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THE INFLUENCE OF THE DESIGN PARAMETERS ON THE PROFILE SLIDING IN AN INTERNAL HYPOCYCLOID GEAR PAIR

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Abstract. While hypogerotor pump working, the profiles of the inner and outer rotors match together following gearing rule of the hypocycloidal gear-set. Therefore, those two opposite profiles matching each other like in generation process, and during action, one will roll and slip in relation with the other. Relative sliding between two profiles in the contact point causes wearing out of the tooth profile. Aiming to evaluate influence of the geometrical dimension parameters of the pump rotor profile on the wear, in this paper, the authors established equation for determining slip coefficient from geometrical dimensions. Furthermore, the authors have investigated and evaluated the phenomenon of the profile slipping to find out the geometrical dimensional parameters for avoiding unequal wearing of the inner and outer rotors of the hypogerotor pump.

Keywords: hypogerotor pump, profile slipping, hypocycloidal gear.

Classification numbers: 5.5.1, 5.6.1, 5.10.1.

1. INTRODUCTION

Hypogerotor pump is designed by internal matching principle of the hypocycloidal gear-train. In that pump, the tooth profile of outer gear is hypocycloidal, and that of the matching inner gear is circular. Also, the relation between the number of teeth of the outer gear (z_2) and inner gear (z_1) can be expressed as $z_2 = z_1+1$ [1]. On the other hand, because of the matching characteristics of the gear-train, the chambers in the pump are formed by the profiles of the gears and the flange, as shown in the Figure 1 [2, 3]. Also in this gear-train, the outer gear participates in matching process with its whole

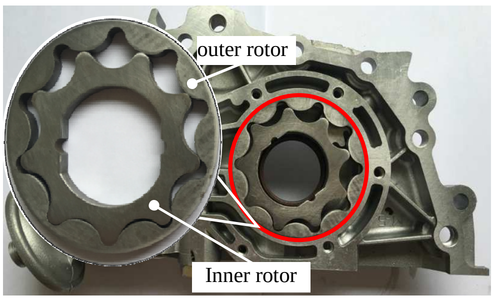

Figure 1. Hypogerotor pump.

hypocycloidal profile (from dedendum to addendum), meanwhile only the addendum part of the circular-arc profile of the inner gear has involved into this process. Following page 60 [4], for the cycloidal gear pair, the contact stress clearly will increase when two convex profiles are matching each other. And logically, it leads to wear effect as in [5, 6], where the authors tried to find the sliding velocity between the profiles of the epicycloidal gear pair. Therefore, it is necessary to select an appropriate set of parameters when designing gear profiles (R_1, r_{cl}), to ensure that both matching profiles will be worn equally and simultaneously. This is the main goal of this research.

2. KINEMATIC ANALYSIS OF HYPOGEROTOR PUMP

In [8], the hypogerotor pump consists of the pair of internal matching hypocycloidal gears with five parameters: E, R_1, R, r_{cl}, z_1. Those parameters are shown in Figure 2.

