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APPLICATION OF DATA ASSIMILATION FOR PARAMETER CORRECTION IN SUPER CAVITY MODELLING

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ABSTRACT

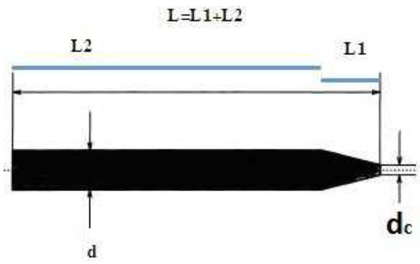
On the imperfect water entry, a high speed slender body moving in the forward direction rotates inside the cavity. The super cavity model describes the very fast motion of body in water. In the super cavity model the drag coefficient plays important role in body's motion. In some references this drag coefficient is simply chosen by different values in the interval 0.8-1.0. In some other references this drag coefficient is written by the formula $k = C_{D0}(1+\sigma)\cos^2\alpha$ with σ is the cavity number, α is the angle of body axis and flow direction, C_{D0} is a parameter chosen from the interval 0.6-0.85. In this paper the drag coefficient $k = k_1 C_{D0}(1+\sigma)\cos^2\alpha$ is written with fixed $C_{D0} = 0.82$ and the parameter k_1 is corrected so that the simulation body velocities are closer to observation data. To find the convenient drag coefficient the data assimilation method by differential variation is applied. In this method the observing data is used in the cost function. The data assimilation is one of the effected methods to solve the optimal problems by solving the adjoin problems and then finding the gradient of cost function.

Keywords: data assimilation, optimal, Runge-Kutta methods.

1. INTRODUCTION

When slender body running very fast under water (velocity is higher than 50 m/s) the cavity phenomena is happened. Cavity may have a variety of cause. The most common example is boiling water, where the vapor pressure is increased by raising the water temperature. In hydrodynamics applications cavitation is the appearance of vapor bubbles and pockets inside homogeneous liquid medium. This phenomenon occurs because the pressure is reduced to the vapor pressure limit. In this paper we will study super cavity appearing by the very fast movement of slender body in water that makes uncontrolled gun-launched slender body. Except the body head called by cavitator is directly touching with water, the gas layer can be covered partial or full body depending on the design of body form. The body rotates about its nose. The form of body's nose can be differently chosen such as: sharp, hemisphere, plate disk... For simple calculation we choose cavitator formed by the plate disk with diameter d_c (Figure 1).

The body is consisted of two parts: the cone top and cylinder part with the diameter d .



- L is the length of the slender body;
- L_2 is the body's length of cylinder part
- L_1 is the body's length of cone top part
- d is the body's diameter
- d_c the body's nose diameter

Figure 1. Slender body geometer.

In the super cavity model the following assumptions are ([1, 2]):

- The motion of the projectile is confined to a plane;

- The slender body rotates about its nose ([1 - 4]);

- The effect of gravity on the dynamics of this body is negligible;

- The motion of the slender body is not influenced by the presence of gas, water vapor or water drops in the cavity;

The super cavity problems are studied in [1, 2, 5 - 11]. To study the motion problems of slender body running under water there are basic approaches:

- The experimental approach consisting in observing and measuring motion by remote sensing.

- The modeling approach based on mathematical models of the flow and of the body motion.

- The models of body's motion under water include some parameters that have not a clear physical meaning because they are a synthetic representation of several physical effects such as sub-grid turbulence that can't be explicit in the model because of a necessary truncation for numerical purposes.

None of these approaches is sufficient to predict the evolution of body motion. They have to be combined to retrieve the body motion under water. All the techniques used to combine the information provided by observations and the information provided by models are named by Data Assimilation methods and have known an important development during these last decades. The Data Assimilation method using differential variation is based on the theory of optimal control for partial differential equation by Lions et al. [12, 13] and Marchuk et al. [14]. This method is applied to correct coefficients, solve the inverse problems, simulate the air and fluid pollution processes ([14 - 21]).

-In this paper we will concentrate the study on the identification coefficient parameter k_1 of the drag coefficient $k = k_1 C_{D0}\left(1+\sigma\right)\cos^2\alpha$ ($C_{D0} = 0.82$). In the second section we will describe the abstract definition of an inverse problem via variation methods. The unknown coefficient is defined as the solution of an optimization problem. In the third section we will formulate the model of the problem of body's fast motion under water problem. The 4-th section is devoted to the application of optimal control to the identification of model's coefficient.

