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INTERACTION BETWEEN THE ELEMENTS CHARACTERIZING THE FORCED AND PARAMETRIC EXCITATIONS

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ABSTRACT. In nonlinear systems, the first order of smallness terms of nonresonance forced and parametric excitations have no effect on the oscillation in the first approximation. However, they do interact one with another in the second approximation.

Using the asymptotic method of nonlinear mechanics [1] we obtain the equations for the amplitudes and phases of oscillation. The amplitude curves are drawn by means of a digital computer. The stationary oscillations and their stability are of special interest.

1. Construction of approximate solutions

The nonlinear system under consideration in this paper is governed by the differential equation

$$\ddot{x} + \omega^2 x = \varepsilon\big[q\cos(2\omega t + \chi) + px\cos\omega t\big] + \varepsilon^2(\Delta x - 2h\dot{x} - \beta x^3), \tag{1.1}$$

where $\varepsilon^2\Delta = \omega^2 - 1$ and 1 is natural frequency. The terms with q and p represent the forced and parametric excitations, respectively. Both of them are in nonresonance. The forced excitation will be in resonance when it has frequency ω instead of 2ω. In contrary, the parametric excitation will be in the principal resonance when it has frequency 2ω instead of ω.

The solution of the equation (1.1) is found in the form

$$x = a\cos\theta + \varepsilon u_1(a,\psi,\theta) + \varepsilon^2 u_2(a,\psi,\theta) + \varepsilon^3\ldots,\quad \theta = \omega t + \psi, \tag{1.2}$$

where a and ψ must be determined from the following differential equations

$$\begin{aligned}\frac{da}{dt} &= \varepsilon A_1(a,\psi) + \varepsilon^2 A_2(a,\psi) + \ldots,\\ \frac{d\psi}{dt} &= \varepsilon B_1(a,\psi) + \varepsilon^2 B_2(a,\psi) + \ldots.\end{aligned} \tag{1.3}$$

The functions $u_i(a,\psi,\theta)$ are periodic with period 2π with respect to both angular variables ψ and θ and do not contain the first harmonics $\sin\theta$, $\cos\theta$ and $A_i(a,\psi)$, $B_i(a,\psi)$ are periodic functions with period 2π with respect to the angular variable ψ. For determination of these functions, we will use the procedure of direct differentiations and substitutions into the original equation (1.1) and subsequently equating firstly terms with equal powers of ε and then terms with equal harmonics $\sin\theta$, $\cos\theta$.

Comparing the coefficients of ε^1 in (1.1) we obtain

$$-2\omega A_1 \sin\theta - 2a\omega B_1\cos\theta + \omega^2\Big(\frac{\partial^2 u_1}{\partial\theta^2} + u_1\Big) = q\cos\big[2(\theta-\psi)+\chi\big] + ap\cos(\theta-\psi)\cos\theta. \tag{1.4}$$

Comparing the harmonics in (1.4) gives:

$$A_1 = B_1 = 0, \tag{1.5}$$

$$u_1 = \frac{pa}{2\omega^2}\cos\psi - \frac{1}{3\omega^2}\Big[q\cos(2\psi-\chi) + \frac{pa}{2}\cos\psi\Big]\cos 2\theta - \frac{1}{3\omega^2}\Big[q\sin(2\psi-\chi) + \frac{pa}{2}\sin\psi\Big]\sin 2\theta. \tag{1.6}$$

Comparing the coefficients of ε^2 in (1.1) we get

$$-2\omega A_2\sin\theta - 2\omega a B_2\cos\theta + \omega^2\Big(\frac{\partial^2 u_2}{\partial\theta_2} + u_2\Big) = pu_1\cos\omega t + \Delta a\cos\theta + 2h\omega a\sin\theta - \beta a^3\cos^3\theta. \tag{1.7}$$

Equating the coefficients of the first harmonics $\sin\theta$ and $\cos\theta$ in (1.7) we obtain

$$\begin{aligned} A_2(a,\psi) &= -ha - \frac{p^2 a}{8\omega^3}\sin 2\psi + \frac{pq}{12\omega^3}\sin(\psi-\chi),\\ B_2(a,\psi) &= -\frac{\Delta}{2\omega} - \frac{p^2}{12\omega^3} + \frac{3\beta}{8\omega}a^2 - \frac{p^2}{8\omega^3}\cos 2\psi + \frac{pq}{12\omega^3 a}\cos(\psi-\chi). \end{aligned} \tag{1.8}$$

