

APPLICATION OF HEDGE ALGEBRAS FOR CONTROLLING MECHANISMS OF RELATIVE MANIPULATION

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ABSTRACT

This paper presents a method for controlling a mechanism of relative manipulation (MRM robot) that based on an algebraic approach to linguistic hedges in the fuzzy logic. The proposed model of MRM robot is introduced as a two-component mechanism of which two serial robots co-operate with each other to realize technological manipulations. MRM robot has complex structure; therefore, the robot system's mathematical equations describing dynamical behaviors are voluminous. Furthermore, the external components affect MRM robot's dynamics that are difficult to determine adequately and exactly. Applying the well-known methods (based on dynamical equations) such as PD/PID, computed torque algorithm etc. for robot control is difficult, especially with MRM robot. By dint of the human-like inference mechanism, designing controller via the fuzzy logic can overcome the mentioned drawbacks. However, the linguistic variables in the fuzzy logic are not represented by any physical values; and hence, the comparison among linguistic variables is unable. Moreover, the composition of fuzzy relations and the defuzzification use approximation functions which trigger error in data process. Hedge Algebras (HA) gives favorable conditions to restrict the fuzzy logic's drawbacks because the linguistic labels in Hedge Algebras are represented by the semantic values; and, the composition of fuzzy relations and the defuzzification are processed by simple interpolation and mapping functions. The obtained results from HA controller are compared to the obtained results from two methods which are fuzzy controller and computed torque controller.

Keywords: mechanism of relative manipulation (MRM robot), hedge algebras.

1. INTRODUCTION

The mechanism of relative manipulation (MRM) robot includes two serial/parallel robots which has complex structure [1, 2]; thus, the robot's dynamical equations are complicated [3, 4] and [5]. It causes the difficulty when applying classical algorithms for robot control. In [6], the fuzzy logic is applied to control MRM robot that overcomes the mentioned problem. Besides,

the obtained results from fuzzy controller are compared with the obtained results from computed torque controller to evaluate the reliability. The comparison shows the benefits when applying the fuzzy logic to control MRM robot. However, the application of fuzzy logic still exists several limitations, namely the fuzzy logic's linguistic variables cannot be compared with each other, the use of fuzziness measure and composition of fuzzy relations is still complicated; the defuzzification must be processed by approximation functions; and hence, it leads to the error in implementation process. With specific features, Hedge Algebras (HA) can overcome aboved drawbacks. Firstly, the linguistic labels in HA are represented by the physical values (namely the semantic values of each linguistic label is calculated by the semantically quantifying mapping), so they can be compared with each other. Moreover, the input and output do not need the defuzzification because the semantically quantifying mapping of hedge algebras maps input/output's physical values to the value domain $[0, 1]$ to process; and therefore, to calculate the physical values of controller's outputs, we just need to map their semantic value (belongs to $[0, 1]$) to their physical domain. The calculation of output's semantic value is conducted by an interpolation function. The product operator and the average operator are usually used to transform m-dimension SAM table (the table of the semantic relationship between inputs and outputs) to 2-dimension SAM table, and then building the interpolation function. By doing this, the composition of fuzzy relations, the fuzziness measure and the defuzzification are avoided. As a results, the complexity is reduced and the accuracy is enhanced. Several papers applied HA in control and obtained reasonable results, as [7 - 16]. However, applying HA to robot control is relatively new. This paper proposes a method for controlling a mechanism of relative manipulation that based on an algebraic approach to linguistic hedges in fuzzy logic. To evaluate the proposed method, the obtained results from HA controller are compared to the obtained results from two methods which are presented in [6].

2. MRM ROBOT'S KINEMATIC AND DYNAMICAL MODEL

Figure 1 presents 5-DOF MRM robot model which is used for welding (or lazer machining), including 3-DOF tool-robot (T), 2-DOF jig-robot (B), and technological object (W), (as presented in [6]). The tool-robot consists of unmovable base L_{10} , movable links are denoted by L_{11} , L_{12} and L_{13} . The jig-robot consists of unmovable base L_{20} and movable links L_{21} , L_{22} .

2.1 MRM robot's manipulation motion

The motion of the links:

Links L_{11} and L_{12} rotate around axes z_{10} and z_{11} respectively which are performed by corresponding angles Θ_{11} and Θ_{12} ; link L_{13} translates along axis z_{12} which is performed by d_{13} .

Links L_{21} and L_{22} rotate around axes z_{20} and z_{21} respectively which are performed by corresponding angles Θ_{21} and Θ_{22} .

The adequately generalized coordinates of MRM robot is presented by:

$$\mathbf{q} = [q_1, \dots, q_5]^T = [\theta_{11}, \theta_{12}, d_{13}, \theta_{21}, \theta_{22}]^T \quad (2.1)$$

MRM robot's kinematic parameters are taken as given in [6] and presented in Table 1.

Parameters d_{11} , a_{11} , a_{12} are kinematic dimensions of the tool-robot, Θ_{20} , d_{20} , a_{20} , α_{20} , are Denavit-Hartenberg parameters describing the position and the orientation of the jig-robot's base frame in the general base frame.

Table 1. Parameters describe the robot structure.

d_{11}	a_{11}	a_{12}	Θ_{20}	d_{20}	a_{20}	α_{20}	d_{22}
0.82(m)	0.33(m)	0.33(m)	0	0.3(m)	0.45(m)	-90^0	0.1(m)

During the implementation process of a technological manipulation, link L_{13} brings the tool performing the relative motion to the workpiece (set-up on movable jig-table L_{22}). The welding is conducted by the tool's manipulation motions which work on the workpiece. These motions are represented through robot's manipulation coordinates in the moving space. The manipulation coordinates are determined by the position and the orientation of Cartesian coordinates $x_{13}y_{13}z_{13}$ (on link L_{13}) in Cartesian coordinates $x_{22}y_{22}z_{22}$ (on movable jig-table L_{22}). The robot's manipulation coordinates are presented by:

$$\mathbf{p} = [p_1, p_2, \dots, p_6]^T = [{}^{22}x_{p13}, {}^{22}y_{p13}, {}^{22}z_{p13}, {}^{22}\alpha_{p13}, {}^{22}\beta_{p13}, {}^{22}\eta_{p13}]^T \quad (2.2)$$

The bottom right indexes of elements are represented by index of L_{13} , the upper left indexes are represented by index of L_{22} .