2. GENERAL VARIATION APPROACH

Because In the model's parameters are a synthetic representation of several physical effects, they can't be directly estimated. They depend both on the model and on the data. They will be evaluated as the solution of an "Inverse Problem", basically as the solution of an optimization problem. The advantage is that there exist many efficient algorithms for solving these problems. Most of them require to compute the gradient of the function to be minimized. The cost function is done by solving an "Adjoin Model". The method is described in many papers together with the computational developments ([14 - 21]). It can be summarized as follows:

Let $X(t)$ the state vector describing the evolution of a system governed by the abstract equation:

$$\begin{cases} \dfrac{dX}{dt} = F\left(X, E_1, ..., E_n\right) \\ X\left(0\right) = X_0 \end{cases} \tag{2.1}$$

where: $E_1,...,E_n$ are the equation's parameters with n is the number of parameters; $X(t)$ is a unknown state vector belonging for any t to a Hilbert space $\Im$, $X_0 \in \Im$; F is a nonlinear operator mapping $Y \times Y_p$ to Y with $Y = L_2\left(0, T, \Im\right)$, $\|.\|_Y = (.,.)_Y^{1/2}$, Y_p is Hilbert space (the space of model's parameters); Suppose that for given initial value $X(0) = X_0 \in \Im$ and $(E_1,...,E_n) \in Y_p$ there exists a unique solution $X \in \Im$ to (2.1). In case the values of $E = (E_1,...,E_n)$ are unknown and there are some observation data $X_{obs} \in \Im_{obs}$ with $\Im_{obs}$ is a Hilbert space (observation space) we introduce the functional called cost function:

$$J(E) = \frac{1}{2}\int_0^T \left(H\left(CX - X_{obs}\right), CX - X_{obs}\right)_{\Im_{obs}} dt + \frac{1}{2}\left(E - E_0\right)^2 \tag{2.2}$$

where $(E_{0,1},...,E_{0,n})$ are priori approximation evaluations of $E_1,...,E_n$; $C: \Im \to \Im_{obs}$ is a linear bounded operator, $H: \Im_{obs} \to \Im_{obs}$ is symmetric positive definite operator; The problem is to determine $E^* = \left(E_1^*,..,E_n^*\right)$ by minimizing J. The second and the third terms in J are a regularization term in the sense of Tykhonov, have a well posed problem (see [15, 17]). The optimal solutions are characterized by $\vec{\nabla}.J(E_1^*,...,E_n^*)$, where $\vec{\nabla}.J$ is the gradient of J. To compute this gradient we introduce e_i $(i = 1, 2, ..., n)$, the directions in the space Y_p. We will compute the Gateaux derivative of the cost function J by $E = \left(E_1,...,E_n\right)$ in the directions of $e = (e_1,...,e_n)$. The Gateaux derivative of the cost function J in the directions of $e = (e_1,...,e_n)$ will be:

$$\begin{aligned}\hat{J}(E_1,...,E_n) &= \sum_{i=1}^{n}\int_0^T \left(C^T H\left(CX - X_{obs}\right), \hat{X}^{(i)}\right)_{\Im} dt + \sum_{i=1}^{n}\left\langle E_i - E_{i,0}, e_i\right\rangle \\ &= \sum_{i=1}^{n}\int_0^T \left(C^T H\left(CX - X_{obs}\right), \hat{X}^{(i)}\right)_{\Im} dt + \sum_{i=1}^{n}\left\langle E_i - E_{i,0}, e_i\right\rangle \\ &= \left(\hat{J}_{E_1}\left(E_1,..,E_n\right),...\hat{J}_{E_n}\left(E_1,..,E_n\right)\right)\left(e_1,...,e_n\right)^T\end{aligned} \tag{2.3}$$

where: $\hat{X}^{(i)}$, $\hat{J}_{E_i}\left(E_1,..,E_n\right)$ respectively are the Gateaux derivatives of X and J with respect to E_i in the directions e_i. Here <,> is the dot product associated with the norm operator $\| \ \|$. The optimal solution of problem is characterized by $\hat{J}\left(E_1,...,E_n\right) = \vec{\nabla}.J.\left(e_1,...,e_n\right)^T = 0$ where $\vec{\nabla}.J = \left(J^{'}_{E_1},...,J^{'}_{E_n}\right)$ is the gradient of J with respect to $E_1,..,E_n$; The superscript T indicates the transpose of the vector.

