is the static moment acting on the system and keeping it in the equilibrium state, the following relations can be derived:

$$\theta_0 = 0, \quad c_2 u_2 \Delta_2^0 = mg, \quad c_2 R_2 \Delta_2^0 = M_2^0,$$

$$-\frac{1}{2}m_c g\ell_0\cos\varphi_0 - c_2 u_2 \ell_0 \Delta_2^0 \cos\varphi_0 + (c_1 u_1 \Delta_1^0 + c_2 \Delta_2^0)\frac{b\ell_0\sin(\phi_0+\varphi_0)}{\ell_{(\varphi_0)}} = 0. \tag{5.1}$$

Equation (5.1) can be rewritten in the following form:

$$\Big(m+\frac{1}{2}m_c\Big)g\cos\varphi_0-\Big(c_1u_1\Delta_1^0+\frac{mg}{u_2}\Big)\frac{b\sin(\phi_0+\varphi_0)}{\ell_{(\varphi_0)}}=0. \tag{5.2}$$

For a given value of φ_0, the static elasticity of rodlifting rope Δ_1^0 is calculated by:

$$\Delta_1^0=\frac{1}{c_1u_1}\Big\{\Big(m+\frac{1}{2}m_c\Big)\frac{g\ell_{(\varphi_0)}\cos\varphi_0}{b\sin(\phi_0+\varphi_0)}-\frac{mg}{u_2}\Big\} \tag{5.3}$$

The static moment acting on the body lifting drum is:

$$M_2^0=\frac{mgR_2}{u_2}\,. \tag{5.4}$$

The coordinate ψ_2 satisfies the following relation:

$$\psi_2^0=ar\cos\frac{d^2+a^2-b^2}{2da}+ar\cos\frac{R_2-r}{d}\,. \tag{5.5}$$

Other requirements are:

$$90^\circ>\varphi\geq 0,\quad S_{\max}\geq S>0.$$

Beside, the moment acting on the winding rope drum must satisfy the following condition $M_2\geq M_2^0$. At an equilibrium state, the static elasticity of the bodylifting rope is calculated as follow:

$$\Delta_2^0=\frac{mg}{c_2u_2}\,. \tag{5.6}$$

6. Numerical computation

a) Needed data

The problem under consideration is solved for the automobile crane K-162 with the parameters:

$a=2.4\,(\text{m})$; $d=2.0\,(\text{m})$; $b=3.0\,(\text{m})$; $R_2=0.175\,(\text{m})$; $r=0.15\,(\text{m})$;

$m=8000\,(\text{kg})$; $m_c=1000\,(\text{kg})$; $J_c=125000\,(\text{kg.m}^2)$; $J_2=2079\,(\text{kg.m}^2)$;

$u_1=4$; $u_2=2$; $c_1=728000\,(\text{N/m})$; $c_2=1020000\,(\text{N/m})$; $S_0=12\,(\text{m})$;

$\varphi_0=30^\circ$; $M_{kd}=17506\,(\text{N.m})$.

b) Numerical method

The method of Runge-Kutta of the 4-th order is applied for solving the problem. The initial conditions are taken as follows:

$$\varphi_{(0)} = \varphi_0, \ (90° > \varphi \geq 0) \quad \dot{\varphi}_{(0)} = 0;$$
$$S_{(0)} = S_0, \ (S_{\max} \geq S > 0), \quad \dot{S}_{(0)} = 0;$$
$$\theta_{(0)} = \dot{\theta}_{(0)} = 0;$$
$$\psi_{2(0)} = \psi_2^0 = \arccos \frac{d^2 + a^2 - b^2}{2da} + \arccos \frac{R_2 - r}{d}; \quad \dot{\psi}_{2(0)} = 0.$$

c) Results of computation

A computer programme written on FORTRAN-77 language has been developed, tested and applied for the above problem. The calculation is conducted with the time step of 0.3s and the results are illustrated in the Fig. 2-6.