So, in the second approximation one has:

$$x = a\cos\theta + \varepsilon\Big\{\frac{pa}{2\omega^2}\cos\psi - \frac{1}{3\omega^2}\Big[q\cos(2\psi-\chi) + \frac{pa}{2}\cos\psi\Big]\cos 2\theta - \frac{1}{3\omega^2}\Big[q\sin(2\psi-\chi) + \frac{pa}{2}\sin\psi\Big]\sin 2\theta\Big\}, \tag{1.9}$$

with a and ψ determined by the equations

$$
\begin{aligned}
\frac{da}{dt} &= -\frac{\varepsilon^2}{2\omega}\left[2ha\omega + \frac{p^2 a}{4}\sin 2\psi - \frac{pq}{6}\sin(\psi - \chi)\right], \\
a\frac{d\psi}{dt} &= -\frac{\varepsilon^2}{2\omega}\left[\left(\Delta + \frac{p^2}{6}\right)a - \frac{3}{4}\beta a^3 + \frac{p^2 a}{4}\cos 2\psi - \frac{pq}{6}\cos(\psi - \chi)\right].
\end{aligned} \tag{1.10}
$$

2. Stationary solutions

By putting $R = \dfrac{p^2}{4}$, $E = \dfrac{-pq}{6}$, we have the following equations for stationary solutions:

$$f_0 = 0, \qquad g_0 = 0. \tag{2.1}$$

where

$$
\begin{aligned}
f_0 &= 2\omega h a_0 + Ra\sin 2\psi_0 + E\sin(\psi_0 - \chi), \\
g_0 &= \left(\Delta + \frac{p^2}{6}\right)a_0 - \frac{3}{4}\beta a_0^3 + Ra_0\cos 2\psi_0 + E\cos(\psi_0 - \chi),
\end{aligned}
$$

or equivalently

$$
\begin{aligned}
f_0\cos\psi_0 - g_0\sin\psi_0 &= 0, \\
f_0\sin\psi_0 + g_0\cos\psi_0 &= 0.
\end{aligned}
$$

From here we obtain

$$
\begin{aligned}
&2\omega h a_0\sin\psi_0 - \left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{2} + R\right)\right]a_0\cos\psi_0 + E\cos\chi = 0, \\
&\left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right)\right]a_0\sin\psi_0 + 2\omega h a_0\cos\psi_0 - E\sin\chi = 0.
\end{aligned} \tag{2.2}
$$

Note. The equations (2.2) have the form

$$A\sin\psi_0 + B\cos\psi_0 = C. \tag{1}$$

From (1) it follows

$$A^2\sin^2\psi_0 = C^2 + B^2\cos^2\psi_0 - 2BC\cos\psi_0. \tag{2}$$

Substituting $\sin^2\psi$ by $1 - \cos^2\psi$ we obtain the quadratic equation with respect to $\cos\psi$:

$$(A^2 + B^2)\cos^2\psi_0 - 2BC\cos\psi_0 + C^2 - A^2 = 0. \tag{3}$$

The reality condition of $\cos\psi$ is

$$\Delta_* = B^2C^2 - (A^2+B^2)(C^2-A^2) = A^2(A^2+B^2-C^2) \geq 0,$$

or

$$A^2 + B^2 \geq C^2. \tag{2.3}$$

Applying the reality condition (2.3) to the equations (2.2) we have:

$$a_0^2\left\{4\omega^2h^2 + \left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} + R\right)\right]^2\right\} \geq E^2\cos^2\chi, \tag{2.4}$$

and

$$a_0^2\left\{4\omega^2h^2 + \left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right)\right]^2\right\} \geq E^2\sin^2\chi. \tag{2.5}$$

3. System without friction (h=0)

In this case, the equations (2.2) take the form

$$\begin{aligned} \left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} + R\right)\right] a_0\cos\psi_0 &= E\cos\chi, \\ \left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right)\right] a_0\sin\psi_0 &= E\sin\chi. \end{aligned} \tag{3.1}$$

The following subcases should be identified:

a) Subcase 1.

