

STUDY ON IMPACT FORCES OF THE UNDERWATER CAVITY PROJECTILE

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ABSTRACT

The motion of the underwater projectile with cavity effect includes two motions: First, the projectile moves in the forward direction. Second, the projectile's center of mass rotates about its tip. Because of this rotation, its tail impacts on the cavity wall. Accordingly, the impact forces occur, they include the drag force at its tip and the impact force at the impact point. The paper studies the forces occurring during motion of the underwater cavity projectile. This paper also investigates the effect of the length of projectile and the mass distribution of the projectile to the magnitude of impact force and the drag force of the underwater cavity projectile.

Keywords: cavity, number cavitation, wetted length, underwater cavity projectile.

1. INTRODUCTION

A cavity can be maintained by one of the two ways: (1) achieving such a high speed that the water vaporizes near the tip of the body; (2) supplying gas to the cavity at nearly ambient pressure. The first technique is known as vaporous or natural cavity. The second is termed ventilation or artificial cavitation [1]. The underwater projectile moves with high - speed, the local pressure at its tip will decrease. Until this local pressure is less than or equal to the vapor pressure, the bubbles appear from its tip and they surround a part of projectile. If the velocity of projectile is more than 50 m/s, a field of bubble covers over the projectile. This phenomenon is called the underwater cavity projectile (Figure 1a). During the motion of projectile, only its nose contacts water. This explains why the drag force reduces greatly. After a certain time, because of the rotation motion of projectile, the projectile tail impinges on the cavity wall, causing the impact force F_n and the drag impact force F_{pr} , as shown in Figure 1b.

The rotational velocity of the projectile depends on its length, mass distributions and drag force at its nose. The increasing velocity makes the projectile tail impinge rapidly on the cavity wall causing the impact force and drag force. The impact force causes the projectile tail to impinge on the opposite cavity wall, This phenomenon repeats itself making the projectile motion stabilize on the initial trajectory [2]. On the other hand, the projectile tail sink deeper in the water, the drag impact force and drag force at the projectile nose makes the velocity of the projectile drop quickly, leading to unstable motion of the projectile. The cavity is broken down and finally disappeared.

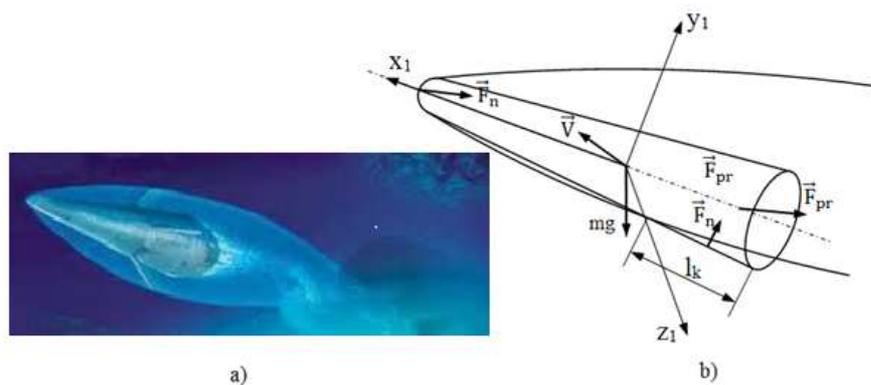


Figure 1. The underwater cavity projectile (a),
The projectile tail impact on the cavity wall (b).

2. THE ASSUMPTIONS AND THE EQUATIONS OF MOTION OF THE PROJECTILE

2.1. The assumptions

Study on dynamics of the underwater cavity projectile, Salil S. Kulkarni and Rudra Pratap [2, 3] assumed that:

- The moving of projectile occurs only on a plane. The coordinate systems for motion is shown in Figure 2. $(O.X_0Y_0Z_0)$ is the inertial reference frame with origin at O, $(A.X_1Y_1Z_1)$ is the noninertial reference frame with origin at A, the tip of the projectile. The X_1 – axis coincides with the longitudinal axis of the projectile. The components of velocity V of point A along the X_1 and Z_1 directions are U and W , respectively. The rotational velocity about the Y_1 – axis is Q .
- At the time the projectile tail impinges on the cavity wall, the cavity axis is very little to change. The angle of the projectile axis with the cavity wall is θ .
- The effect of gravity on the dynamics of the projectile is negligible [4];
- The projectile rotates about its nose in a restrictive plane;
- The motion of projectile is divided into two distinct phases:

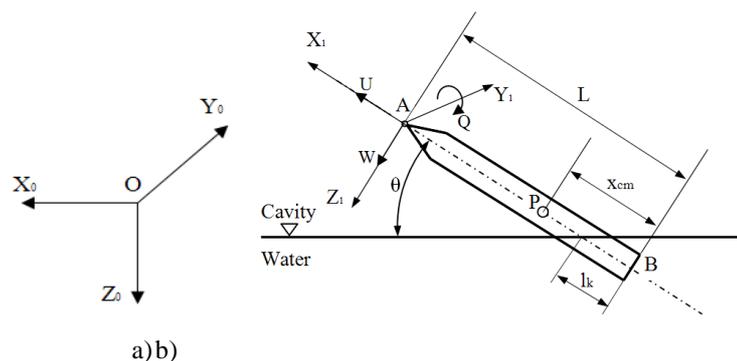


Figure 2. The axes selection for study on motion of projectile and
Scheme of forces acting at the impact point.

- + Phase 1: The projectile moves within the cavity without interaction of the cavity wall;
- + Phase 2: The projectile moves within the cavity and its tail impinges on the cavity wall.

2.2. The equation of motion

The equation of motion is written for Phase 1, with condition $U^2 \gg W^2, \rho A_c k U^2 \gg 2mLQ^2$

$$\left\{ \begin{array}{l} \frac{dU}{dt} = -\frac{1}{2m} \rho A_c U^2 C_D \\ \frac{dW}{dt} = QU \\ \frac{dQ}{dt} = 0 \\ U_{(0)} = U_0 \\ W_{(0)} = W_0 \\ Q_{(0)} = Q_0 \end{array} \right. \quad (2.1)$$

The equation of motion is written for Phase 2, with condition $U^2 \gg W^2, \rho A_c k U^2 \gg 2mLQ^2$

$$\left\{ \begin{array}{l} \frac{dU}{dt} = -\frac{1}{2m} \rho C_D U^2 f(A_c, r, l_k, \theta) \\ \frac{dW}{dt} = W^2 \left[(M_1 l_k) + M_2 (l_k x_{cm})(L - x_{cm}) \right] + \\ \quad 2W \left[Q M_2 (L l_k x_{cm})(L - x_{cm}) \right] + QU \\ \frac{dQ}{dt} = -M_2 \left[W^2 (l_k x_{cm}) + 2QW (L l_k x_{cm}) \right] \end{array} \right. \quad (2.2)$$

in which: $M_1 = -\frac{K\rho d}{m}$; $M_2 = \frac{K\rho d}{I}$; $f(A_c, r, l_k, \theta) = A_c + r^2 \cos^{-1} \left(\frac{r - l_k \operatorname{tg} \theta}{r} \right) - (r - l_k \operatorname{tg} \theta) \sqrt{d l_k \operatorname{tg} \theta}$

A_c - is the area contact water;

$r = \frac{d}{2}$; d - is the diameter of projectile;

θ - An angle created the axis of projectile and the cavity boundary;

K - is constant, its value depends on the cross section of projectile. If the cross section of projectile is circle, its value is 2π ;

l_k - is the wetted length.

The above equation of motion determines that, the acceleration of projectile in direction X_1 depends on the drag force at its tip and the immersed area, the wetted length l_k and the angle θ . The angular acceleration depends on the inertia I , the wetted length l_k and x_{cm} , which is measured from the base to the center of mass of projectile. Note, x_{cm} depends on the length and mass distributions of the projectile.

3. THE IMPACT FORCES AND THEIR FORMULAS

As is known, the projectile while moving in the forward direction also starts rotating about its tip on the vertical plane. This rotation leads to impacts between the projectile tail and the

cavity wall. Because of this impingement, the impact forces appear at the impingement point. The impact forces are shown in Figure 3.

During the impingement in Phase 1, the projectile contacts water by its nose, so only one component drag force $F_{D_{NOSE}}$. In Phase 2, there are additional forces acting at the point of impact of the projectile tail, the impact forces appear at the impact point. F_{IMP} is the impact force, it is the reason why the projectile tail impinges on the opposite cavity wall. This restitution impingement explains why the trajectory of projectile is stable. F_{IMP} is the drag force at the impact point, it includes two components $F_{X_{IMP}}$ and $F_{Z_{IMP}}$. The magnitude of moment generated by $F_{Z_{IMP}}$ has negligible influence to the motion of projectile [3].