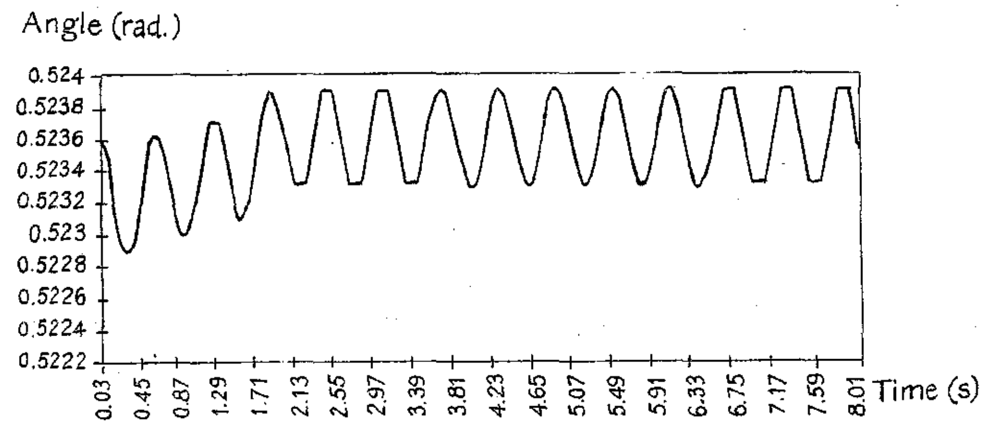

Fig. 2. Vibration of the rod under the operation of the bodylifting mechanism

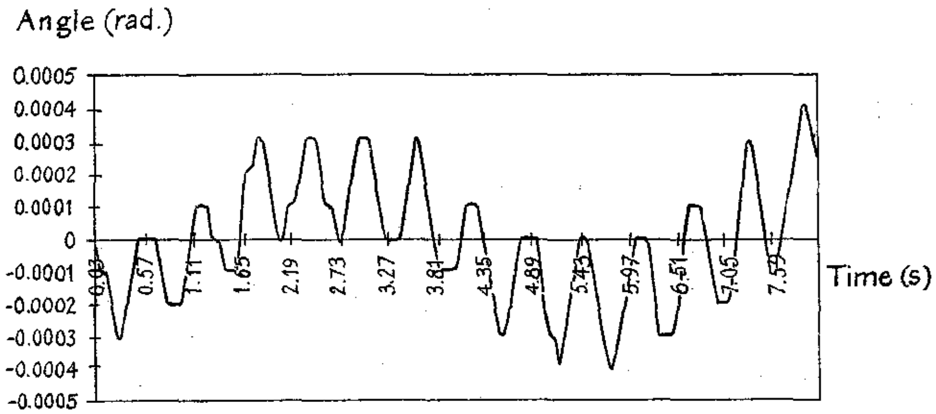

Fig. 3. Vibration of the body under the operation of the bodylifting mechanism

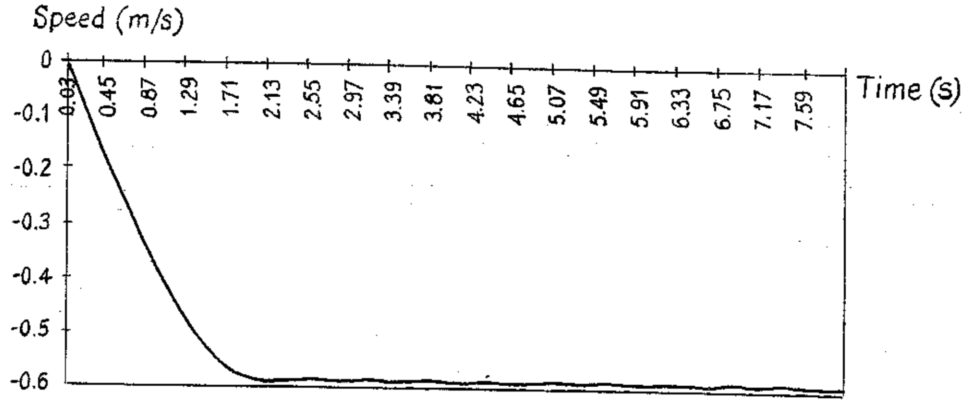

Fig. 4. **Bodylifting speed**

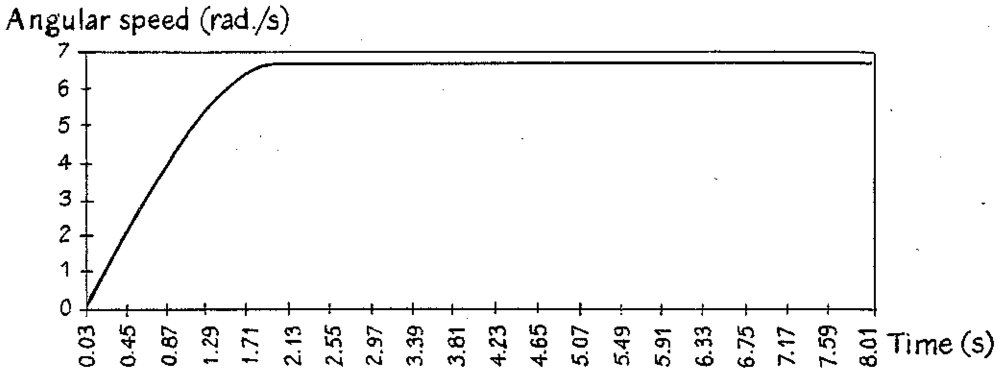

Fig. 5. **Angular speed of the bodylifting drum**

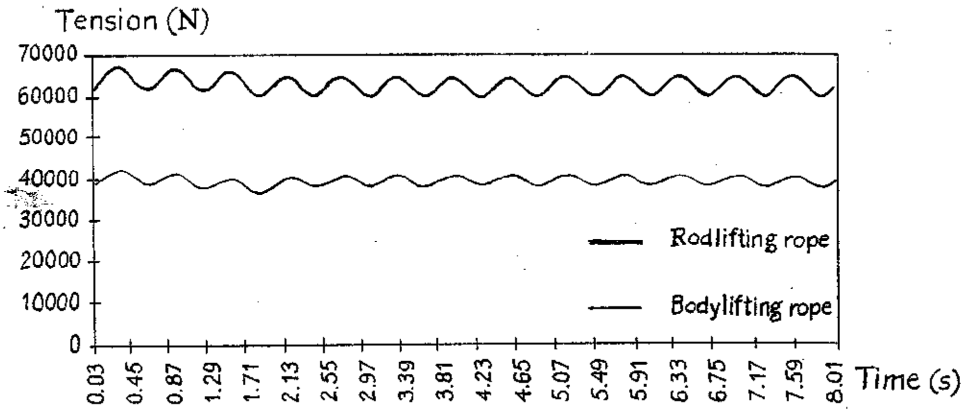

Fig. 6. **Dynamic tension in the ropes**

From the results of computation and figures presented above, it can be seen that:

• Although the rod lifting mechanism does not work, the rod vibrates due to the presence of elasticity of the rodlifting rope, therefore the shake of the rod has the quasiharmonic character with not large amplitude (approximately 3×10^{-4} m) and the period of about 0.6s (Fig. 2).

• Due to the slight shake of the rod, the vibration of the body consists of two oscillations with different periods (Fig. 3) and with the amplitudes increasing with time.

• Body lifting speed and the speed of the rope winding drum increase from 0 to the values of 0.6 m/s and 6.6 rad/s, respectively during the starting period and remain constant for the following steady motion (Figs. 4, 5).

• The dynamic tension in the rope also varies harmonically with the mean value above that of static tension.

7. Conclusion