$$\left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} + R\right)\right]\left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right)\right] \neq 0. \tag{3.2}$$

Eliminating the phase ψ_0 from (3.1) we obtain the equation for the resonance curve C_1:

$$W(\omega^2, a_0^2) = 0, \tag{3.3}$$

where

$$W(\omega^2, a_0^2) = \frac{E^2\cos^2\chi}{\left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} + R\right)\right]^2} + \frac{E^2\sin^2\chi}{\left[\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right)\right]^2} - a_0^2. \tag{3.4}$$

b) Subcase 2.

$$\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} + R\right) = 0. \tag{3.5}$$

So, the resonance curve C_2 is given by the equation

$$\frac{3}{4}\beta a_0^2 = \omega^2 - 1 + \frac{p^2}{6} + R. \tag{3.6}$$

In this case, the equations (2.2) become

$$\begin{aligned} 0.a_0 \cos\psi_0 &= E\cos\chi, \\ 2Ra_0 \sin\psi_0 &= E\sin\chi, \end{aligned}$$

and therefore,

$$\begin{aligned} &\cos\chi = 0 \Rightarrow \chi = \frac{\pi}{2}, \frac{3\pi}{2}, \\ &\sin\chi = \pm 1 \quad \text{and} \quad \psi_0 = \pm \arcsin\frac{E}{2Ra_0}, \end{aligned}$$

$$\left(\frac{E}{2Ra_0}\right)^2 \leq 1 \Rightarrow a_0^2 \geq \frac{E^2}{4R^2}. \tag{3.7}$$

c) Subcase 3.

$$\frac{3}{4}\beta a_0^2 - \left(\Delta + \frac{p^2}{6} - R\right) = 0. \tag{3.8}$$

The resonance curve C_3 has form:

$$\frac{3}{4}\beta a_0^2 = \omega^2 - 1 + \frac{p^2}{6} - R. \tag{3.9}$$

From the equations (2.2) we obtain

$$\begin{aligned} O.a \sin\psi_0 &= E\sin\chi, \\ 2Ra_0 \cos\psi_0 &= -E\cos\chi, \end{aligned}$$

and therefore,

$$\begin{aligned} &\sin\chi = 0 \Rightarrow \chi = 0,\ \pi, \\ &\cos\chi = \pm 1, \quad \psi = \arccos\frac{\pm E}{2Ra_0} \Rightarrow a_0^2 \geq \frac{E^2}{4R^2}. \end{aligned} \tag{3.10}$$

Last two subcases show that, if $\chi \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, the resonance curves C_2, C_3 do not exist. If $\chi = \frac{\pi}{2}, \frac{3\pi}{2}$, then beside the resonance curve C_1 there is still a semi-straight line C_2 in the plane (a^2, ω^2) with $a_0^2 \geq \dfrac{E^2}{4R^2}$ · If $\chi = 0,\ \pi$, then beside the curve C_1 there is still a semi-straight line C_3 in the plane (a_0^2, ω^2) with $a_0^2 \geq \dfrac{E^2}{4R^2}$ ·

4. System with friction $(h \neq 0)$

Now, we consider the equations (2.2), denoting

$$\begin{aligned} D &= 4\omega^2 h^2 + \Big[\frac{3}{4}\beta a_0^2 - (\Delta + \frac{p^2}{6})\Big]^2 - R^2, \\ D_1 &= \Big\{\Big[\frac{3}{4}\beta a_0^2 - \Big(\Delta + \frac{p^2}{6} + R\Big)\Big]\sin\chi - 2\omega h \cos\chi\Big\}E, \\ D_2 &= \Big\{2\omega h \sin\chi + \Big[\frac{3}{4}\beta a_0^2 - \Big(\Delta + \frac{p^2}{6} - R\Big)\Big]\cos\chi\Big\}E. \end{aligned} \tag{4.1}$$

a) Subcase 1. $D \neq 0$

In this subcase we have

$$a_0 \sin\psi = \frac{D_1}{D}, \quad a_0 \cos\psi = \frac{D_2}{D},$$

$$W = D_1^2 + D_2^2 - a_0^2 D^2 = 0. \tag{4.2}$$

For $\chi = 0$, the equation (4.2) takes the form

$$\begin{aligned} &\Big\{4\omega^2 h^2 + \Big[\Delta + \frac{p^2}{6} - \frac{3}{4}\beta a_0^2 - R\Big]^2\Big\}E^2 \\ &- a_0^2\Big\{R^2 - \Big[\frac{3}{4}\beta a_0^2 - \Big(\Delta + \frac{p^2}{6}\Big)\Big]^2 - 4\omega^2 h^2\Big\}^2 = 0. \end{aligned} \tag{4.3}$$