The Gateaux derivative equations of (2.1) by E_i in the directions of e_i ($i = 1,2,..,n$) are:

$$\begin{cases} \dfrac{d\hat{X}}{dt} = \dfrac{\partial F\left(X, E_1,..,E_n\right)}{\partial X}\cdot \hat{X}^{(i)} + \dfrac{\partial F}{\partial E_i}\cdot e_i \\ \hat{X}^{(i)}(0) = 0 \end{cases} \tag{2.4}$$

Let us introduce $P^{(i)}$, the adjoin variable in the same space as X. Multiplying equation (2.4) by $P^{(i)}$ in space $\Im$ we integrate by time between 0 and T. It comes:

$$\int_0^T \left(\frac{d\hat{X}^{(i)}}{dt}, P^{(i)}\right)_{\Im} dt = \int_0^T \left(\frac{dF}{dX}\cdot \hat{X}^{(i)}, P^{(i)}\right)_{\Im} dt + \int_0^T \left(\frac{dF}{dE_i}\cdot e_i, P^{(i)}\right)_{\Im} dt \tag{2.5}$$

$$\text{or}\left(\hat{X}^{(i)}(T), P^{(i)}(T)\right)_{\Im} - \left(\hat{X}^{(i)}(0), P^{(i)}(0)\right)_{\Im} = \int_0^T \left(\hat{X}^{(i)}, \frac{dP^{(i)}}{dt} + \left[\frac{dF}{dX}\right]^t \cdot P^{(i)}\right)_{\Im} dt + e_i\int_0^T \left[\frac{dF}{dE_i}\right]^t .P^{(i)} dt \tag{2.6}$$

$$i = 1,2,..,n$$

The superscript ^{t} indicates the transpose of the matrix.

Summing n equations of (2.6) we have

$$\begin{aligned}&\sum_{i=1}^{n}\left[\left(\hat{X}^{(i)}(T), P^{(i)}(T)\right)_{\Im} - \left(\hat{X}^{(i)}(0), P^{(i)}(0)\right)_{\Im}\right] \\ &= \sum_{i=1}^{n}\left[\int_0^T \left(\hat{X}^{(i)}, \frac{dP^{(i)}}{dt} + \left[\frac{dF}{dX}\right]^t \cdot P^{(i)}\right)_{\Im} dt + e_i\int_0^T \left[\frac{dF}{dE_i}\right]^t .P^{(i)} dt\right]\end{aligned} \tag{2.7}$$

If $P^{(i)}$ is the solution of:

$$\begin{cases} \dfrac{dP^{(i)}}{dt} + \left[\dfrac{dF}{dX}\right]^t \cdot P^{(i)} = C^T H\left(CX - X_{obs}\right) \\ P^{(i)}\left(T\right) = 0 \end{cases} \tag{2.8}$$

then (2.7) becomes:

$$\sum_{i=1}^{n}\int_{0}^{T}\left(\hat{X}^{(i)},\frac{dP^{(i)}}{dt}+\left[\frac{dF}{dX}\right]^{t}\cdot P^{(i)}\right)_{\Im}dt=\sum_{i=1}^{n}\int_{0}^{T}\left(\hat{X}^{(i)},C^{T}H\left(CX-X_{obs}\right)\right)_{\Im}dt$$

$$=-\sum_{i=1}^{n}e_i\int_{0}^{T}\left[\frac{dF}{dE_i}\right]^{t}.P^{(i)}dt \qquad (2.9)$$

Therefore, from (2.3), (2.9), we have

$$\hat{J}\left(E_1,...,E_n\right)=\sum_{i=1}^{n}\left(-\int_{0}^{T}\left[\frac{dF}{dE_i}\right]^{t}\cdot P^{(i)}dt+E_i-E_{i,0}\right)e_i$$

$$=\vec{\nabla}.J.\left(e_1,...,e_n\right)^{T} \qquad (2.10)$$

with $$\vec{\nabla}J=\left(J'_{E_1}\left(E_1,...,E_n\right),...,J'_{E_n}\left(E_1,...,E_n\right)\right) \qquad (2.11)$$

where: $J'_{E_i}\left(E_1,...,E_n\right)=-\int_{0}^{T}\left[\frac{\partial F}{\partial E_i}\right]^{t}P^{(i)}dt+E_i-E_{i,0}$.