• A more comprehensive model describing the dynamics of automobile crane system has been established, which allows to assess the impacts of the rod and the tope elasticity to the working bodylifting mechanism.

• The corresponding numerical model and computer programe have been developed. The numerical results and their illustration by graphs show good agreement with the experiment carried out in the Military Technical Academy of Vietnam in the framework of the author thesis and allows to assess the interaction between the parameters of the system.

• Qualitatively, the elasticity of the rope influences on the working mechanism of the automobile crane system leading to the change in value of its dynamic parameters. This influence, however, is not large in quantity. The above obtained results describing the interaction between parameters of the study system show the qualitative satisfactory of the established model. They can be the basic for solving the optimization problems in designing cranes.

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ĐỘNG LỰC HỌC CẦN TRỤC Ô TÔ CÓ KỂ ĐẾN SỰ CO GIÃN CỦA DÂY CÁP

Việc tính toán động lực học hệ cần trục ô tô cho đến nay còn ít được quan tâm và hầu như vẫn sử dụng những mô hình rất đơn giản của động lực học. Trong bài báo này các tác giả đưa ra mô hình tương đối đầy đủ để mô tả động lực học của cần trục ô tô và giải các phương trình thiết lập được bằng phương pháp số trong một trường hợp riêng: khi chỉ cơ cấu nâng vật làm việc.