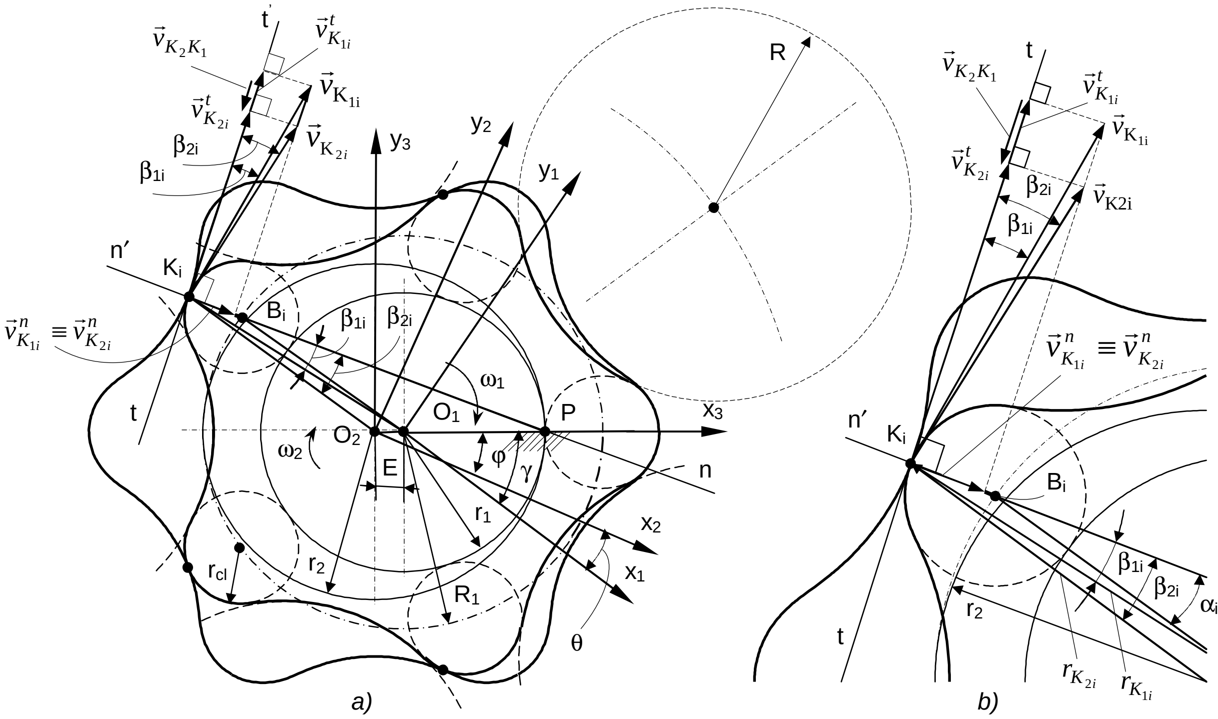

Figure 2. Calculating scheme of sliding velocity at matching point K.

Where:

E: eccentricity between two rotation centers of the inner and outer gears (center distance),

R_1: radius of the locus of the centers of the addendum circular arcs on the inner rotor,

R: radius of the dedendum arc of the inner rotor (mating with two consecutive addenda of the inner rotor),

z_1: number of teeth of the inner rotor,

r_{cl}: radius of the addendum arc of the inner rotor.

Following matching principle of the hypocycloidal gear-train, let P be the contact point between circle of radius r_1 $\Sigma^1(O_1, r_1)$ and circle of radius r_2 $\Sigma^2(O_2, r_2)$. Then P will be the ***pitch point*** (stationary in this case) and $r_1 = Ez_1$, $r_2 = Ez_2$.

K_i: are the contact point between the profiles of the outer and inner rotors, which K_{1i}, K_{2i} belong to the inner and outer rotor, respectively.

B_i: are the center of the circular arc of the inner rotor adendum.

nn′, tt′: are the common normal and tangent at the arbitrary contact point K_i (*matching point*), and nn′ always goes through P, B_i, K_i.

When the inner rotor is driven clockwise with the angular velocity ω_1 around O_1 (Figure 2), it also makes the outer rotor rotate around O_2 with the angular velocity ω_2 in the same direction as ω_1. The velocities of points K_i in the absolute motion can be written as:

$$\begin{cases} v_{K_{1i}}(\gamma_i) = \omega_1 r_{K_{1i}}(\gamma_i) \\ v_{K_{2i}}(\gamma_i) = \omega_2 r_{K_{2i}}(\gamma_i) \end{cases} \tag{1}$$

where $r_{K_{1i}}(\gamma_i) = O_1K_{1i}$, $r_{K_{2i}}(\gamma_i) = O_2K_{2i}$

Projecting $v_{K_{1i}}(\gamma_i)$, $v_{K_{2i}}(\gamma_i)$ onto tangent tt′ results in:

$$\begin{cases} v^t_{K_{1i}}(\gamma_i) = v_{K_{1i}}(\gamma_i)\cos[\beta_{1i}(\gamma_i)] \\ v^t_{K_{2i}}(\gamma_i) = v_{K_{2i}}(\gamma_i)\cos[\beta_{2i}(\gamma_i)] \end{cases} \tag{2}$$

in which $\beta_{1i}(\gamma_i)$, $\beta_{2i}(\gamma_i)$ are the angles between $\vec{v}_{K_{1i}}(\gamma_i)$, $\vec{v}_{K_{2i}}(\gamma_i)$ and tangent tt′ at K_i during matching process. Subtituting (1) into (2) results in:

$$\begin{cases} v^t_{K_{1i}}(\gamma_i) = \omega_1 r_{K_{1i}}(\gamma_i)\cos[\beta_{1i}(\gamma_i)] \\ v^t_{K_{2i}}(\gamma_i) = \omega_2 r_{K_{2i}}(\gamma_i)\cos[\beta_{2i}(\gamma_i)] \end{cases} \tag{3}$$

where $v^t_{K_{1i}}(\gamma_i)$, $v^t_{K_{2i}}(\gamma_i)$: are components of the sliding velocity at K_i, with K_{1i} belongs to the profile of the inner rotor, and K_{2i} lies on the profile of the outer rotor. The parameters $r_{K_{1i}}(\gamma_i)$, $r_{K_{2i}}(\gamma_i)$, $\beta_{1i}(\gamma_i)$, $\beta_{2i}(\gamma_i)$ in equation (3) will be calculated in the sections 2.1, 2.2, 2.3.

2.1. Calculation of $r_{K_{1i}}(\gamma_i)$, $r_{K_{2i}}(\gamma_i)$

From equation (6) of [8], the coordinates of the point K_i in the fixed coordination system $\vartheta_3(o_2y_3x_3)$ can be expressed as:

$$\begin{cases} {}^3x_{K_i}(\gamma_i) = R_1\cos\gamma_i + r_{cl}\cos[\alpha_i(\gamma_i) + \gamma_i] + E \\ {}^3y_{K_i}(\gamma_i) = -R_1\sin\gamma_i - r_{cl}\sin[\alpha_i(\gamma_i) + \gamma_i] \end{cases} \tag{4}$$

in which $\alpha_i(\gamma_i) = \tan^{-1}\left[\dfrac{Ez_1\cos(\gamma_i)}{R_1 - Ez_1\sin(\gamma_i)}\right]$

From the equation (4), one can obtain:

$$r_{K_{2i}}(\gamma_i) = \sqrt{[{}^3x_{K_i}(\gamma_i)]^2 + [{}^3y_{K_i}(\gamma_i)]^2} \tag{5}$$

and

$$r_{K_{1i}}(\gamma_i) = \sqrt{[{}^3x_{K_i}(\gamma_i) - E]^2 + [{}^3y_{K_i}(\gamma_i)]^2} \tag{6}$$

2.2. Calculation of cosβ$_{1i}$(γ$_i$), cosβ$_{2i}$(γ$_i$)

Applying the law of cosines to the triangle PO_1K_i results in:

$$\cos\beta_{1i}(\gamma_i) = \frac{[r_{K_{1i}}(\gamma_i)]^2 + [PK_i(\gamma_i)]^2 - [Ez_1]^2}{2r_{K_{1i}}(\gamma_i)PK_i(\gamma_i)} \tag{7}$$

and

$$\cos\beta_{2i}(\gamma_i) = \frac{[r_{K_{2i}}(\gamma_i)]^2 + [PK_i(\gamma_i)]^2 - [E(z_1+1)]^2}{2r_{K_{2i}}(\gamma_i)PK_i(\gamma_i)} \tag{8}$$

where

$$PK_i(\gamma_i) = \sqrt{[{}^3x_{K_i}(\gamma_i) - Ez_2]^2 + [{}^3y_{K_i}(\gamma_i)]^2} \tag{9}$$

2.3. Transmission ratio of the rotors

From equation (2) of [1], the gear ratio can be expressed as:

$$\begin{cases} i_{12} = \dfrac{\omega_1}{\omega_2} = \dfrac{z_1+1}{z_1} \\ i_{21} = \dfrac{\omega_2}{\omega_1} = \dfrac{z_1}{z_1+1} \end{cases} \tag{10}$$

Case study

For the illustrative purpose, the parameters of the considered gearset are as follows: $z_1 = 5$, $r_{cl} = 7$ mm, $R_1 = 26.25$ mm, $R = 30$ mm, $E = 3.5$ mm.