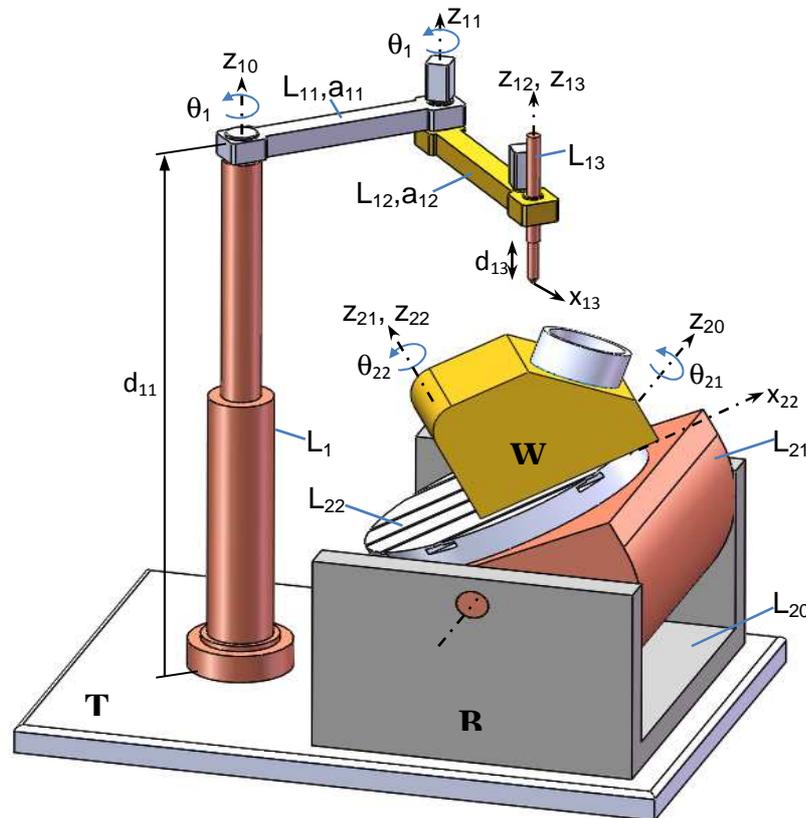


Figure 1. MRM robot – workpiece.

Depending on the technological object and the kind of technological manipulation, the elements of the vector \mathbf{p} are determined as the function of time or numeric data. As mentioned, to compare the proposed method with the computed torque controller and the fuzzy controller (as presented in [6]), we consider the technological manipulation that MRM robot welded tube part to machine part.

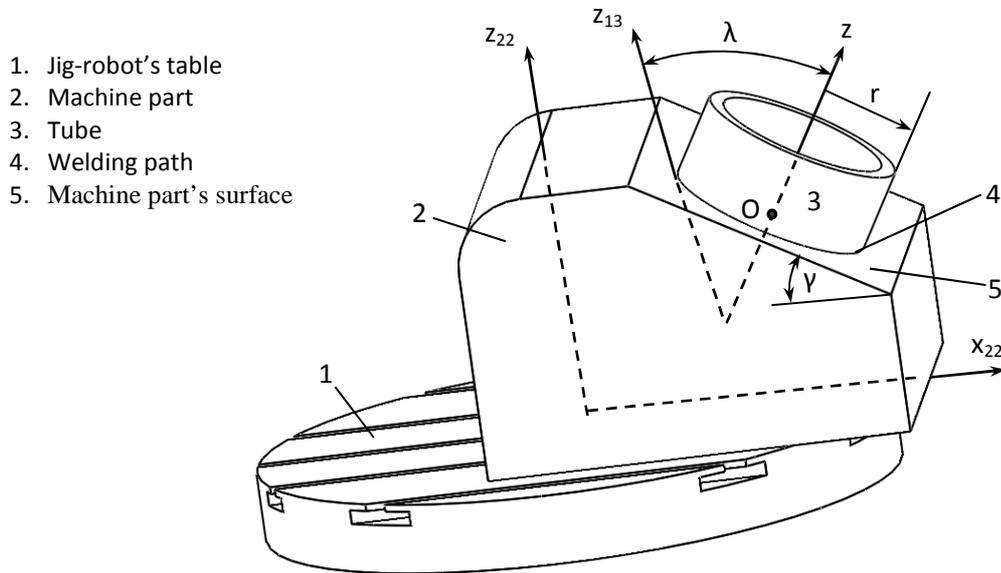


Figure 2. Welding path is realized by MRM robot.

The method of kinematic investigation in [6] is used for this paper; and, the welding process is shown in Figure 2. Machine part 2 is on jig-robot's Table 1 which is welded with tube 3 following butt-weld 4, with parameters:

- The path of the butt-weld 4 is intersection of tube surface with machine part's surface 5, which is a parallel plane with y_{22} axis and slope an angle γ to x_{22} axis.
- The tube axis z is perpendicular to the surface 5.
- Center O of welding path's circle have coordinates (x_o, y_o, z_o) in the frame $x_{22}y_{22}z_{22}$, radius of the tube is r .
- The axis of welding tool (welding gun) z_{13} is coplanar and sloping an angle λ with tube axis.
- The velocity v_h of welding tool's head along the welding path which is given based on the welding technics.

Table 2 shows the parameters appearing in this application which are the same in [6] except for v_h (in [6], $v_h = 0.2$ m/s).

Table 2. Parameters describe the machining object.