Equations 2.1 - 2.9 and the condition for the gradient (2.11) to be null are the Optimality System (O.S). The adjoin model will be run back word to get the gradient which are used to carry out an algorithm of optimization [14 - 21].

3. MATHEMATICAL MODEL FOR THE BODY MOTION

To describe the motion of body, a body fixed coordinate system as shown in Figure 2 is chosen. $\left(X_0,Y_0,Z_0\right)$ is the inertial reference frame with origin at O and $\left(X_1,Y_1,Z_1\right)$ is the non-inertial reference frame with origin at A, the tip of the slender body. The X_1-axis coincides with the longitudinal axis of the slender body. The components of velocity of point A along X_1 and Z_1 direction are U and W respectively. The components of velocity of point A along X_0 and Z_0 direction are U_F and W_F respectively.The angular velocity and rotating angular about Y_0 axis are Q and □ respectively.

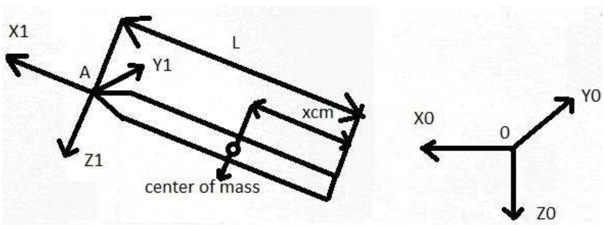


Figure 2. Axes of body and inertial frames.

The relationships between body and inertial fixed velocities are described by the following formulas:

$$U_F = U\cos\vartheta + W\sin\theta;\quad W_F = -U\sin\vartheta + W\cos\vartheta;\quad \dot{\vartheta} = Q;\quad \vartheta(0) = \vartheta_0$$

The mathematic cavity model [1] is used to describe the motion of slender body under water in cavity. The motion of slender body in both phases is written by the following equations:

Phase 1: For $U^2 >> W^2$ and $\rho A_c k(U,W,h)U^2 >> 2mLQ^2$ the equation can be written as:

$$\begin{aligned}
&\frac{\partial U}{\partial t} = -\frac{1}{2m}\rho k(U,W,h)A_c U^2\\
&\frac{\partial W}{\partial t} = QU\\
&\frac{\partial Q}{\partial t} = 0\\
&\frac{\partial h}{\partial t} = -U\sin\vartheta + W\cos\vartheta\\
&\frac{\partial \vartheta}{\partial t} = Q\\
&U(0)=U_0; W(0)=W_0; Q(0)=Q_0; Q(0)=Q_0; h(0)=h_0, \square(0)=\square_0
\end{aligned} \tag{3.1}$$

Phase 2: For $U^2 >> W^2$ and $\rho A_c k(U,W,h)U^2 >> 2mLQ^2$ the equation can be written as:

$$\begin{aligned}
&\frac{\partial U}{\partial t} = -\frac{1}{2m}\rho k(U,W,h)F(A_c,r,l_k,\theta)U^2\\
&\frac{\partial W}{\partial t} = KW^2\left[M_1 l_k + M_2 l_k x_{cm}(L-x_{cm})\right] + 2KW\left[QM_2 L x_{cm} l_k (L-x_{cm})\right] + QU\\
&\frac{\partial Q}{\partial t} = -KM_2\left[W^2 l_k x_{cm} + 2WQLl_k x_{cm}\right],\\
&\frac{\partial h}{\partial t} = -U\sin\vartheta + W\cos\vartheta\\
&\frac{\partial \vartheta}{\partial t} = Q
\end{aligned} \tag{3.2}$$

where:

- θ is the angle of slender body during impact with the cavity boundary,

$$\tan\theta \approx \frac{W}{U} \text{ or } \theta \approx \arctan\frac{W}{U}$$

- $M_1 = -\frac{\rho d}{m}, M_2 = \frac{\rho d}{I}$
- $F(A_c,r,l_k,\theta) = A_c + r^2\cos^{-1}\left(\frac{r - l_k\tan\theta}{r}\right) - (r - l_k\tan\theta)\sqrt{dl_k\tan\theta}$
- $k(U,W,h) = k_1 C_{D0}(1+\sigma)\cos^2\alpha$
- $C_{D0} = 0.82$