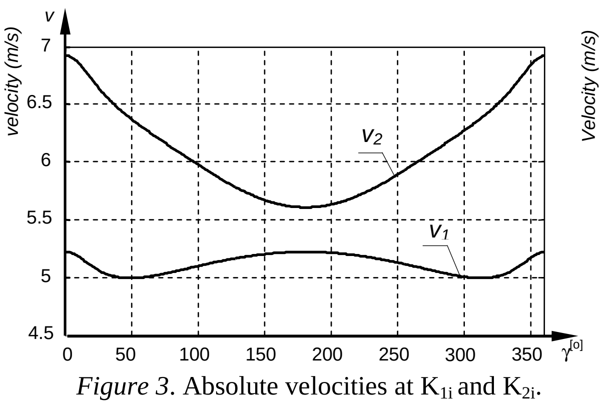

Figure 3. Absolute velocities at K_{1i} and K_{2i}.

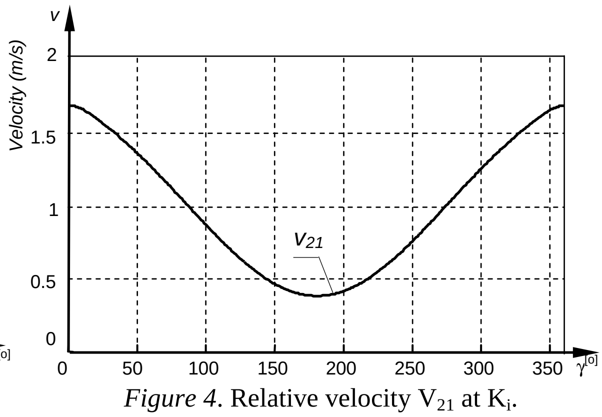

Figure 4. Relative velocity V_{21} at K_i.

When the inner rotor is driven by $\omega_1 = 157$ (rad/s), the outer rotor is rotated with the same direction $\omega_2 = 130.8$ (rad/s). In that case, velocities $v_{K_{1i}}(\gamma_i)$, $v_{K_{2i}}(\gamma_i)$ at the contact points K_i are shown in Figure 3. Figure 4 provides the graph of the relative sliding velocity $v_{K_{21}}(\gamma_i)$.

3. PROFILE SLIP COEFFICIENT

3.1. Equation for calculation of the profile slip coefficient

During matching process at K_i (*contact point*), one profile rolls and slides against the other. The relative sliding velocity at the point K_i lies on the tangent tt′ and causes wear effect of the profiles:

$$\begin{cases} v_{tr12i}(\gamma_i) = v^t_{K1i}(\gamma_i) - v^t_{K2i}(\gamma_i) \\ v_{tr21i}(\gamma_i) = v^t_{K2i}(\gamma_i) - v^t_{K1i}(\gamma_i) \end{cases} \tag{11}$$

Let ξ_{1i} and ξ_{2i} be the slip coefficients of the inner and outer rotors, respectively. The sliop coefficients can be defined as:

$$\begin{cases} \xi_{1i} = \dfrac{v_{tr12i}(\gamma_i)}{v^t_{K1i}(\gamma_i)} \\ \xi_{2i} = \dfrac{v_{tr21i}(\gamma_i)}{v^t_{K2i}(\gamma_i)} \end{cases} \tag{12}$$

Substituting equations (3, 6, 8 – 11) into (12), the slip coefficients can be r as:

$$\begin{cases} \xi_{1i} = 1 - i_{21}\dfrac{r_{K2i}(\gamma_i)\cos[\beta_{2i}(\gamma_i)]}{r_{K1i}(\gamma_i)\cos[\beta_{1i}(\gamma_i)]} \\ \xi_{2i} = 1 - i_{12}\dfrac{r_{K1i}(\gamma_i)\cos[\beta_{1i}(\gamma_i)]}{r_{K2i}(\gamma_i)\cos[\beta_{2i}(\gamma_i)]} \end{cases} \tag{13}$$

Using equations (13), the profile slip coefficients between the addendum of the inner rotor and the dedendum of the outer rotor, as well as sliding coefficient between the dedendum of the inner rotor and the addendum of the outer rotor can be computed.

Case study

Using equations (13), figures 5 and 6 show the variation of ξ_1 and ξ_2, respectively. In these figures, the parameters of the hypogerotor pump are: z_1 = 5, E = 3.5 mm, r_{cl} = 5.25 mm, R_1 = 26.25 mm, R = 20 mm.