γ	x_o	y_o	z_o	r	λ	v_h
30^0	0.1(m)	0.1(m)	0.1(m)	0.07(m)	45^0	0.0275(m/s)

By virtue of technological object, the technical requirements for welding manipulation as aboved and parameters in Table 2; the relative position of welding gun need to be determined, namely the elements of \mathbf{p} (2.2). The kinematics and plan motion for robot is investigated by applying the same method in [2] and [6]. The obtained trajectory will be the input data (desired trajectory) for the controller. These data are saved to files **Position.txt**, **Velocity.txt**, **Acceleration.txt** which contain robot's trajectory, including corresponding position, velocity and acceleration.

2.2. MRM robot's dynamical equations

As given in [6], the robot's dynamical equations are written in the matrix form:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{Q} = \mathbf{U} \quad (2.3)$$

where

$$\mathbf{M}(\mathbf{q}) = \left[\sum_{i=1}^5 \left(\mathbf{J}_{Ti}^T m_i \mathbf{J}_{Ti} + \mathbf{J}_{Ri}^T \Theta_{Ci} \mathbf{J}_{Ri} \right) \right]_{5 \times 5} \quad (2.4)$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = [c_1, c_2, \dots, c_5]^T, \quad c_j = \sum_{k,l=1}^5 (k, l; j) \dot{q}_k \dot{q}_l, \quad (k, l; j) = \frac{1}{2} \left(\frac{\partial m_{kj}}{\partial q_l} + \frac{\partial m_{lj}}{\partial q_k} - \frac{\partial m_{kl}}{\partial q_j} \right) \quad (2.5)$$

$$\mathbf{G}(\mathbf{q}) = [g_1, g_2, \dots, g_5]^T, \quad g_j = \frac{\partial \Pi}{\partial q_j} \quad (2.6)$$

The values m_i , \mathbf{J}_{Ti} and \mathbf{J}_{Ri} are the mass, the translational Jacobian, and the rotational Jacobian respectively of link I ; Θ_{Ci} is the inertia tensor of the i^{th} link about center of mass C_i which is expressed in the frame with origin at the i^{th} link's center of mass; $i = 1, \dots, 5$; Π is the potential energy of the system; $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of the joint positions (2.1), velocities, accelerations, respectively; $(k, l; j)$ is Christoffel notations; m_{kl} ($k, l = 1, \dots, 5$) are elements of the mass matrix $\mathbf{M}(\mathbf{q})$.

The vector \mathbf{U} is the expression of the calculated force/torque which matchs the programed motion of the robot [17, 18].

$$\mathbf{U} = [U_1, U_2, \dots, U_5]^T \quad (2.7)$$

The vector \mathbf{Q} is the expression of the generalized force of friction, the disturbance as well as other non-conservative forces applying on the robot.

$$\mathbf{Q} = [Q_1, Q_2, \dots, Q_5]^T \quad (2.8)$$

In general, it is hard to build up exactly the robot's dynamical equations because of the difficulty of their determination. To solve these drawbacks, a method based on the fuzzy logic which is proposed in [6] for controlling MRM robot. However, the fuzzy method still exists some shortcomings. This paper proposes a method based on HA to overcome above drawbacks and restrict the shortcoming of the fuzzy controller.

We assume that \mathbf{Q} can be determined exactly to have comparable base between the methods. The vector \mathbf{Q} is the combination of friction and disturbance forces. It depends on the generalized position and the generalized velocity:

$$\mathbf{Q}_{fr} = \mu[\dot{q}_1, \dots, \dot{q}_5]^T \quad (2.9)$$

$$\mathbf{Q}_{dis} = \delta[\sin(q_1)+1, \cos(q_2)+1, \sin(q_3), \sin(q_4)\cos(q_4), \sin(2q_5)]^T \quad (2.10)$$

then

$$\mathbf{Q} = \mathbf{Q}_{fr} + \mathbf{Q}_{dis} \quad (2.11)$$

with $\mu = 0.003$ and $\delta = 0.5$.

Based on this condition, the computed torque method is conducted to obtain the clear control results.

\mathbf{Q} is not used for the fuzzy method and the HA method. The obtained results from three methods will be compared with each other to evaluate the HA controller.

3. HEDGE ALGEBRAS CONTROLLER

The technical requirements are described in the previous section which are the position, orientation and the velocity of the welding gun in welding technics. To control the welding gun following the desired welding path in jig-table's Cartesian coordinates, MRM robot's joints are controlled following their trajectory which are received from the inverse kinematics. Throughout this paper, $q_i(t)$ and $\dot{q}_i(t)$ present actuated joint's position and velocity of MRM robot, $q_i^d(t)$, $\dot{q}_i^d(t)$ present desired position and velocity.

The controller is designed so that $q_i(t)$ and $\dot{q}_i(t)$ follow $q_i^d(t)$ and $\dot{q}_i^d(t)$. By doing this, the welding gun follows the desired welding path in the Cartesian coordinate system of the jig-robot's table. In other words, the control purpose is to adjust the driving torque at actuated joints so that the position error $e(t)$ and the velocity error $\dot{e}(t)$ are small as desire.

The position error and the velocity error are computed by:

$$\begin{cases} e_i(t) = q_i(t) - q_i^d(t) \\ \dot{e}_i(t) = \dot{q}_i(t) - \dot{q}_i^d(t) \end{cases} \quad (3.1)$$

In [6], the fuzzy controller is proposed for controlling MRM robot which aims to reduce the difficulty of the dynamical identification. In addition, the computed torque controller is used to compare with the fuzzy controller. This paper uses these methods to get the results which are used to compared with the obtained results from proposed controller. The control rules and the parameters of fuzzy controller and computed torque controller are similarly chosen in [6].

3.1. Background of hedge algebras

An algebra structure called hedge algebras - HA which is defined $AX = (\mathbf{X}, \mathbf{G}, \mathbf{H}, \leq)$ therein $\mathbf{X} = \text{Dom}(X)$ is a term-set of a linguistic variable X , $\mathbf{G} = \{0, \mathbf{c}^-, \mathbf{W}, \mathbf{c}^+, \mathbf{1}\}$ is the set of the primary terms of linguistic variables (\mathbf{W} is neutral element), $\mathbf{H} = \{h^-, h^+\}$ is a set of unary operations representing linguistic hedges and " \leq " is a partially ordering relation in \mathbf{X} . [12], [13].

