

## **EFFICIENT ANALYSIS OF MIMO-OFDM SYSTEM IN NAKAGAMI- $m$ FADING CHANNEL**

**Van-Cuu Ho, Hoang-Hiep Le<sup>\*</sup>, Thu-Hang Thi Nguyen**

*Faculty of Electronics – Telecommunications, Saigon University, 273 An Duong Vuong Street,  
Ward 3, District 5, Ho Chi Minh City, Vietnam*

Email: [lehoanghiep@gmail.com](mailto:lehoanghiep@gmail.com)

Received: 9 May 2016; Accepted for publication: 31 July 2016

### **ABSTRACT**

Bit error rate (BER) and symbol error rate (SER) performance response fading parameter for multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) system over Nakagami- $m$  fading channel would be provided in this study. Some channel simulations and evaluations were for  $2 \times 2$  MIMO-OFDM system in which data are sent from two transmit antennas to two receive antennas. In this study, authors have used binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK) modulation techniques to modulate the data stream. In addition, numerical results have been presented and compared for several different environmental propagations such as Nakagami- $m$ , Rician, Rayleigh multipath fading channels, and non-fading additive white Gaussian noise (AWGN) channel. Based on such obtained presentation and comparison, the paper would discuss further about relation between value of parameter  $m$  and quality of system.

*Keywords:* MIMO, OFDM, Nakagami- $m$  Fading, Rayleigh Fading, Rician Fading, AWGN.

### **1. INTRODUCTION**

MIMO-OFDM is one of interesting issues which have been discussed more and more nowadays. It has also played as an important technique for high speed wireless transmission in recent years. MIMO system with OFDM gives higher gain by using the direct and the reflected signals, thus facilitating the transmission at high data rate. Another main reason is that it presents a good efficiency for demand of increasing channel capacity in wireless and mobile communication due to reliable high speed data and multimedia services. Early works on MIMO-OFDM studied performance of this technique over some environmental propagations, including fading and non-fading channel, such as AWGN, Rayleigh and Rician. They shown that as result of using multiple antennas, MIMO wireless technology is able to considerably increase the capacity of a given channel while still obeying Shannon's law. Furthermore, since number of receive and transmit points is increased, the throughput of channel is also possible to be linearly increased. In addition, combination MIMO and OFDM is useful because OFDM technique now

can support for more antennas and larger bandwidths, this modulation method also simplifies equalization significantly in MIMO systems.

There are some pioneer researches may be listed here. First of all, performance of MIMO-OFDM system over AWGN, Rayleigh and Rician channel models implemented by using Least Square (LS), LS-Modified and Minimum Mean Square Error (MMSE) algorithm was presented in [1]. Similarly, in [2] the authors provided significant BER performance of the MIMO-OFDM system with two different equalizers as Zero-Forcing (ZF) Equalization, a linear detection technique, and MMSE estimation algorithm for various modulation techniques i.e. BPSK, QPSK, 16-QAM and 64-QAM. This study also run over channels AWGN, Rayleigh and Rician. Definition of 3D MIMO-OFDM, so called 3D pilot aided channel estimation (PACE), was proposed and discussed deeply in [3], as well as, bandwidth efficient pilot design for a MIMO-OFDM downlink was addressed. Besides, BER prediction of coded MIMO-OFDM systems and performance of Golden coded MIMO-OFDM system which were respectively discussed in [4] and [5] are some interesting researches as well.

In this research, authors investigate the effective for the case of MIMO-OFDM in the Nakagami- $m$  fading channel. Implementation was in several environmental propagations such as Rayleigh and Rician fading channel, and AWGN channel as well. Thence, some comparisons may obtained between different environmental propagations each other. In this study, BPSK and QPSK modulation methods were applied to modulate the data stream. The MATLAB tool was used to simulate for this system. Based on estimated BER results in the cases of fading parameter  $m$ , authors might figure out at which value of  $m$  is, an approximate minimum error for such conventional MIMO-OFDM system would be obtained.

The outline of the paper is as follows. In section 2, authors review basic theory of Nakagami- $m$  distribution. The section 3 describes for a typical MIMO-OFDM system. Some simulation results would be shown in section 4. Finally, concluding remarks are made in section 5.