- α is the angle between flow direction and body's direction in moving

$\cos\alpha \approx \frac{U}{\sqrt{U^2+W^2}}$

- $p_\infty = \rho g h + P_{atm}$ - Ambient pressure
- l_k is the wetted length of the body
- k_1 , K are parameters; For the circular section $K = 2\pi$ ([1])
- h is the water depth between the body's position and water free surface
- ρ is the mass density of water
- x_{cm} is the distance between body's tail and its centre of mass;
- m is the mass of the slender body
- σ is the cavitation number $\sigma = \frac{p_\infty - p_c}{0.5\left(U^2+W^2\right)}$
- I is the moment of inertia of the body about an axis parallel to the Y_1 axis and passing through its centre of mass
- $r = d / 2$ is the radius of slender body
- $A_c = \frac{\pi d_c^{\ 2}}{4}$ is the area of the cavitator
- $r_c = \frac{d_c}{2}$ is the cavitator radius
- $g = 9.81$ m/s is the gravity acceleration
- p_c is the vapour pressure of water

To get the above equations the following condition is needed: $\frac{l_k}{L} << 1$

The geometry of the cavity is given by ([1, 2, 8]):

$$\frac{\left(x - l/2\right)^2}{\left(l/2\right)^2} + \frac{y^2}{\left(D_k/2\right)^2} = 1$$

where the maximum diameter D_k and length l of the cavity shape are given by the following formulas:

$$D_k = d_c\sqrt{\frac{k_1 C_{D0}\left(1+\sigma\right)}{\sigma}}, l = \frac{d_c}{\sigma}\sqrt{\log\frac{1}{\sigma}}$$

The equation (3.1) - (3.2) can be rewritten as follows:

$$\begin{cases} \frac{\partial X}{\partial t} = A(X) \\ X(0) = X_0 \end{cases} \quad (3.3)$$

where: $$X = \left(U, W, Q, h, \vartheta\right)^T \tag{3.4}$$

is an unknown state function vector of the equations (3.1)-(3.2) and

$$X_0 = \left(U_0, W_0, Q_0, h_0, \vartheta_0\right)^T$$

$$A(X) = \left[A_1(X), A_2(X), A_3(X), -U\sin\vartheta + W\cos\vartheta, Q\right]^T \tag{3.5}$$

$$A_1(X) = \begin{cases} -\dfrac{1}{2m}\rho k\left(U, W, h\right) A_C U^2 & \text{in the first phase} \\ -\dfrac{1}{2m}\rho k\left(U, W, h\right) F\left(A_C, r, l_K, \theta\right) U^2 & \text{in the second phase} \end{cases}$$

$$A_2(X) = \begin{cases} QU & \text{in the first phase} \\ KC_1 W^2 + KC_2 W + QU & \text{in the second phase} \end{cases}$$

$$A_3(X) = \begin{cases} QU & \text{in the first phase} \\ C_3 W^2 + C_4 WQ & \text{in the second phase} \end{cases}$$

$$C_1 = M_1 l_k + M_2 l_k x_{cm}\left(L - x_{cm}\right); C_2 = 2M_2 L x_{cm} l_k\left(L - x_{cm}\right); C_3 = -M_2 l_k x_{cm}; C_4 = -M_2 L l_k x_{cm}$$

The equation 3.3 is solved by Runge Kutta method.

4. CORECTION OF k_1 COEFFICIENT

We have priori approximations $k_{1,0}$ of k_1 and measurement $X_{obs} = \left(U_{obs}, W_{obs}, Q_{obs}, h_{obs}, \vartheta_{obs}\right)$ of the motion velocity of body. Using the cost function (see formula 4.1) the continuous problem is to determine k_1^* minimizing J :

$$J(k_1) = \frac{1}{2}\int_0^T \left(CX - X_{obs}, CX - X_{obs}\right)_{\Im_{obs}} dt + \frac{1}{2}\left(k_1 - k_{1,0}\right)^2 \tag{4.1}$$

C is an operator, that is Diract's matrix, from the space of the variable X to the space of observation with point wise measurement. Therefore, we have an optimal control problem with respect to the coefficient k_1. The first step is to exhibit the Euler-Lagrange equation- necessary equation for an optimum in order to exhibit the gradient of J with respect to k_1. Then, we will be able to carry out some optimization algorithm.