If \mathbf{X} and \mathbf{H} are linearly ordered sets then AX is the linear hedge algebras. By dint of the action-effect of the hedges $h \in \mathbf{H}$ on element $x \in \mathbf{X}$, we observe $\mathbf{H}(x)$ is a set of all elements $hx \in \mathbf{X}$ and $hx = h_n \dots h_1 x$ with $h_n, \dots, h_1 \in \mathbf{H}$; and therefore, $\mathbf{H}(x)$ is a linearly ordered set [14].

If \mathbf{X} is created from \mathbf{G} and hedges therein \mathbf{G} is a linearly ordered set, then \mathbf{X} is also a linearly ordered set. Moreover, with u and v belong to \mathbf{X} , if $u < v, u \notin \mathbf{H}(v)$ and $v \notin \mathbf{H}(u)$, then $\mathbf{H}(u) < \mathbf{H}(v)$. In addition, if $u = h_n \dots h_1 x$ and $v = h_m \dots h_1 x (m < n)$, then u is more special than v . Elements in \mathbf{H} are comparable; and, if $h \neq k$ and $hx = kx$ then x is a fixed point [13], [14], [15].

Fuzziness measure: a given fuzziness measure of element τ called $fm(\tau)$, $\tau \in \mathbf{X}$ that the values of $fm(\tau)$ always belong to $[0, 1]$ [16].

$$(1) fm(\tau) = 0, \text{ if } \tau \text{ is clear.} \tag{3.2}$$

(2) If h is a hedge and τ is a fuzzy value then $h\tau$ is more featured than τ , so we have

$$fm(h\tau) < fm(\tau) \text{ and } fm(h\tau) = \mu(h)fm(\tau), \text{ with } \forall \tau \in \mathbf{X}. \tag{3.3}$$

(3) If $c+, c-$ are two primary terms in \mathbf{X} , then

$$fm(c+) + fm(c-) = 1. \tag{3.4}$$

Semantically quantifying mapping: By fm is a fuzziness measure of \mathbf{X} , semantically quantifying mapping $v: \mathbf{X} \rightarrow [0,1]$ is determined [12]:

$$(1) \square(\mathbf{W}) = \theta = fm(c), \square(c^-) = \theta - \alpha fm(c^-) = \beta fm(c^-), \square(c^+) = \theta + \alpha fm(c^+) \tag{3.5}$$

$$(2) \square(h_j x) = \square(x) + \text{Sign}(h_j x) \cdot \left\{ \sum_{i=\text{Sign}(j)}^j fm(h_i x) - \omega(h_j x) fm(h_j x) \right\} \tag{3.6}$$

where $\omega(h_j x) = \frac{1}{2}[1 + \text{Sign}(h_j x)\text{Sign}(h_p h_j x)(\beta - \alpha)]$, and

$$j \in \{-q \leq j \leq p \& j \neq 0\} = [-q \dots p]. \sum_{i=-q}^{-1} \mu(h_i) = \alpha \text{ and } \sum_{i=1}^p \mu(h_i) = \beta,$$

with $\alpha, \beta > 0$ and $\alpha + \beta = 1$.

\mathbf{W} is neutral element.

In the fuzzy logic, the composition of fuzzy relations is demonstrated by m dimensional FAM table (the table of the fuzzy rules among the inputs and outputs), we will use the semantically quantifying mapping to convert FAM table to semantic table SAM.

3.2. Applying hedge algebras for controlling MRM robot

Step 1: Determining inputs/outputs

The inputs of hedge algebras controller including $\mathbf{q}^d(t), \dot{\mathbf{q}}^d(t), \mathbf{q}(t), \dot{\mathbf{q}}(t)$ which are desired trajectory, velocity and real trajectory, velocity of MRM robot. The output is adjusted torque $\mathbf{u}(t)$ at actuated joints so that $\mathbf{q}(t), \dot{\mathbf{q}}(t)$ follow $\mathbf{q}^d(t), \dot{\mathbf{q}}^d(t)$, in other words, \mathbf{e} and $\dot{\mathbf{e}}$ are small as desire:

$$u_i = \tau_i - \tau_i^d$$

here in τ_i is real driving torque set at the i^{th} actuated joints to e_i and \dot{e}_i come to 0; τ_i^d is theoretical driving torque set at the i^{th} actuated joints to e_i and \dot{e}_i come to 0.

Now, we can consider controller's inputs of $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t)$, output of $\mathbf{u}(t)$

$$\mathbf{e} = [e_1, \dots, e_5]^T$$

$$\mathbf{de} = [\dot{e}_1, \dots, \dot{e}_5]^T$$

$$\mathbf{u} = [u_1, \dots, u_5]^T$$

where e_i, \dot{e}_i, u_i are position error, velocity error and adjusted torque at the i^{th} actuated joint of MRM robot, respectively. Approximately physical domain of inputs/outputs are later used in Table 6.

Step 2: Choosing hedge parameters

Owing to the number of linguistic terms in the fuzzy controller, we chose hedge parameters so that each input/output e_i, \dot{e}_i, u_i will be demonstrated by five linguistic labels with five semantic values. This gives the same condition when designing two controllers for the purpose of comparing the proposed controller and the fuzzy controller. The set of the primary terms of linguistic variables and hedges are chosen such that if applying hedges to the set of the primary terms of linguistic variables, then we observe five linguistic values which present for each input/ output e_i, \dot{e}_i, u_i of each joint:

$$\mathbf{G} = \{0, S, W, B, 1\} \text{ with } S = \text{Small}; B = \text{Big}.$$

$$\mathbf{H} = \{H^-, H^+\} \text{ with } H^- = L; H^+ = V \Rightarrow q=1, p=1 \text{ herein } L = \text{Little}; V = \text{Very}.$$

Now, each input/output e_i, \dot{e}_i, u_i are presented by one hedge algebras which is created from the set of the primary terms of the linguistic variable \mathbf{G} and the set of hedges \mathbf{H} through a linearly ordering relation. However, we just use the partition $k = 2$ when creating linguistic values in each hedge algebras which aims to compare with the fuzzy controller. So, the linguistic terms in hedge algebras are presented by $\mathbf{X} = \{VS, LS, W, LB, VB\}$. In the fuzzy controller, the linguistic values are the same for inputs/ outputs, but their physical domains are different. In the same way, with each hedge algebras present e_i, \dot{e}_i and u_i , their linguistic values are the same; and therefore, they will have the same semantic values. However, those values just describe semantics in individual input/ output's physical domains; and hence, by mapping their semantic values to their real physical domain, we observe different values for different inputs/ outputs.