To solve this equation on digital computer, it is convenient to write it in the form of an algebraic equation relatively to the variable δ:

$$\begin{aligned} &a_0^2\delta^4 - 3\beta a_0^4\delta^3 + \Big[-E^2 + a_0^2\Big(\frac{27}{8}\beta^2 a_0^4 + 8h^2\omega^2 - 2R^2\Big)\Big]\delta^2 \\ &+ \Big[2E^2\Big(\frac{3}{4}\beta a_0^2 + R\Big) + 3\beta a_0^4\Big(R^2 - \frac{9}{16}\beta^2 a_0^4 - 4h^2\omega^2\Big)\Big]\delta \\ &+ a_0^2\Big(R^2 - 4h^2\omega^2 - \frac{9}{16}\beta^2 a_0^4\Big)^2 - E^2\Big[4h^2\omega^2 + \Big(\frac{3}{4}\beta a_0^2 + R\Big)^2\Big] = 0, \end{aligned} \tag{4.3a}$$

where $\delta = \Delta + \dfrac{p^2}{6}$, $\omega^2 \approx 1$.

The relation (4.3a) gives the dependence of the amplitude a on the frequency ω (through δ) and is presented for the parameters: $h = 10^{-3}$, $R = 0.02$, $E = 10^{-2}$, $\beta = 0.08$ (Fig. 1), $\beta = -0.08$ (Fig. 2) and $h = 0$, $R = 0.02$, $E = 10^{-2}$, $\beta = 0.08$ (Fig. 3), $\beta = -0.08$ (Fig. 4). When $h = 0$, the equation (4.3) degenerates into a double equation $\delta - \frac{3}{4}\beta a_0^2 - R = 0$ (curve 2, Fig. 3, 4) and $E^2 - a_0^2\left(\delta - \frac{3}{4}\beta a_0^2 + R\right)^2 = 0$ (curve 1, Fig. 3, 4).

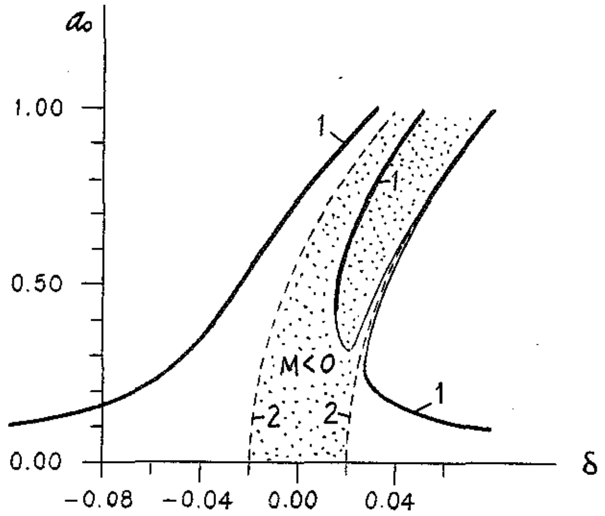

Fig. 1. Amplitude curves for the case $\beta > 0$, $h > 0$.
Curves 2 serve as the boundary of stability zone $M = z_0^2 + u_0^2 - v_0^2 = 0$.

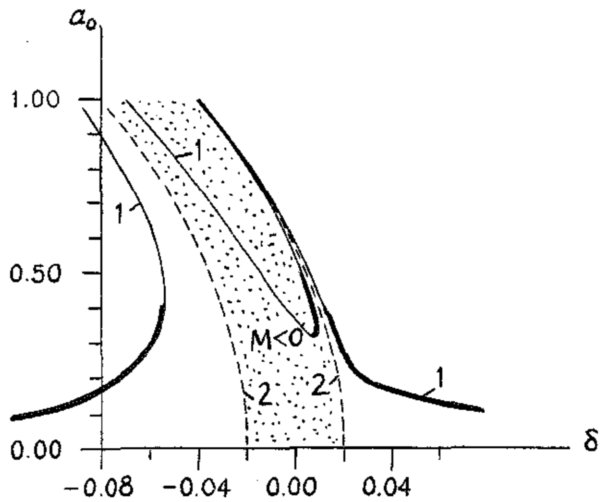

Fig. 2. Amplitude curves for the case $\beta < 0$, $h > 0$.
Curves 2 serve as the boundary of stability zone $M = z_0^2 + u_0^2 - v_0^2 = 0$.