The data assimilation problem is written in the form:

$$\begin{cases} \dfrac{\partial X}{\partial t} = A(X) \\ X(0) = X_0 \\ J\left(k_1^*\right) = \inf\limits_{k_1^*} J\left(k_1\right) \end{cases} \tag{4.2}$$

here $X=(U,W,Q,h,\vartheta)^T$, $A(X)$ is the vector function defined by the formula (3.4)- (3.5), and the cost function $J(k_1)$ is defined by the formula (4.1). To solve the problem (4.2) we will define the formula of function $J'_{k_1}(k_1)$ in the next subsection.

4.1. Computation of Gateaux derivative for the cost function J

Let k_1 being a value in the space of the control. Let us introduce the Gateau derivative $\hat{X}=(\hat{U},\hat{W},\hat{Q},\hat{h},\hat{\vartheta})^T$ of $X=(U,W,Q,h,\vartheta)^T$ by k_1 in the directions of $\bar{k}_1$ as follows ([22]):

$$\hat{X}=\lim_{\alpha\to 0}\frac{X(k_1+\alpha\bar{k}_1)-X(k_1)}{\alpha}$$

Then the Gateaux derivative of the cost function J with respect to k_1 in the directions of $\bar{k}_1$ will be:

$$\hat{J}(k_1)=\int_0^T\left(C^T(CX-X_{obs}),\hat{X}\right)_{\Im}dt+(k_1-k_{1,0})\bar{k}_1 \tag{4.3}$$

Firstly, we will compute Gateaux derivatives $\hat{J}_{k_1}(k_1)$ of the cost function J with respect to k_1 in the directions of $\bar{k}_1$.

The Gateau derivative equations of (3.3) with respect to k_1 in the direction of $\bar{k}_1$ are written as follows:

$$\begin{cases}\dfrac{\partial\hat{X}}{\partial t}=N(X)\hat{X}+B(X)\bar{k}_1\\ \hat{X}(0)=0\end{cases} \tag{4.4}$$

where:

$$N(X)=\begin{bmatrix}N_{11}(X) & N_{12}(X) & 0 & N_{14}(X) & 0\\ N_{21}(X) & N_{22}(X) & N_{23}(X) & 0 & 0\\ 0 & N_{32}(X) & N_{33}(X) & 0 & 0\\ -\sin\vartheta & \cos\vartheta & 0 & 0 & -U\cos\vartheta-W\sin\vartheta\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}; \tag{4.5}$$

$$N_{ij}=\begin{cases}N_{ij}^{(1)} & in\ the\ first\ phase\\ N_{ij}^{(2)} & in\ the\ second\ phase\end{cases}\qquad (i=1..3;\ j=1..4)$$

$$N_{11}^{(1)}=-\frac{1}{2m}\rho k_1C_{D0}\left(1+\frac{p_\infty-p_c}{0.5\rho(U^2+W^2)}\right)\frac{(2U^4+3U^2W^2)}{(U^2+W^2)^{3/2}}A_c+\frac{1}{m}k_1\rho C_{D0}\frac{p_\infty-p_c}{0.5\rho(U^2+W^2)^{5/2}}U^4A_c$$

$$N_{12}^{(1)}=-\frac{1}{2m}\rho k_1C_{D0}\left(1+\frac{p_\infty-p_c}{0.5\rho(U^2+W^2)}\right)\frac{U^3W}{(U^2+W^2)^{3/2}}A_c+\frac{1}{m}k_1\rho C_{D0}\frac{p_\infty-p_c}{0.5\rho(U^2+W^2)^{5/2}}WU^3A_c$$

$$N_{14}^{(1)} = -\frac{\rho}{2m} k_1 C_{D0} \frac{g}{0.5\left(U^2+W^2\right)^{3/2}} U^3 A_c$$

$$N_{21}^{(1)} = Q;\ N_{22}^{(1)} = 0;\ N_{23}^{(1)} = U;\ N_{32}^{(1)} = 0;\ N_{33}^{(1)} = 0$$

$$N_{11}^{(2)} = -\frac{1}{2m} k_1 \rho C_{D0}\left(1+\frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)}\right)\frac{\left[\left(2U^4+3U^2W^2\right)\right]F_c}{\left(U^2+W^2\right)^{3/2}} + \frac{1}{m} k_1 \rho C_{D0} \frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)^{5/2}} U^4 F_c$$