The values 0 and 1 in \mathbf{G} present elements which have the smallest semantic value and the biggest semantic value, respectively; and, they are fixed points. The values 0 and 1 will be used when processing the input/ output by merging operators to avoid the loss of the information.

By virtue of \mathbf{G} and \mathbf{H} , the fuzziness of the primary terms and hedges are measured through (3.2), (3.3) and (3.4).

$$fm(S) = \theta = 0,5 \cdot \mu(L) = \mu(V) = 0,5 \rightarrow \alpha = \beta = 0,5 \text{ and } fm(B) = 1 - fm(S) = 0,5.$$

In [6], the fuzzy terms are AL, AN, Z, DN and DL of which A, D, Z stand for *negative*, *positive* and *zero*, respectively; whereas N and L stand for *small* and *big*, respectively. Now, the fuzzy terms are transformed to HA-terms:

$$AL \Rightarrow VS \quad AN \Rightarrow LS \quad Z \Rightarrow W \quad DN \Rightarrow LB \quad DL \Rightarrow VB$$

Step 3: The semantic representation of linguistic labels

The usual fuzzy rule base is converted into a linguistic rule base to convert the ordinary linguistic labels into HA-terms. By virtue of above parameters, (3.5) and (3.6), the FAM table is transformed to SAM table. In fuzzy controller, the composition of fuzzy relations is presented via Table 3 [6], including linguistic values which describe inputs/ outputs e_i, \dot{e}_i, u_i of controller. However, in hedge algebras, a set of rules is demonstrated by semantic Table 4 (m-SAM table), including the semantic values of the linguistic labels which describe e_i, \dot{e}_i, u_i . If inputs e_i, \dot{e}_i

belong to linguistic domains, with semantic value in $[0, 1]$, then the controller will respond the output signal u_i belong to the linguistic domain, with suitable semantic value to adjust e_i, \dot{e}_i to 0.

Table 3. FAM table (GTNN: linguistic values).

GTNN	AL	AN	Z	DN	DL
AL	DL	DL	DL	DN	Z
AN	DL	DN	DN	Z	AN
Z	DL	DN	Z	AN	AL
DN	DN	Z	AN	AN	AL
DL	Z	AN	AL	AL	AL

Table 4. m-SAM table (SV: semantic values).

SV	$v(VS) = 0,125$	$v(LS) = 0,375$	$v(W) = 0,5$	$v(LB) = 0,625$	$v(VB) = 0,875$
$v(VS) = 0,125$	0,875	0,875	0,875	0,625	0,5
$v(LS) = 0,375$	0,875	0,625	0,625	0,5	0,375
$v(W) = 0,5$	0,875	0,625	0,5	0,375	0,125
$v(LB) = 0,625$	0,625	0,5	0,375	0,375	0,125
$v(VB) = 0,875$	0,5	0,375	0,125	0,125	0,125

Table 5. 2-SAM table.

0	1
0.0156	0.875
0.0469	0.875
0.0625	0.875
0.0781	0.625
0.1094	0.5
0.1406	0.625
0.1875	0.625
0.2344	0.5
0.3281	0.375
0.25	0.5
0.3125	0.375
0.4375	0.125
0.3906	0.375
0.5469	0.125
0.7656	0.125
1	0

m-dimensional SAM table presents the semantic relationship between the input and the output; and, it represents a super surface in 3-D space. Merging operator will be used to exchange m dimensional SAM table to 2-dimensional SAM table which aims to simplify the interpolation. This paper use Product operator.

Table 5 represents 2-dimensional SAM table which is a curve in 2-D space R^2 . The horizontal axis is the integration of the semantic values of the linguistic labels which demonstrate e_i and \dot{e}_i and the vertical axis is the semantic value of the linguistic label which describes u_i . We can interpolate polynomial easily. In addition, a set of rules is chosen to be symmetry about semantics, so there is no point which has the similar horizontal degree with other ones.

No case in interpolation should be left out, if input's physical value is on the left of linguistic value AL's physical domain then its semantic value is $v(0)=0$. If input's physical value is on the right of linguistic value DL's physical domain then its semantic value is $v(1)=1$

Step 4: Determining output's physical value

The controller's outputs are adjusted torques at actuated joints; however, the interpolation based on 2-dimensional SAM table which gives the semantic values of the linguistic variables. We need to determine real physical values of outputs via their semantics. To receive outputs' real physical values, we map their semantic values from [0, 1] to their individual physical domains.

The simulink model for HA-controller is presented in Figure 3.