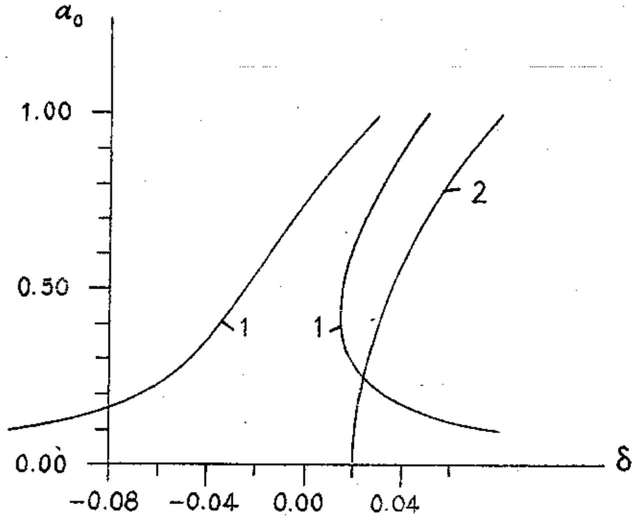

Fig. 3. Amplitude curves in absence of friction and for the case $\beta > 0$.

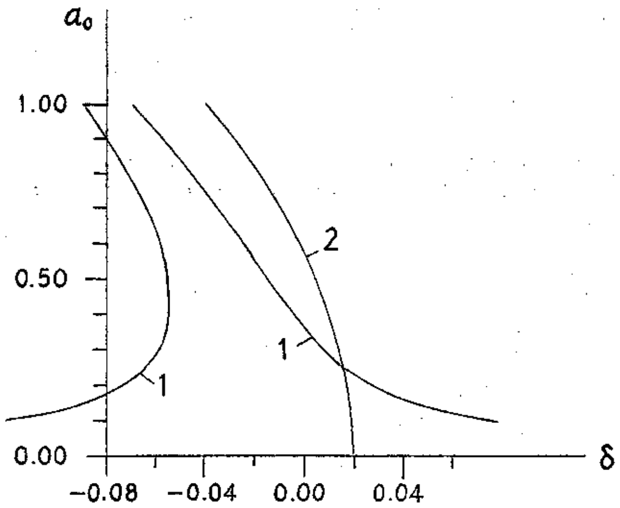

Fig. 4. Amplitude curves in absence of friction and for the case $\beta < 0$.

b) Subcase 2. $D = 0$

In this subcase we have the following equation for the resonance curve:

$$\frac{3}{4}\beta a_*^2 = \omega^2 - 1 + \frac{p^2}{6} \pm \sqrt{R^2 - 4\omega^2 h^2}\ . \tag{4.4}$$

The expressions (4.2) give $D_1 = D_2 = 0$, or equivalently,

$$D_1 \cos\chi - D_2 \sin\chi = 0, \quad D_1 \sin\chi + D_2 \cos\chi = 0.$$

Substituting here the values of D_1 and D_2 from (4.1) we get

$$\omega_* = -\frac{R}{2h}\sin 2\chi, \quad \frac{3}{4}\beta a_*^2 = \omega_*^2 - 1 + \frac{p^2}{6} - R\cos 2\chi. \tag{4.5}$$