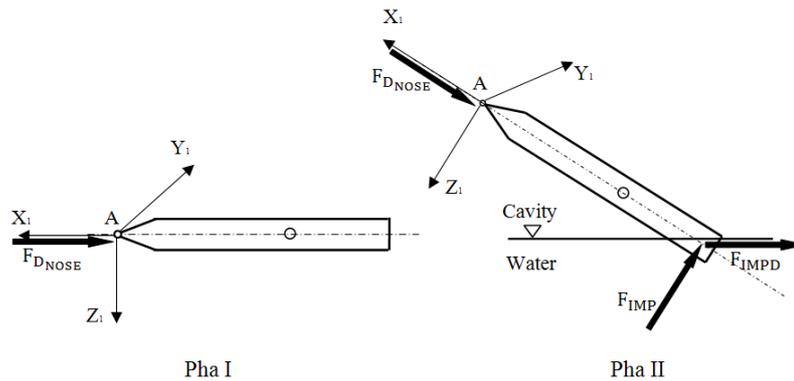


Figure 3. Forces acting on the underwater cavity projectile in the both Phase 1 and Phase 2.

Table 1 shows the forces and moments of on the underwater cavity projectile in both Phase 1 and Phase 2.

Table 1. The relevant forces and moments in Phase 1 and Phase 2.

	$\sum F_{X_1}$	$\sum F_{Z_1}$	$\sum M_{Y_1}$
Phase 1	$F_{D_{NOSE}}$	0	0
Phase 2	$F_{D_{NOSE}} + F_{X_{IMP}}$	$F_{IMP} + F_{Z_{IMP}}$	$M_{F_{IMP}}$

3.1. The drag force at the projectile nose

During moving of the underwater cavity projectile, its nose contacts water causing the drag force at the projectile nose. Its value is calculated by:

$$F_{D_{NOSE}} = \frac{1}{2} \rho A_C U^2 C_D \tag{3.1}$$

in which: $\frac{1}{2} \rho U^2$ - is dynamic pressure [5]; A_C - is the area contact to water; ρ - is the density of liquid; U - is the velocity of flow; $C_D = c_x \cos \alpha$ - is the coefficient drag; α - angle created by the

normal vector of the transverse section of projectile nose and the projectile vector of flow. According to the above assumption, its value is θ .

Epshtein and Tseitlin [6] derived from the experiment results, they confirm $c_x \approx c_{x0}(1+\sigma)$ and the value of c_x is maximum when $\sigma = 0$, this is mean $c_x = c_{x0}$. c_{x0} is different values if the shape of projectile nose is a disk or a cone with different cone angle (see *Table 2*). The cone angle is made by the lateral height of the cone and its diameter

Table 2. The values of c_{x0} when $\sigma = 0$, corresponding the different cone angle.

The cone angle (degrees)	0	5	10	15	20	30	45
c_{x0}	0.82	0.78	0.75	0.715	0.68	0.607	0.465

$$\sigma = \frac{p_\infty - p_c}{\frac{1}{2}\rho U^2} \quad (\text{Number Cavitation});$$

p_∞ - is the pressure of flow, p_c - is the pressure within the cavity.

3.2. The impact force

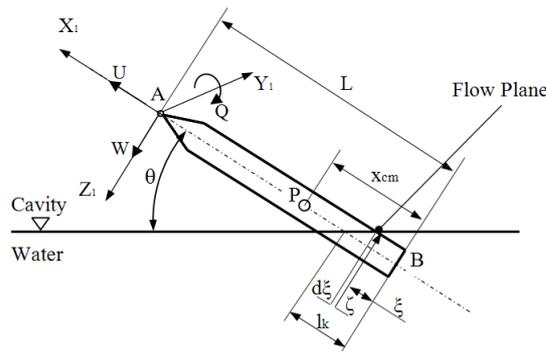


Figure 4. The scheme of impact force calculation

The scheme is used to specify the impact force F_{IMP} [7]. It is assumed that during the impact, the shape and dimensions of the cavity are almost unchanged; The flow is laminar, that includes layers of plane parallel flow. (ξ, ζ) is a new set of axes, it is attached to the base of the projectile at point B. The ξ - axis direction is the same as the X_1 - axis direction while the ζ - axis direction is opposite to the Z_1 - axis direction. The coordinate ξ denotes the distance from point B to any transverse flow plane at the time t , this plane is called the cross section of consideration. The coordinate ζ denotes the penetration into any such flow plane. The wetted length is shown by l_k .