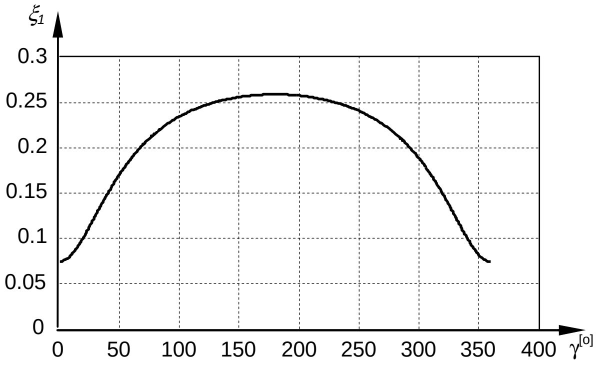

Figure 5. Sliding curve ξ_1.

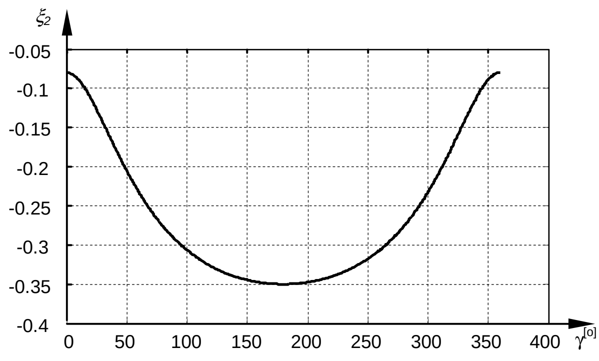

Figure 6. Sliding curve ξ_2.

From Figures 5 and 6, it is noticable that the sliding coefficients are always negative at the tooth dedendum and positive at the tooth addendum.

4. INFLUENCE OF THE KINEMATIC DIMENSION ON THE PROFILE SLIP COEFFICIENT

As mentioned in Section 2, the hypogerotor pump is built of the pair of internal hypocycloidal gears with 5 parameters E, z_1, R_1, R, r_{cl}. However, this paper only presents the influence of two parameters R_1 and r_{cl} on the profile slip coefficients. The two most important parameters in the process of manufacturing hypocycloidal-profile gears are:

$$\lambda = \frac{R_1}{Ez_1} \tag{15}$$

and

$$c = \frac{r_{cl}}{E} \tag{16}$$

In that case, we can re-formulate the problem into evaluating the influence of the parameters λ and c on the profile shift (slip) coefficients ξ_1, ξ_2, which will be solved in the following Sections of 4.1, 4.2, 4.3.

4.1. Influence of λ on ξ_1, ξ_2

In order to evaluate the influence of λ on ξ_1 and ξ_2, the parameters of the internal hypocycloidal gear-train are chosen as E = 3.5 mm, R_1 = 20 mm, z_1 = 5. Setting c = 1.5 [4] results in $\lambda \in$ [1 ÷ 1.57], according to [8].

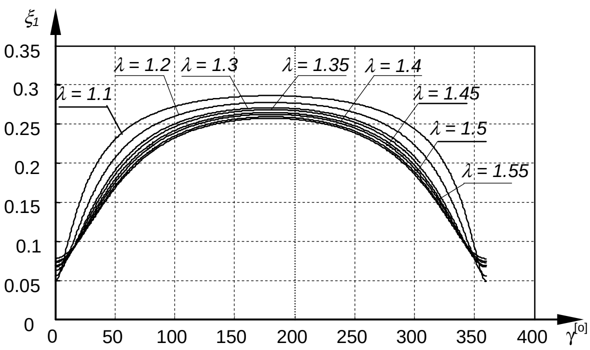

Figure 7. Sliding curve ξ_1 with respect to λ.

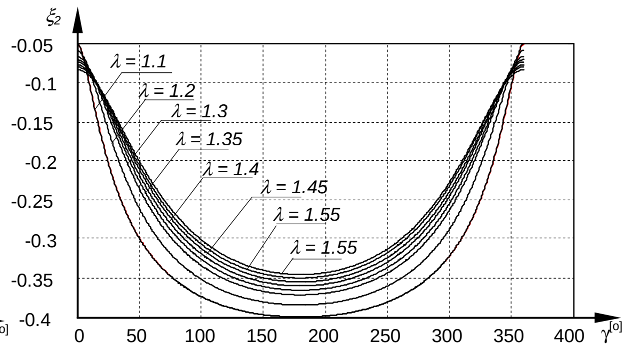

Figure 8. Sliding curve ξ_2 with respect to λ.