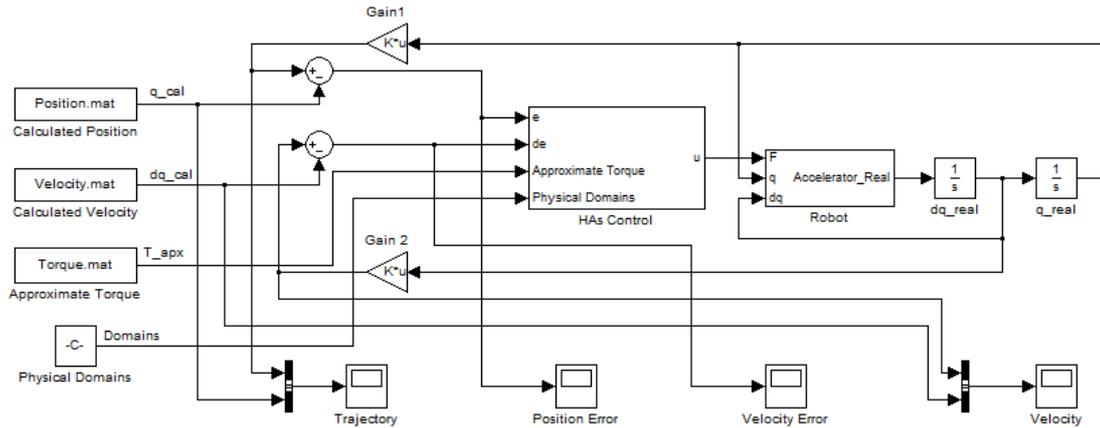


Figure 3. Simulink model for HA controller.

herein **Position.mat** and **Velocity.mat** are desired position and velocity which are obtained from kinematic problem and trajectory planning. **Physical Domain** is approximate domains of inputs and outputs, **Torque.mat** is approximate torque. **HAs Control** is controller via hedge algebras; **Robot** is the robot model, all are programmed in **M-File** which are inserted into **Simulink** by **Matlab Function**.

3. SIMULATION RESULTS

The proposed model of MRM robot is described in section (2); and, its kinematic and dynamical parameters are shown in Table 7. In this paper, the mass, the center of mass and the inertia tensor are received from design software. By doing this, the proposed model of MRM robot is more accurate and closer to real robot model (because robot's links are not homogeneous and its cross sections are not unchanged in reality). Based on the technical requirements in section (2.1) and robot's kinematics and dynamics, we obtain the results in Figure 4 - Figure 11 by applying above control methods.

Table 6. Physical domains of inputs/output of HA controller.

Joint	e_i	\dot{e}_i	u_i
1	[-0.002, 0.002] (rad)	[-0.06, 0.06] (rad/s)	[-42, 41.73] (N.m)
2	[-0.002, 0.002] (rad)	[-0.09, 0.09] (rad/s)	[-47,46.08] (N.m)
3	[-0.0001, 0.0001] (m)	[-0.08, 0.08] (m/s)	[-54.6, 55] (N)
4	[-0.0001, 0.0001] (rad)	[-0.05, 0.05] (rad/s)	[-68, 68] (N.m)
5	[-0.0001, 0.0001] (rad)	[-0.03, 0.03] (rad/s)	[-8.0, 8.1] (N.m)

Table 7. Kinematic and dynamical parameters of MRM robot.

Link		11	12	13	21	22
Weight(kg)		8	8	3.5	50	20
Coordinates of links' barycenter on local Cartesian coordinate system	x(m)	-0.165	-0.165	0	0	0.068845
	y(m)	0	0	0	0	0.022064
	z(m)	0	0	0.2	-0.077511	0.107575
Inertia tensor (kg.m ²)	I _{xx}	0.002818	0.002716	0.041672	1.36253	0.204889
	I _{yy}	0.108298	0.115332	0.041672	1.146869	0.188627
	I _{zz}	0.108825	0.115847	0.000166	1.861916	0.307898
	I _{xy}	0	0	0	0	0.005476
	I _{xz}	0.000381	-0.00002	0	0	0.008992
	I _{yz}	0	0	0	0	-0.002919

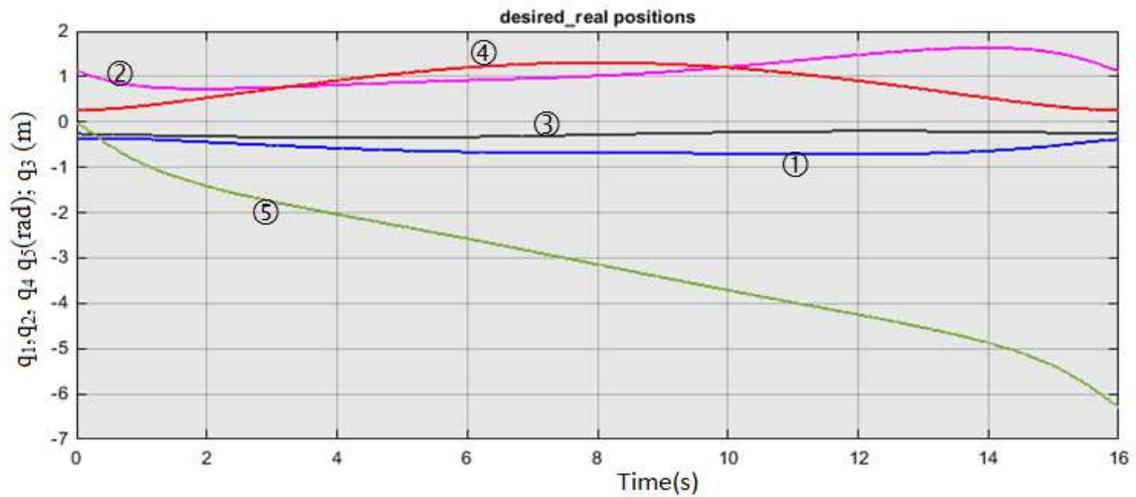


Figure 4. Graph of desired and real trajectory of MRM robot's 5 joints.