At the considered cross section of the projectile, ξ is dimensioned from the projectile bottom to any transverse flow plane at the time t . At this time, the force made by a transverse flow exerted on the projectile tail. It is calculated by [7]:

$$f = -\frac{d(m_{app} W_{\xi})}{dt} \quad (3.2)$$

in which: W_{ξ} - is the magnitude of velocity in direction Z_1 for the cross section part of the projectile at the point under consideration $W_{\xi} = W + (L - \xi)Q = \frac{d\zeta}{dt}$;

m_{app} - is the virtual mass of the cross section part of the projectile at the point under consideration in direction Z_1 . It is dimensioned from point B to the transverse section of consideration, $m_{app} = K\rho\zeta d$

K - is constant, its value depends on the profile of transverse cross section. If the transverse section of the projectile is the circle: $K = 2\pi$; ρ - is density of flow liquid; d - is diameter of the projectile; ζ - is the depth of penetration of the tail in the water under the transverse section consideration, $\zeta = (l_k - \xi)\tan\theta$

Substituting the above component in the equation (3.2), to give the impact force, it exerted by only a transverse flow on the projectile tail:

$$f = -\frac{d(m_{app} W_{\xi})}{dt} = -K\rho d \frac{d\{\zeta[W + (L - \xi)Q]\}}{dt} \quad (3.3)$$

integrate the equation (3.3) on over the wetted length l_k , gives us the total magnitude of impact force:

$$F_{IMP} = -(K\rho d) \left(C_1 + C_2 \frac{dW}{dt} + C_3 \frac{dQ}{dt} \right) \quad (3.4)$$

in which:

$$C_1 = W^2 l_k - Q^2 \left[\frac{(L - l_k)^3}{3} - \frac{L^3}{3} \right] - 2WQ \left[\frac{(L - l_k)^2}{2} - \frac{L^2}{2} \right]$$

$$C_2 = \frac{l_k^2}{2} \tan\theta$$

$$C_3 = \left[Ll_k^2 - \frac{l_k^2}{2(L + l_k)} + \frac{l_k^3}{3} \right] \tan\theta$$

3.3. The moment generated by the impact force

The moment generated by only a transverse flow plan exerted on the projectile is specified by:

$$M_f = -\frac{d(m_{app} W_{\xi})}{dt} (x_{cm} - \xi) \quad (3.5)$$

integrate the equation (3.5) over the wetted length l_k , to give the total moment. It is generated by the impact force F_{IMP} :

$$M_{F_{IMP}} = (K\rho d) \left(E_1 + E_2 \frac{dW}{dt} + E_3 \frac{dQ}{dt} \right) \quad (3.6)$$

in which:

$$E_1 = W^2 \left[\frac{l_k^2}{2} - l_k x_{cm} \right] + 2WQ \left[l_k^2 (L + x_{cm}) - \frac{l_k^3}{3} - L l_k x_{cm} \right] + Q^2 \left[\frac{L^2 l_k^2}{2} - L^2 l_k x_{cm} - 2 \frac{L l_k^3}{3} + 2 \frac{L x_{cm} l_k^2}{2} + \frac{l_k^4}{4} - \frac{l_k^3 x_{cm}}{3} \right]$$

$$E_2 = \left[\frac{l_k^3}{6} - \frac{l_k^2 x_{cm}}{2} \right] \tan \theta$$

$$E_3 = \left[\frac{l_k^3 (L + x_{cm})}{6} - \frac{l_k^4}{12} - \frac{L x_{cm} l_k^2}{2} \right] \tan \theta$$

3.4. The component drag force at the impact point

During the motion, the projectile tail impinges on the cavity wall. Because of this impact, the impact forces occur at the impact point, this magnitude of drag force depends on the depth of penetration of the tail in the water and the angle θ which is made by the projectile axis and the cavity wall. The drag force at the impact point includes two components: $F_{X_{1IMPDP}}$ in the direction X_1 and $F_{Z_{1IMPDP}}$ in the direction Z_1 .