Figure 7 shows the sliding curve of the inner rotor addendum and the outer rotor dedendum. Figure 8 presents the sliding curve of the inner rotor dedendum and the outer rotor addendum with respect to the parameter λ. The gear-trains with relation to λ are presented in Figure 9. From Figures 7, 8 it can be seen that the obtained results matched with the results in page 235 of the reference [9]. In case of the external hypocycloidal gear pair, the profile shift (slip) coefficient is a constant. However, in the internal hypocycloidal gear train, this coefficient is not a constant. When λ increases, meaning that parameter R_1 increases as well, then the profile shift coefficient decreases.

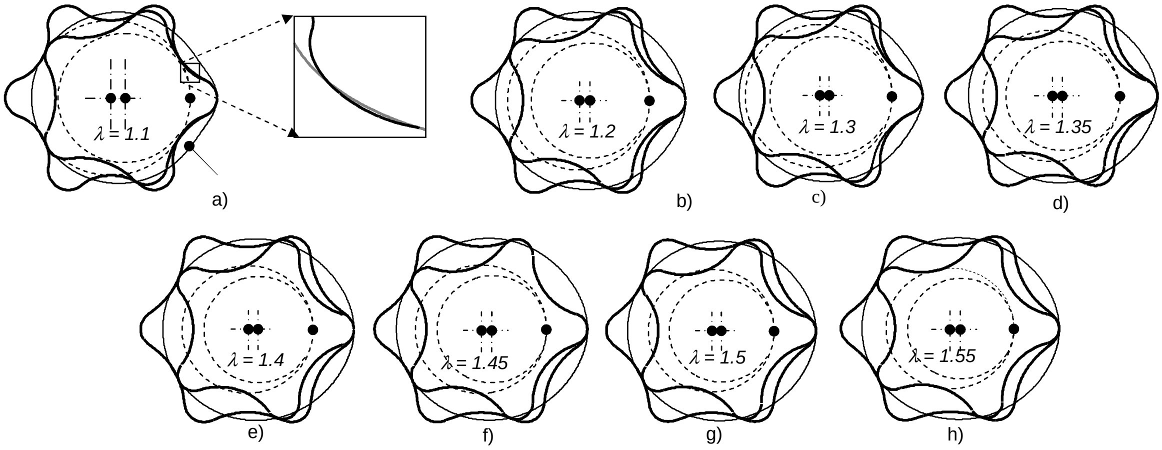

Figure 9. Gear-train with respect to λ.

4.2. Influence of the parameter c on the profile slip coefficient

In the case of E = 3.5 mm, z_1 = 5, λ = 1.5 and R = 20 mm, according to [8], it can be proved that c ∈ [0 ÷ 7.78]. Figure 10 shows the sliding curve of the inner rotor addendum and the outer rotor dedendum, and in Figure 11 is the sliding curve of the outer rotor dedendum and the inner rotor addendum with respect to the parameter *c*. In Figure 12 the pairs of hypocycloidal gears in relation with *c* are depicted.