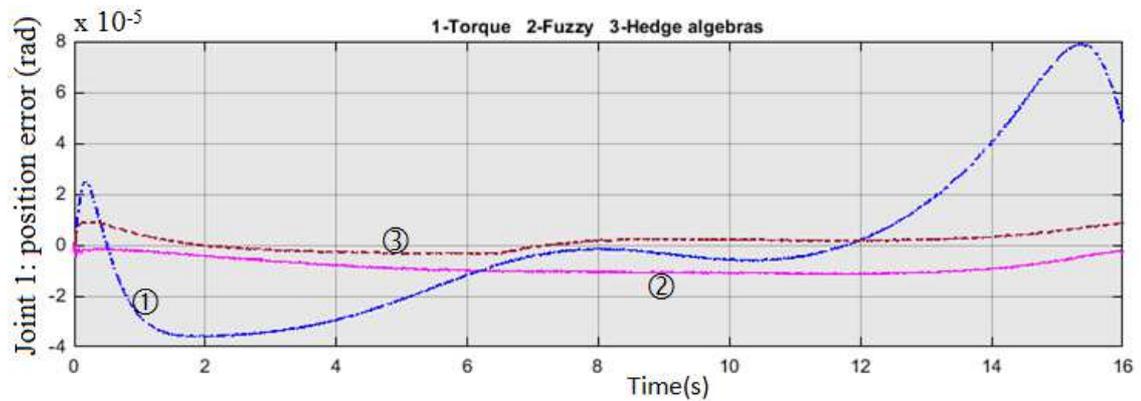


Figure 5. Graph of position error of the joint 11 (e_1).

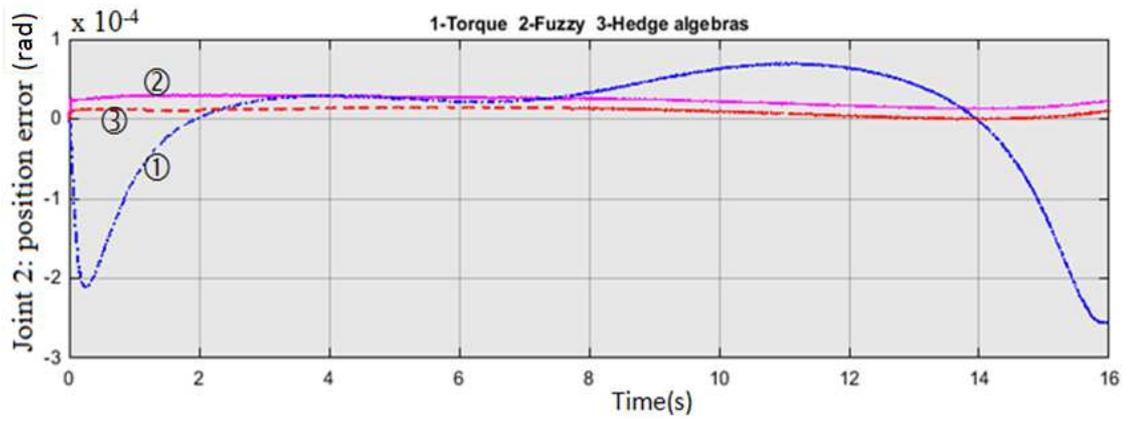


Figure 6. Graph of position error of the joint 12 (e_2).

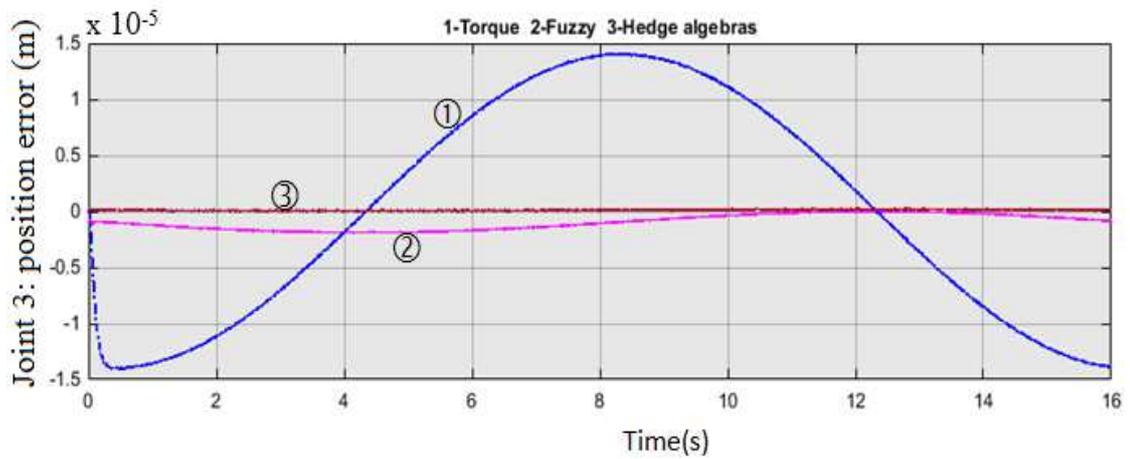


Figure 7. Graph of position error of the joint 13 (e_3).

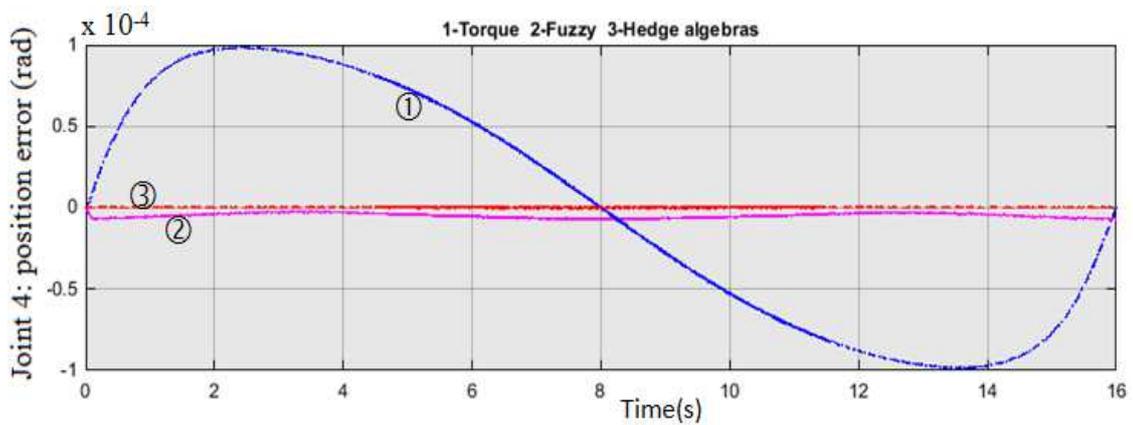


Figure 8. Graph of position error of the joint 21 (e_4).