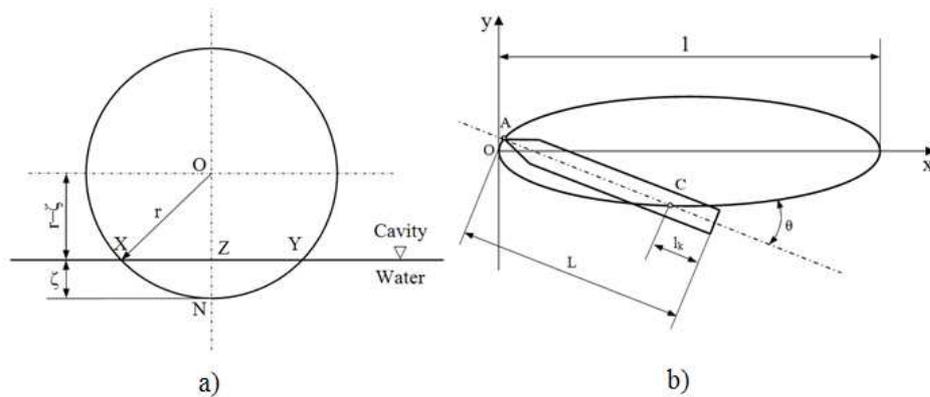


Figure 5. The scheme of drag force calculation F_{IMPDP} (a),
The scheme of wetted length calculation l_k (b).

The magnitude drag force in direction X_1 is specified by:

$$F_{X_{1IMPDP}} = -\frac{1}{2} \rho A_{rea} (XYNX)_B U^2 C_D \quad (3.7)$$

in which: $A_{rea} (XYNX)_B$ is the immersed area of the projectile at point B;

$$A_{rea} (XYNX)_B \approx r^2 \cos^{-1} \left(\frac{r - l_k \tan \theta}{r} \right) - (r - l_k \tan \theta) \sqrt{l_k \tan \theta}$$

U is the magnitude velocity of projectile in direction X_1 .

Calculate the magnitude of component force in direction Z_1 of $F_{\text{IMP D}}$. The first, calculate the magnitude of component forces in direction Z_1 on only the cross section under consideration, its magnitude shown:

$$f_{Z_{\text{IMP D}}} = -\frac{1}{2}\rho W_{\xi}^2 A_p C_D \quad (3.8)$$

in which: C_D is the coefficient drag, its value is the same as the value of the coefficient drag at the projectile

$$A_p = 2\sqrt{d\zeta}; \quad \zeta = (l_k - \xi) \tan\theta$$

$$W_{\xi} = W + (L - \xi)Q$$

The second, substituting the values of A_p and W_{ξ} on the equation (3.8), this force is then integrated over the wetted length to give the total force acting on the projectile at the impact point. The expression for this force is given by:

$$F_{Z_{\text{IMP D}}} = -\rho F_1 C_D \sqrt{d \tan\theta} \quad (3.9)$$

in which:

$$F_1 = \frac{16}{105Q^2 l_k^{7/2}} + \frac{2}{3W^2 l_k^{3/2}} + \frac{4}{3WQ l_k^{3/2}} + \frac{2}{3Q^2 L^2 l_k^{3/2}} - \frac{8}{15Q l_k^{5/2} (W - QL)}$$

3.5. Calculate the wetted length

$$\begin{cases} \left(\frac{x - \frac{1}{2}}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{\left(\frac{D_k}{2}\right)^2} = 1 \right. \\ \left. y = ax + b \right. \end{cases} \quad (3.10)$$

in which: The first equation in the equations (3.10) is the profile equation of ellipse for cavity [8], the second equation is the line equation, and it is made by two points A and C on the cavity (Figure 5b)

$$a = \frac{y_C - y_A}{x_C - x_A}; \quad b = -ax_C + y_C$$

y_A, x_A is 0 in the first step. Then, it receives the value in the steps 1,2,3,...,n of the equation (2.1) when $l_k \leq 0$ and (2.2) when $l_k > 0$.

$$y_C = (W + LQ)\Delta t; \quad x_C = L - U\Delta t$$

the final result: $l_k = L - \frac{x_C}{\cos\theta}$.