Taking into account these values of a and ω, the condition (2.4) takes the form

$$a_*^2 \geq \frac{E^2}{4R^2}\,.$$

5. Stability of Oscillations

We now consider the stability of stationary oscillations with the amplitude a and phase ψ determined by the equation (1.10):

$$\begin{aligned} \frac{da}{dt} &= -\frac{\varepsilon^2}{2\omega}\left[z + v\sin 2\psi + E\sin(\psi-\chi)\right], \\ \frac{ad\psi}{dt} &= -\frac{\varepsilon^2}{2\omega}\left[u + v\cos 2\psi + E\cos(\psi-\chi)\right], \end{aligned} \tag{5.1}$$

where

$$R = \frac{p^2}{4}, \; E = \frac{-pq}{6}, \; \delta = \Delta + \frac{p^2}{6}, \; z = 2h\omega a, \quad v = Ra, \quad u = \delta a - \frac{3}{4}\beta a^3. \tag{5.2}$$

Stationary values a_0, ψ_0 of equations (5.1) are determined from the equations: $f = 0$, $g = 0$, where

$$\begin{aligned} f &= z_0 + v_0\sin 2\psi_0 + E\sin(\psi_0 - \chi), \\ g &= u_0 + v_0\cos 2\psi_0 + E\cos(\psi_0-\chi), \\ z_0 &= 2h\omega a_0, \quad v_0 = Ra_0, \quad u_0 = \delta a_0 - \frac{3}{4}\beta a_0^3. \end{aligned} \tag{5.3}$$

The following equations are equivalent to $f = g = 0$:

$$\begin{aligned} f\sin(\psi_0-\chi) + g\cos(\psi_0-\chi) &= 0, \\ f\cos(\psi_0-\chi) - g\sin(\psi_0-\chi) &= 0, \end{aligned}$$

which give:

$$\begin{aligned} (z_0 - v_0\sin 2\chi)\sin(\psi_0-\chi) + (u_0 + v_0\cos 2\chi)\cos(\psi_0-\chi) &= -E, \\ (-u_0 + v_0\cos 2\chi)\sin(\psi_0-\chi) + (z_0 + v_0\sin 2\chi)\cos(\psi_0-\chi) &= 0, \end{aligned}$$

r

$$
\begin{aligned}
(v_0^2 - z_0^2 - u_0^2)\cos(\psi_0 - \chi) &= E(u_0 - v_0\cos 2\chi) \\
(v_0^2 - z_0^2 - u_0^2)\sin(\psi_0 - \chi) &= E(z_0 + v_0\sin 2\chi)
\end{aligned} \tag{5.4}
$$

or $v_0^2 - z_0^2 - u_0^2 \neq 0$.

Eliminating the phase ψ_0 we obtain

$$
W = 0, \tag{5.5}
$$

where

$$
W = (z_0^2 + u_0^2 - v_0^2)^2 - E^2\left[(u_0 - v_0\cos 2\chi)^2 + (z_0 + v_0\sin 2\chi)^2\right]. \tag{5.6}
$$

Denoting $\tilde{a} = a - a_0$, $\tilde{\psi} = \psi - \psi_0$ we have the following equations in variations:

$$
\begin{aligned}
\frac{d\tilde{a}}{dt} &= -\frac{\varepsilon^2}{2\omega}\left\{(z_0' + v_0'\sin 2\psi_0)\tilde{a} + \left[2v_0\cos 2\psi_0 + E\cos(\psi_0 - \chi)\right]\tilde{\psi}\right\}, \\
a_0\frac{d\tilde{\psi}}{dt} &= -\frac{\varepsilon^2}{2\omega}\left\{(u_0' + v_0'\cos 2\psi_0)\tilde{a} - \left[2v_0\sin 2\psi_0 + E\sin(\psi_0 - \chi)\right]\tilde{\psi}\right\},
\end{aligned} \tag{5.7}
$$

where

$$
z_0' = \left(\frac{dz}{da}\right)_{a=a_0}, \quad v_0' = \left(\frac{dv}{da}\right)_{a=a_0}, \quad u_0' = \left(\frac{du}{da}\right)_{a=a_0}
$$

The characteristic equation of this system of equations is

$$
a_0\lambda^2 + \varepsilon^2 h\lambda - \frac{\varepsilon^4}{4\omega^2}S = 0, \tag{5.8}
$$

where

$$
\begin{aligned}
S &= (z_0' + v_0'\sin 2\psi_0)\left[2v_0\sin 2\psi_0 + E\sin(\psi_0 - \chi)\right] \\
&\quad + (u_0' + v_0'\cos 2\psi_0)\left[2v_0\cos 2\psi_0 + E\cos(\psi_0 - \chi)\right] \\
&= 2v_0v_0' + 2z_0'v_0\sin 2\psi_0 + 2u_0'v_0\cos 2\psi_0 \\
&\quad + E\left[(z_0' + v_0'\sin 2\psi_0)\sin(\psi_0 - \chi) + (u_0' + v_0'\cos 2\psi_0)\cos(\psi_0 - \chi)\right].
\end{aligned} \tag{5.9}
$$