## 2. NAKAGAMI- $m$ FADING CHANNEL

Nakagami- $m$  fading models assume that the magnitude of a signal that has passed through such a communications channel will fade according to a Nakagami- $m$  distribution, which was introduced by Nakagami in [6] to fit empirical data gathered from high frequency ionospheric channels. It provides the best fit for the fading amplitudes of the satellite-to-indoor and satellite-to-outdoor channel [7]. The probability density function (PDF) of this distribution for random variables  $R$  is given by

$$p(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{-(m/\Omega)R^2} \quad (1)$$

where  $R \geq 0$  denotes the channel amplitude,  $\Omega = E(R^2)$  is average fading power,  $E(\cdot)$  is the expectation operator, and  $\Gamma(\cdot)$  is gamma function, and  $m \geq 0.5$  is the inverse of the normalized variance of  $R^2$ , it is also called fading severity parameter and given by

$$m = E^2(R^2) / \text{Var}(R^2) \quad (2)$$

where  $\text{Var}(\cdot)$  is the variance operator.

Notice that parameter  $m$  indicates various fading conditions, e.g.  $m = 0.5$ , it represents a deeply fading channel,  $m = 1$ , it represents a Rayleigh fading channel, and  $m = \infty$ , it represents a non-fading (AWGN) channel. Respectively, the Fig. 1 and Fig. 2 below shows PDF and CDF of Nakagami- $m$  distribution with several different values of  $m$  and  $\Omega$ .

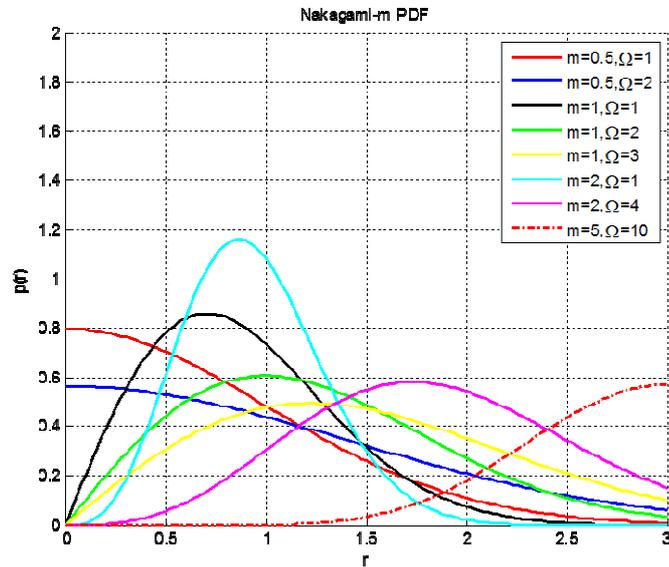


Figure 1. The PDF plot of Nakagami- $m$  distribution.

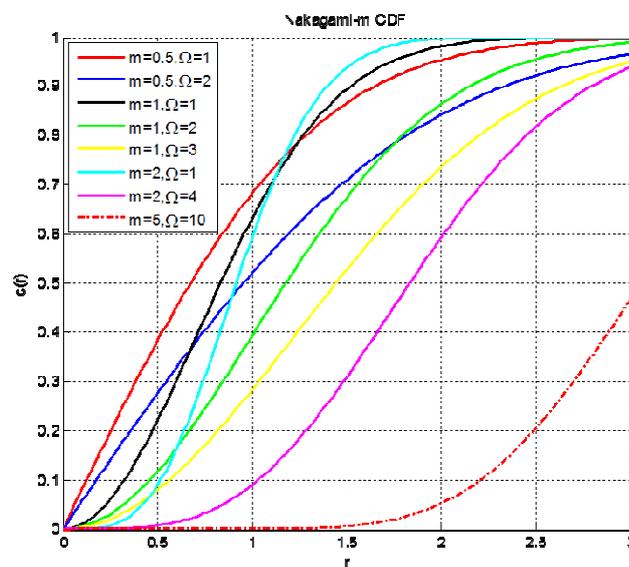


Figure 2. The CDF plot of Nakagami- $m$  distribution.

Since the value of  $m$  measures the channel quality, it is very important to obtain an accurate estimate of  $m$ , in advanced receiver implementations and in channel data analyses. There were dozen of articles proposed methods to simulate for a Nakagami model. A renowned one may be listed here is [8] where authors proposed a mathematical model for the case  $m < 1$ . Next, an

efficient method for generating correlated Nakagami- $m$  ( $0.5 \leq m \leq 10$ ) fading envelope samples was presented in another famous research [9]. Recently, a novel moment-based estimation of value of  $m$  between 0.5 and 5 using noisy channel samples has been provided in [10].