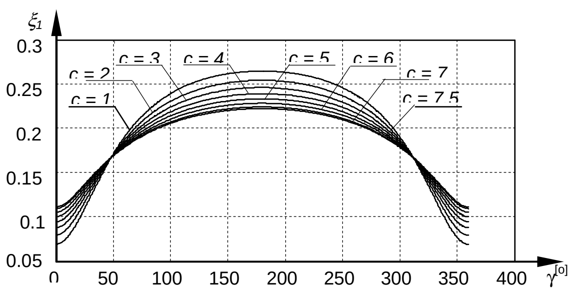

Figure 10. Sliding curve ξ_1 with respect to c

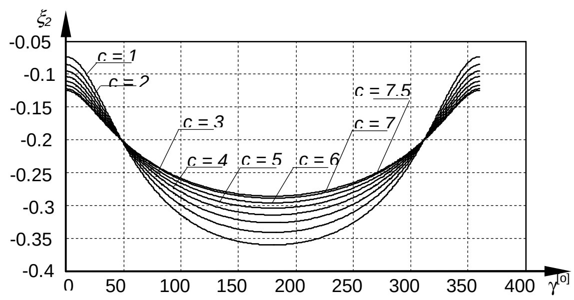

Figure 11. Sliding curve ξ_2 with respect to c

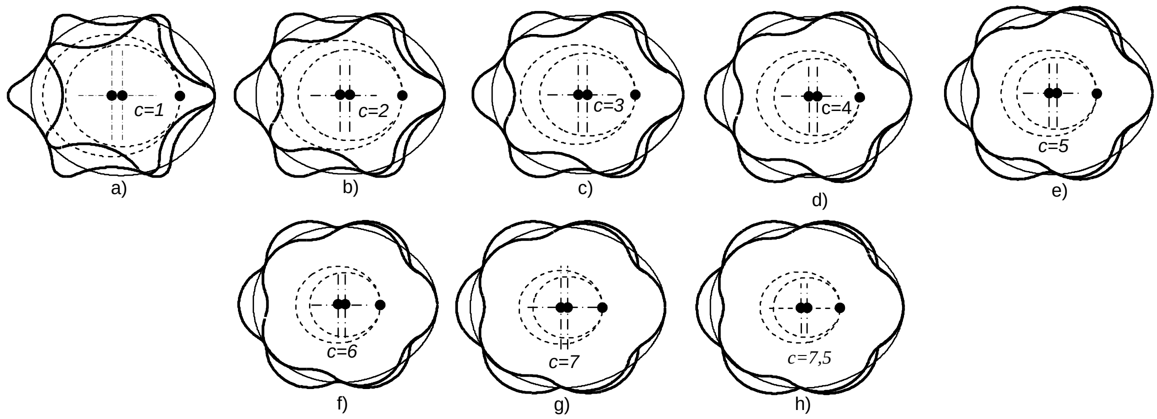

Figure 12. The gear pairs with respect to c.

It can be easily seen that when *c* increases (also r_{cl} increases), the dedendum width of the outer rotor increases, meanwhile, addendum of the outer rotor get smaller. It causes the enlargement of radial dimension, and the reduction of the profile slip coefficient of the pump.

4.3. Influence of the parameters λ and c on the profile slip coefficient

Suppose that the generating parameters of the hypocycloidal gear-pair and λ are taken from section 4.1, on the other hand, the parameter c is calculated in section 4.2.

Figure 13 shows the sliding curve of the inner rotor addendum and the outer rotor dedendum, and in Figure 14 presents the sliding curve of the outer rotor dedendum and the inner rotor addendum with respect to the parameter *c*. In Figure 15, the pairs of hypocycloidal gears in relation with λ and *c* are generated. Reduction of the parameter c could lead to lower bending strength of the inner rotor (*because of thinner dedendum*). However, the area of pump chamber expands in that case.

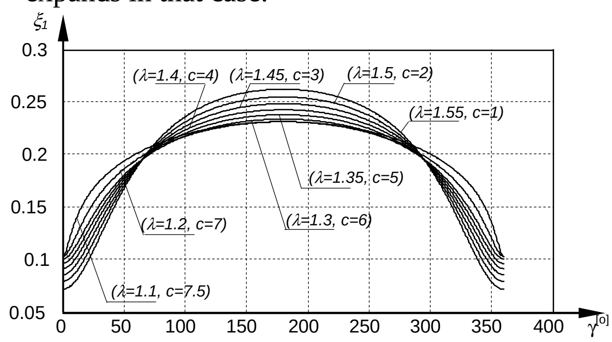

Figure 13. Sliding curve ξ_1 with respect to λ, c.

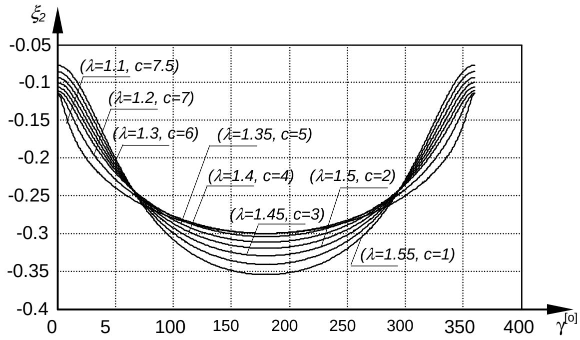

Figure 14. Sliding curve ξ_2 with respect to λ, c.