4. THE RESULTS

4.1. Data input for calculation

The shape, dimension and parameters of projectile shown in Figure 6.

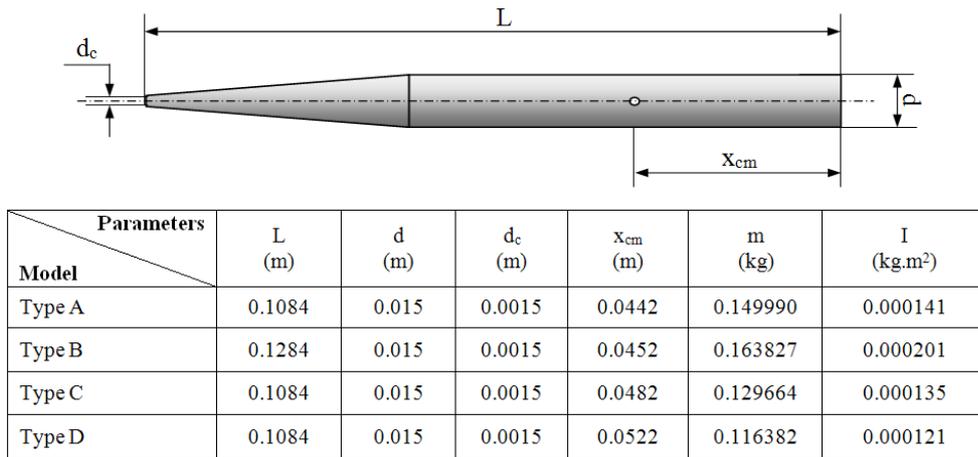


Figure 6. Shape, dimension and parameters of projectile.

- density of water $\rho = 10^3 \text{ kg / m}^3$,
- pressure within the cavity $p_c = 2,5 \text{ kPa}$ [9],
- flow pressure at the depth of 1m $p_t = 111,131 \text{ kPa}$,
- depth from the water surface to point launch $h = 1 \text{ m}$,
- initial velocity in X_1 direction $U_0 = 272 \text{ m/s}$,
- initial velocity in Z_1 direction $W_0 = 0 \text{ m/s}$,
- initial angle velocity $Q_0 = 1 \text{ rad/s}$.

The simulation is carried out for 0.2 sec.

4.2. The calculation results

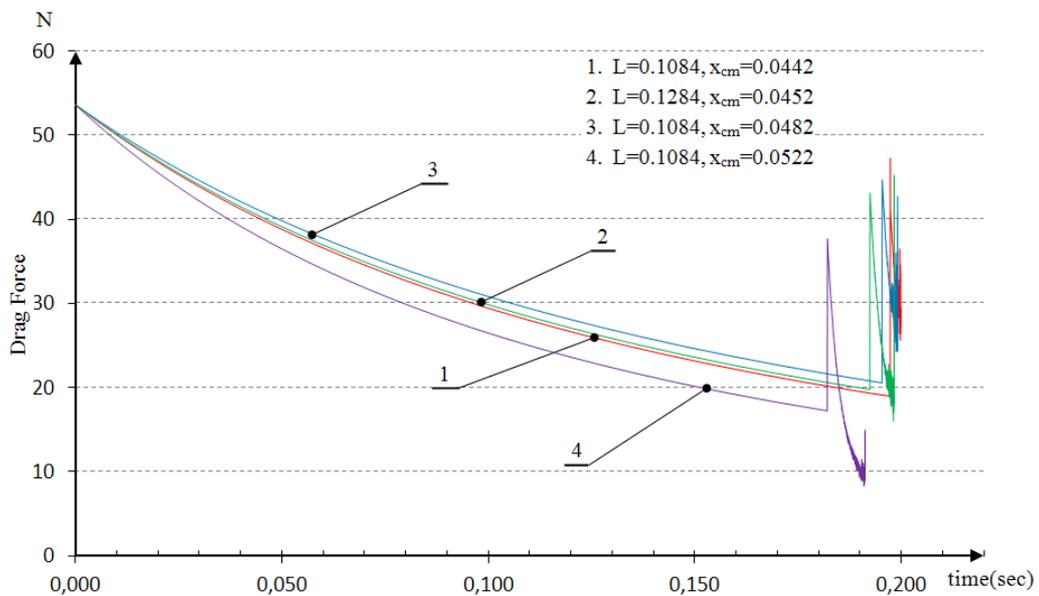


Figure 7. Time evolution of the drag force.