### 3. MIMO-OFDM SYSTEM MODEL

MIMO-OFDM is mentioned in more and more researches in wireless transmission field nowadays. It is spotted as a key technology for next-generation cellular communications due to its significant increase in capacity and spectral efficiency. One of main reasons is that this novel technique has made use of advantages from OFDM and MIMO system, such as, enables support of more antennas and larger bandwidths. This is useful for indoor wireless systems to reach up to several hundreds of Mb/s in data rate and achieve several tens of bits/Hz/s in spectral efficiencies. Besides, a MIMO-OFDM system also may increase spectral efficiency to attain throughput of more than one Gb/s, as well as improves link reliability. MIMO-OFDM system model provides high speed links that offer good quality of service, this will minimize the probability of error [11].

A simple model for a  $2 \times 2$  MIMO-OFDM system is shown in Fig. 3. We may consider the transmitting side first. In modulation block, signal are modulated applying some modulation techniques. In this study, authors used the BPSK and QPSK modulation methods. At the OFDM block broadband signal stream is transformed into a multiplicity of parallel narrow-band single channels. The cyclic prefix (CP) between the individual symbols are inserted in this block as well. Finally, OFDM signal would be conveyed to the receiver through two antennas over a fading channel. On the receiving side, in MIMO decoder block, the ZF detector, which invert the channel matrix is applied to convert collected signal.

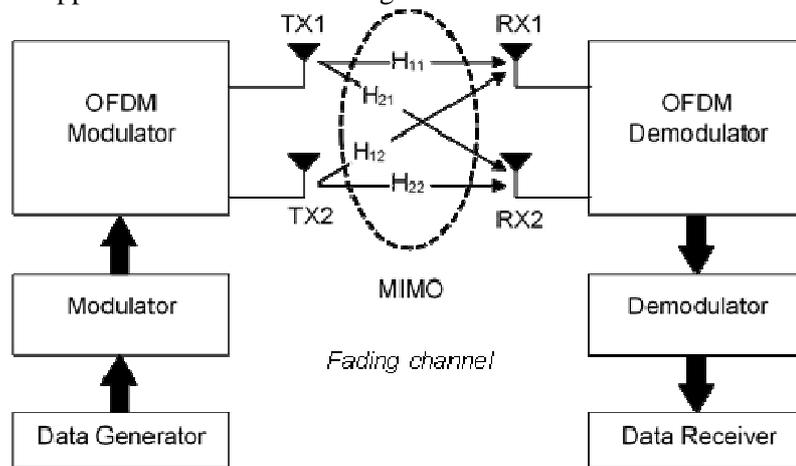


Figure 3. A conventional  $2 \times 2$  MIMO-OFDM system over Nakagami- $m$  fading channel.

The received signal can be represented by using the linear model as:

$$\mathbf{Y} = \mathbf{H} \times \mathbf{X} + \mathbf{n} \quad (3)$$

The equation above can be represented in matrix form as follow

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad (4)$$

where

- $y_1, y_2$  are received symbols on the first and second antenna respectively,
- $H_{11}$  is the Nakagami fading channel from first transmit antenna to the first receive antenna,
- $H_{12}$  is the Nakagami fading channel from second transmit antenna to the first receive antenna,
- $H_{21}$  is the Nakagami fading channel from first transmit antenna to the second receive antenna,
- $H_{22}$  is the Nakagami fading channel from second transmit antenna to the second receive antenna,
- $x_1, x_2$  are transmitted symbols on the first and second antenna, respectively;
- $n_1, n_2$  are the AWGN noise on the first and second receive antennas respectively.