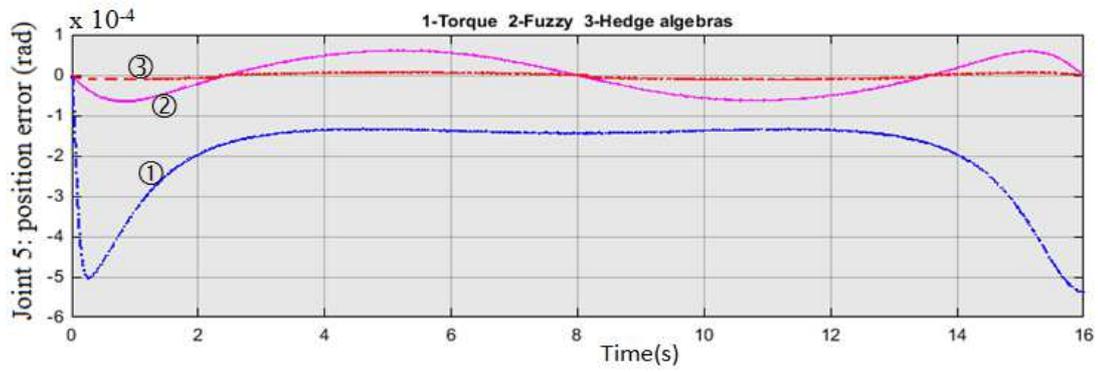


Figure 9. Graph of position error of the joint 22 (e_5).

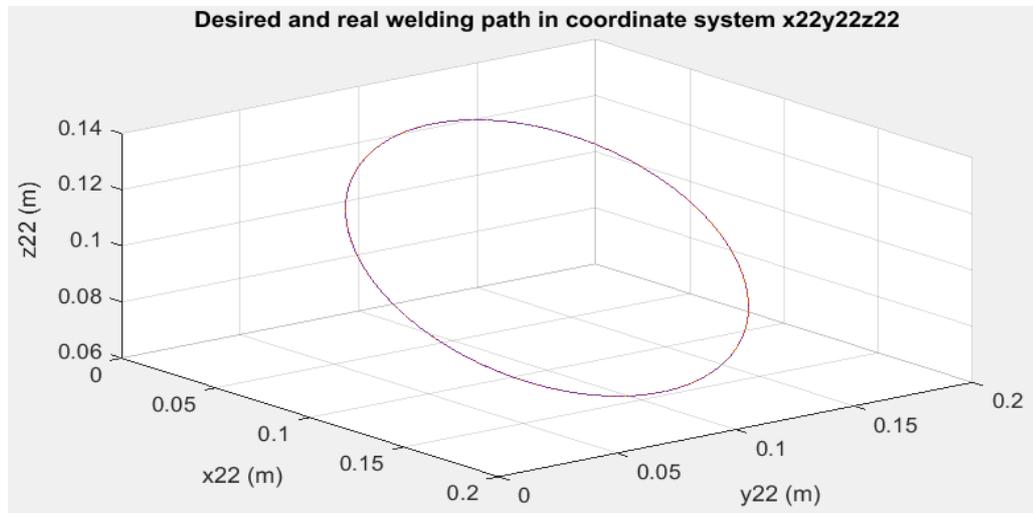


Figure 10. Desired and real welding path in Cartesian coordinate system $x_{22}y_{22}z_{22}$ of jig-robot's table (Fig. 1).

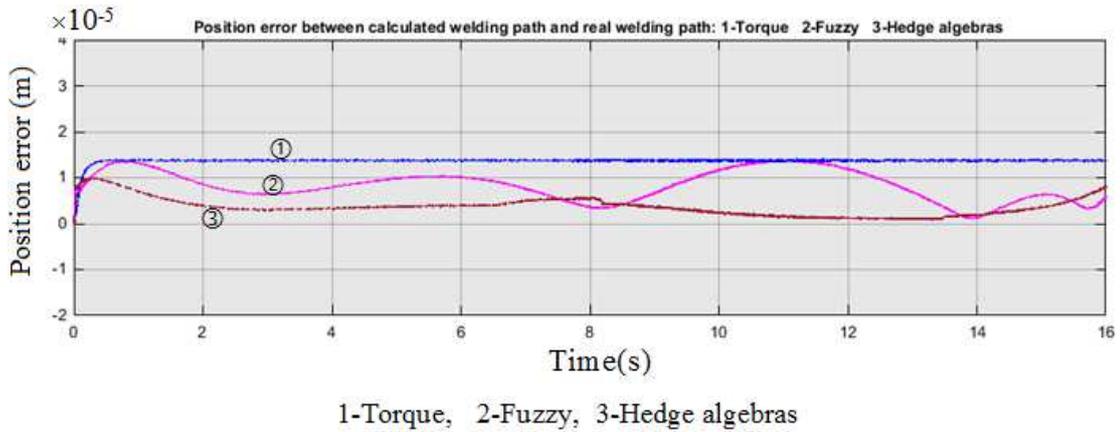


Figure 11. Position error between calculated welding path and real welding path

The desired and real welding path, received by torque, fuzzy and HAs controller overlap each other because the position error is very small (Fig. 10). Figure 11 shows the position error of each path.

5. CONCLUSION

The received results show that the proposed method is more accurate than the classical fuzzy method. Moreover, the application of HA in control will be easier to understand and simpler than the use of fuzzy logic. It brings several benefits for controlling engineering systems. This paper used 5 linguistic values for the fuzzification of inputs and outputs. To compare HA method with the fuzzy controller in the same condition, we just use the partition $k=2$ for inputs/output. The analysis and results contribute a meaningful part in the expansion of research and application of modern theories for controlling robots which have complex structure, such as MRM robot. However, the use of Product operator to convert m-SAM table to 2-SAM table can lead to the loss of the control information, so this problem will be analyzed in the next studies.

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