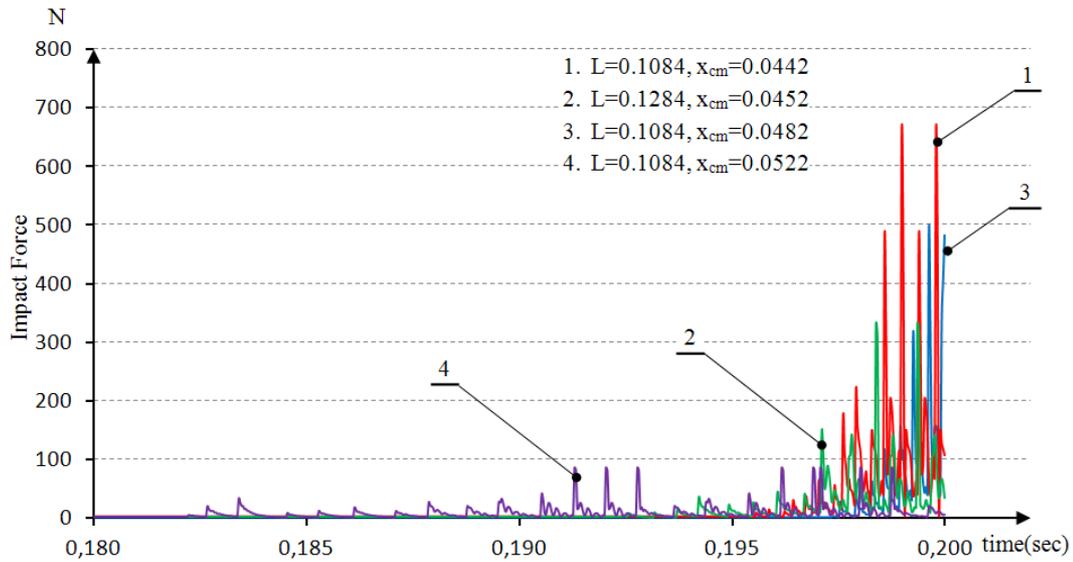


Figure 8. Time evolution of the impact force.

5. CONCLUSION

The authors considered the magnitude of drag force in four cases with the projectile length and mass distribution of different. The results show that if the length x_{cm} is longer, the drag force will reduce promptly and the rotational velocity will increase rapidly, that leads to the projectile tail impacts rapidly on the cavity wall, while the impact force is generated. The impact force is the cause of the projectile tail impacts on the opposite of cavity wall. In the case when the length x_{cm} increases longer, which makes the magnitude of impact force on the opposite cavity wall smaller compared to the case when the center of mass nearer bottom of the projectile.

We also considered the cases, when the projectile lengths are different. The results indicated that the projectile with a longer length will soon appear the impact of the projectile tail on the cavity wall. On the other hand, the magnitude of impact force is smaller, making the depth of penetration of the tail in the water greatly decreased. This is the reason why the trajectory of projectile is more stable.

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TÓM TẮT

NGHIÊN CỨU CÁC LỰC TƯƠNG TÁC KHI VIÊN ĐẠN CHUYỂN ĐỘNG DƯỚI NƯỚC VỚI HIỆU ỨNG CAVITY

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Chuyển động của viên đạn dưới nước với hiệu ứng khoang rỗng bao gồm hai chuyển động: Chuyển động của viên đạn về phía trước và chuyển động quay của khối tâm quang mũi đạn, do các chuyển động này nên đuôi đạn sẽ va chạm với thành của khoang rỗng. Vì thế sẽ sinh ra các lực va chạm, các lực này bao gồm lực kéo tại mũi đạn và lực va chạm tại điểm va chạm. Bài báo này nghiên cứu các lực sinh ra trong quá trình viên đạn chuyển động dưới nước với hiệu ứng khoang rỗng. Ngoài ra, bài báo còn xét đến ảnh hưởng của chiều dài và phân bố của viên đạn đến độ lớn của lực kéo và lực va chạm của viên đạn chuyển động dưới nước với hiệu ứng khoang rỗng.

Từ khóa: khoang rỗng, số khoang rỗng, chiều dài ướt, viên đạn có hiệu ứng khoang rỗng dưới nước.