In this considered  $2 \times 2$  MIMO-OFDM system, the transmitted signal  $x_u$  ( $u = 1, 2$ ) at output antenna is an OFDM given by

$$x_u^n(t) = \sum_{k=0}^{L-1} X_{n,k} g_k(t - nT) \quad (5)$$

where  $X_{n,k}$  denotes the transmitted signal of the  $k$ -th subcarrier at the  $n$ -th OFDM symbol interval,  $T$  is one OFDM symbol duration excluding the CP length,  $L$  is the number of OFDM subcarriers, and

$$g_k(t) = \begin{cases} e^{j2\pi f_k t}, & t \in [0, T) \\ 0 & \text{others} \end{cases} \quad (6)$$

and

$$f_k = f_0 + \frac{k}{T}, \quad k = 0, 1, \dots, L-1 \quad (7)$$

where  $f_k$  is the frequency of the  $k$ -th subcarrier,  $f_0$  is the lowest frequency. Thus, the received signal  $y_v$  ( $v = 1, 2$ ) at input antenna should be

$$\begin{aligned} y_v &= \sum_{u=1}^2 H_{vu} x_u^n(t) \\ &= \sum_{u=1}^2 \sum_{k=0}^{L-1} H_{vu} X_{n,k} g_k(t - nT) \end{aligned} \quad (8)$$

#### 4. NUMERICAL SIMULATION

In this section, the BER performance of a MIMO-OFDM system over Nakagami- $m$  channel and some other fading environments is provided. The MATLAB tool was used to simulate for the system. We may notice that since Nakagami- $m$  distribution related to Gamma distribution, thus in practical programming, we may generate the noise values of Nakagami- $m$  random variables by applying function *gamrnd(.)* in MATLAB. In particular, if the distribution of random variables are characterized as Nakagami ( $m, \Omega$ ), then they may be generated by the square root of which are characterized as Gamma ( $m, \Omega/m$ ). Besides, we may consider some methods which were described in [12] and [13] are also efficient generations for random variables of Nakagami- $m$  distribution. Some simulation results for Rayleigh and Rician fading channel are provided to

comparing. Random variables of these channels were generated by using function *randn()* in MATLAB combine with their own parameters, respectively.

In addition, to set up parameters for OFDM modulation subpart, authors followed the IEEE 802.11a specifications as the Table 1 below.

Table 1. Parameters for OFDM modulation.

Parameters	Value
FFT size	64
FFT sampling frequency	20 MHz
Number of OFDM subcarriers ( $L$ )	52
Subcarrier spacing ( $k/T$ )	312.5 KHz
Cyclic prefix (CP) duration	0.8us
Data symbol duration ( $T$ )	3.2 us

In this simulation, Monte Carlo method was used. Number of errors which were calculated by comparing the transmitted and the received data. The results of simulations are drawn and described below. First of all, Fig. 4 provides BER performance for 2x2 MIMO-OFDM system using BPSK modulation over Nakagami- $m$  channel.

In this simulation, value of  $\Omega$  was kept fixed and equal to 2. It is shown that, for the case  $m$  belongs to  $[0.5, 0.7]$ , bigger value  $m$  is, less error is. However, performance are not increasing when  $m \geq 1$ , but decreasing. These interesting results are not unreasonable. They were mentioned in [14] that in a multi-tap Nakagami fading communication system with  $m \geq 1$ , if we increase  $m$ , no reduction in BER has been reported rather it starts increasing.

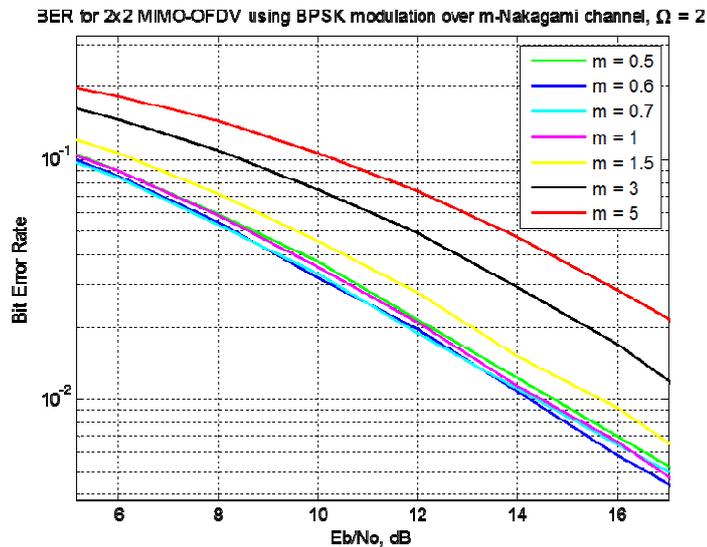


Figure 4. BER performance for 2x2 MIMO-OFDM using BPSK modulation over Nakagami fading channel,  $\Omega = 2$ .