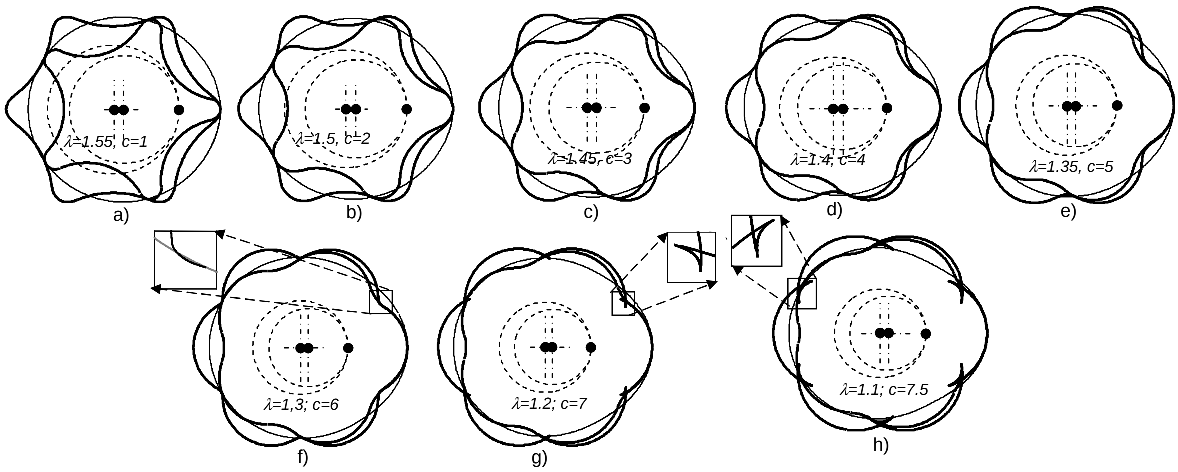

Figure 15. The gear pairs with respect to λ and c.

Figures 10, 11, 13 and 14 show that the parameter c has greater influence on the sliding coefficient than the parameter λ does. Therefore, in order to lower the sliding coefficient, it is recommended to increase the addendum radius of the inner rotor r_{cl}. On the other hand, from Figures 12 and 15, we can see that if we can not choose an appropriate parameter c, it can not only lead to undercutting of the dedendum of the outer rotor, but also can cause the jamming effect between the teeth of the inner and outer rotors (Fig. 15f), as well as the interference of profiles (Figs.15g, h). The smaller value of c can weaken the dedendum, but also leads to enlargement of the pump chambers.

5. CONCLUSION

From this work, if we choose *($c = 3$, $\lambda = 1.5$)* as referred from pages (38 – 40) of [4] when designing the internal hypocycloidal gear-train, only criterium of balanced distribution of dendendum and adendum of the outer rotor was satisfied. The parameters ξ, ξ_2 were not seriously affected.

Through notes in section 4, we can see that the parameter c impacts on the profile slip coefficient more than the parameter λ does. Therefore, to reduce the sliding phenomenon, the designer should prefer tuning parameter r_{cl} more than tuning λ. When λ increases, the profile shift coefficient does not clearly decrease, but the radial dimension will increase rapidly.

From the research results, we find that proper selection of λ and c could impact a lot on designing process of the internal hypocycloidal gear-train. Therefore, it is necessary to take into consideration following points: (i) Selection of λ, c needs to avoid undercutting and interfering of the hypocycloidal gears: $1 < \lambda < 1 + \frac{z_1 - 1}{z_1 + 2}$ and $0 < c \le z_1 \left(\frac{3}{z_1 - 1} \right)^{3/2} \sqrt{(\lambda^2 - 1)(z_1 + 1)}$, this problem was presented in [8]. (ii) The set of equations (13) allows designers to assess and select parameters *(λ, c)* in order to guarantee balanced profile wear during engagement. It means that the designers should select *(λ, c)* so that ξ_1 *(the inner rotor addendum against the outer rotor dedendum)* and ξ_2 *(the outer rotor dedendum against the inner rotor addendum)* will have nearly same value.

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