Substituting here the expressions for $v_0\sin 2\psi_0$, $v_0\cos 2\psi_0$ from equations $f = 0$, $g = 0$ (5.3) we have

$$
S = 2v_0v_0' - 2z_0'z_0 - 2u_0'u_0 + E\left[v_0'\cos(\psi_0 + \chi) - z_0'\sin(\psi_0 - \chi) - u_0'\cos(\psi_0 - \chi)\right]
$$

or

$$S = 2v_0v_0' - 2z_0z_0' - 2u_0u_0' + \\ E\big[(v_0'\cos 2\chi - u_0')\cos(\psi_0 - \chi) - (v_0'\sin 2\chi + z_0')\sin(\psi_0 - \chi)\big]. \tag{5.10}$$

Taking into account expressions (5.4) we can write

$$S = \frac{d}{da}\{v_0^2 - z_0^2 - u_0^2\} - \frac{E^2}{2(v_0^2 - z_0^2 - u_0^2)}\cdot\frac{d}{da}\{(u_0 - v_0\cos 2\chi)^2 + (z_0 + v_0\sin 2\chi)^2\}$$

or from (5.6):

$$S = \frac{1}{2(v_0^2 - z_0^2 - u_0^2)}\,\frac{\partial W}{\partial a_0}\,, \quad v_0^2 - z_0^2 - u_0^2 \neq 0. \tag{5.11}$$

Thus, the stability condition takes the form:

$$M\cdot\frac{\partial W}{\partial a_0} > 0, \tag{5.12}$$

where $M = z_0^2 + u_0^2 - v_0^2$.

The resonance curve ($W = 0$) divides the plane (a_0, ω) into regions, in each of which the expression W has a define sign(+ or −). If moving up along the straight line parallel to the axis a_0, we pass from a region $W < 0$ to a region $W > 0$, then at the point of intersection between the straight line and the resonance curve the derivative $\partial W/\partial a_0$ is positive. So, this point corresponds to a stable state of oscillation if $M > 0$ and to an unstable one if $M < 0$. On the contrary, if we pass from a region $W > 0$ to a region $W < 0$, then the point of intersection corresponds to a stable of oscillation if $M < 0$ and to an unstable one if $M > 0$.

In Figs 1 and 2 equations $M = 0$ are presented by curves 2 and in the stippled region the expression M is negative. The heavy lines correspond to a stable state of oscillations, where the stability conditions (5.12) are satisfied.

6. Conclusion

The interaction between the elements characterizing the forced and parametric excitations has been studied. The first order of smallness terms of nonresonance forced and parametric excitations have no effect on the oscillation in the first approximation. The equations (1.10) show that these terms are not in equality. The effect of forced excitation (q) exists only with the presence of parametric excitation

(p), while the effect of parametric excitation will exist even with the absence of forced one ($q = 0$). The stationary oscillations and their stability in the system with and without friction are of special interest.

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TƯƠNG TÁC GIỮA CÁC PHẦN TỬ ĐẶC TRƯNG
CHO KÍCH ĐỘNG CƯỠNG BỨC VÀ THÔNG SỐ

Trong các hệ phi tuyến, những kích động cưỡng bức và thông số có độ bé bậc nhất và không cộng hưởng sẽ không có ảnh hưởng đến dao động trong xấp xỉ thứ nhất. Tuy nhiên, chúng tác động qua lại nhau trong xấp xỉ thứ hai và được nghiên cứu trong bài báo này. Kết quả cho thấy ảnh hưởng của thành phần kích động cưỡng bức chỉ xuất hiện khi có tác động của thành phần kích động thông số. Trong lúc đó, tác động của thành phần thông số tồn tại cả khi vắng mặt kích động cưỡng bức. Dao động dừng và sự ổn định của chúng của hệ trong trường hợp không cản và có cản được đặc biệt quan tâm nghiên cứu.