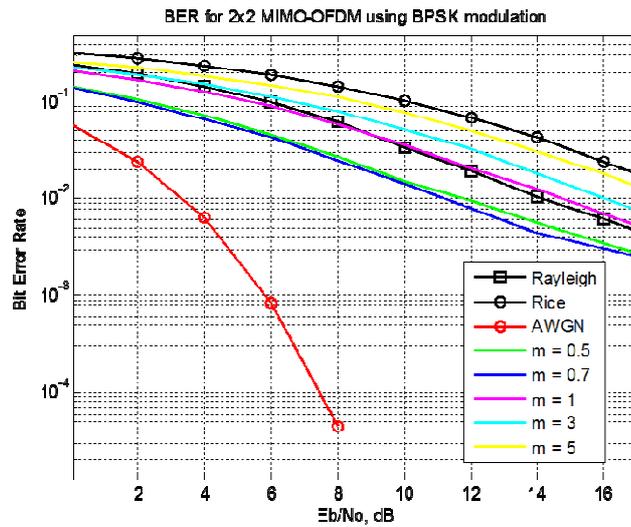


Figure 5. BER performance for 2x2 MIMO-OFDM system using BPSK modulation over different channels.

Furthermore, in order to compare the case of Nakagami- $m$  and some other transmission channels, simulations for Rayleigh, Rician and AGWN respectively applied. Fig. 5 showed that the Rician channel had worst BER value obviously, while the Nakagami- $m$  channel with  $m = 0.7$  performed second lowest error, just behind the AGWN channel. Moreover, when  $m = 1$ , it means there is no single line-of-sight path between receiver and transmitter, the Nakagami- $m$  channel provided a perfect fit to the performance obtained applying in the Rayleigh channel. For  $m = 3$  and 5, worse BER performances of Nakagami- $m$  channel was obtained. Thus, similar to above obtained results that when  $m \geq 1$  if  $m$  is increasing, the quality of received signal would be unimproved.

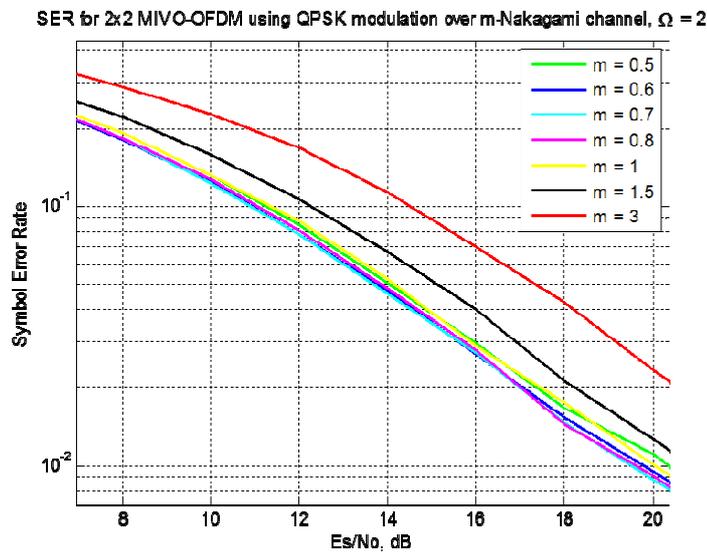


Figure 6. SER performance for 2x2 MIMO-OFDM using QPSK modulation over Nakagami- $m$  fading channel,  $\Omega = 2$ .

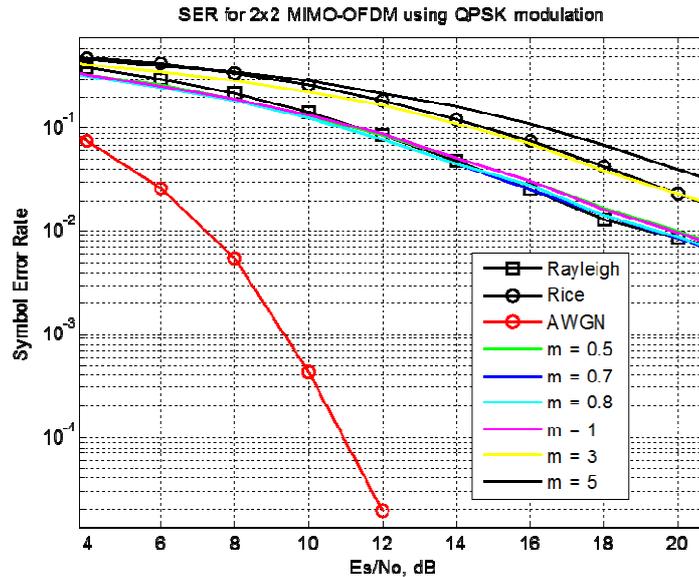


Figure 7. SER performance for 2x2 MIMO-OFDM system using QPSK modulation over different channels.

Next, SER performance for the case of QPSK modulation technique was derived. The Fig. 6 showed error value for 2x2 MIMO-OFDM system using QPSK modulation through Nakagami- $m$  channel only.

The curves in Fig. 7 presented error for some other kind of fading channels, such as Rayleigh, Rician, and non-fading channel AWGN. We may see that, it is similar to the case of BER above, the SER is decreasing when  $m$  grow up from 0.5 to 0.8. However the difference is not very discriminate from each other. In contrary, SER value start to increase if  $m$  is greater than 1.

According to above numerical results, we may see that in a MIMO-OFDM multi-tap fading system, when  $m$  value belong to [0.5, 0.7], the BER performance is directly proportional to the  $m$ . For  $m \geq 1$ , bigger value  $m$  is, worse performance is achieved.

### 5. CONCLUSIONS

A study on MIMO-OFDM system over Nakagami- $m$  fading channel using BPSK and QPSK modulation was presented in this paper. Some discussions and comparisons to other fading and non-fading channels were also provided as well. The BER and SER performance showed that in a multi-tap MIMO-OFDM scheme through a Nakagami- $m$  fading channel, they are not always in direct proportion to the value of  $m$ . In future works, a theoretical BER and SER formula for MIMO-OFDM system would be considered. In addition, figure out at which value of  $m$  the best performance for such scheme would be obtained is also an interesting research direction.

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## TÓM TẮT

### PHÂN TÍCH HIỆU NĂNG TRUYỀN TÍN HIỆU CỦA HỆ THỐNG MIMO-OFDM TRONG KÊNH TRUYỀN FADING NAKAGAMI- $m$

Hồ Văn Cừ, Lê Hoàng Hiệp\*, Nguyễn Thị Thu Hằng

*Khoa Điện tử Viễn thông, Đại học Sài Gòn, Số 273 An Dương Vương, Quận 5, Tp. Hồ Chí Minh*

Email: [lehoanghiiep@gmail.com](mailto:lehoanghiiep@gmail.com)

Trong nghiên cứu này, các tác giả sẽ trình bày về chất lượng truyền tín hiệu bao gồm tỉ lệ lỗi bit (BER) và tỉ lệ lỗi kí hiệu (SER) của một hệ thống MIMO-OFDM (multiple-input multiple output orthogonal frequency division multiplexing) trong kênh truyền Nakagami- $m$ . Cụ thể, việc mô phỏng kênh truyền sẽ được thực hiện và tính toán trên một hệ thống  $2 \times 2$  MIMO-OFDM, nghĩa là tín hiệu được truyền từ đầu phát bằng một bộ gồm 2 anten phát đến đầu thu là một bộ gồm 2 anten thu. Các tác giả đã sử dụng kĩ thuật điều chế pha nhị phân (BPSK) và điều chế pha trực giao (QPSK) cho việc điều chế luồng tín hiệu thông tin. Bên cạnh đó, các kết quả mô phỏng cũng sẽ được trình bày và so sánh giữa các môi trường truyền sóng khác nhau như kênh fading Nakagami- $m$ , Rice, Rayleigh và kênh không fading, hay còn gọi là kênh nhiễu trắng Gauss AWGN (additive white Gaussian noise). Dựa vào các kết quả thu được, các tác giả cũng sẽ thảo luận sâu hơn về mối quan hệ giữa tham số  $m$  với chất lượng kênh truyền của hệ thống MIMO-OFDM.

*Từ khóa:* MIMO, OFDM, kênh fading Nakagami- $m$ , kênh fading Rayleigh, kênh fading Rice, kênh AWGN.