$$+\frac{\rho}{2m} k_1 C_{D0}\left(1+\frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)}\right)\left(r^2\frac{\sin\left(\frac{r-l_k\tan\theta}{r}\right)\frac{l_k}{r}}{\cos^2\left(\frac{r-l_k\tan\theta}{r}\right)} - \frac{r\sqrt{\frac{l_k d}{\tan\theta}}}{2} + \frac{3}{2} l_k \sqrt{dl_k \tan\theta}\right)\frac{UW}{\sqrt{\left(U^2+W^2\right)}}$$

$$N_{12}^{(2)} = \frac{1}{2m} k_1 \rho C_{D0}\left(1+\frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)}\right)\frac{U^3 W F_c}{\left(U^2+W^2\right)^{3/2}} + \frac{1}{m} k_1 \rho C_{D0} \frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)^{5/2}} W U^3 F_c$$

$$-\frac{\rho}{2m} k_1 C_{D0}\left(1+\frac{p_\infty - p_c}{0.5\rho\left(U^2+W^2\right)}\right)\left(r^2\frac{\sin\left(\frac{r-l_k\tan\theta}{r}\right)\frac{l_k}{r}}{\cos^2\left(\frac{r-l_k\tan\theta}{r}\right)} - \frac{r\sqrt{\frac{l_k d}{\tan\theta}}}{2} + \frac{3}{2} l_k \sqrt{dl_k \tan\theta}\right)\frac{U^2}{\sqrt{\left(U^2+W^2\right)}}$$

$$N_{14}^{(2)} = -\frac{\rho}{2m} k_1 C_{D0} \frac{g}{0.5\left(U^2+W^2\right)^{3/2}} U^3 F_c$$

$$N_{21}^{(2)} = Q;\ N_{22}^{(2)} = 2KC_1 W + KC_2 Q;\ N_{23}^{(2)} = KC_2 W + U\quad N_{32}^{(2)} = 2KC_3 W + KC_4 Q$$

$$N_{33}^{(2)} = KC_4 W$$

$$B = \left(B_1, B_2, B_3, 0, 0\right)$$

$$B_1 = \begin{cases} -\frac{1}{2m}\rho C_{D0}\left(1+\sigma\right)\frac{U^4}{U^2+W^2} A_c & \text{for the first phase} \\ -\frac{1}{2m}\rho C_{D0}\left(1+\sigma\right)\frac{U^4}{U^2+W^2} F_c - \frac{1}{2m} k\left(U,W,h\right) U^2 F'_{c,l_k} l'_{k,k_1} & \text{for the second phase} \end{cases}$$

$$F'_{c,l_k} = \left(r^2\frac{\sin\left(\frac{r-l_k\tan\theta}{r}\right)\frac{\tan\theta}{r}}{\cos^2\left(\frac{r-l_k\tan\theta}{r}\right)} - \frac{r\sqrt{\frac{d\tan\theta}{l_k}}}{2} + \frac{3}{2}\tan\theta\sqrt{dl_k \tan\theta}\right)$$

$$B_2 = \begin{cases} 0 & \text{for the first phase} \\ C'_{1,k_1} W^2 + C'_{2,k_1} WQ & \text{for the second phase} \end{cases}$$

$$B_3 = \begin{cases} 0 & \text{for the first phase} \\ C'_{3,k_1} W^2 + C'_{4,k_1} WQ & \text{for the second phase} \end{cases}$$

$C'_{1,k_1}, C'_{2,k_1}, C'_{3,k_1}, C'_{4,k_1}$ are the derivatives of those functions with respect to parameter k_1. Multiplying the equation (4.4) by adjoin variable $P = \left(P_1, P_2, P_3, P_4, P_5\right)^T$ in the same space as X and then integrating by t between 0 and T we have:

$$\left(\hat{X}(T),P(T)\right)_{\Im}-\left(\hat{X}(0),P(0)\right)_{\Im}=\int_0^T\left(\hat{X},\frac{dP}{dt}+F(X,P)\right)_{\Im}dt+\overline{k}_1\int_0^T B\cdot P^T dt \qquad (4.6)$$

where: $F(X,P)=N^T.P$ with $N(X)$ is defined by the formula (4.5).

If P is satisfying the following equation:

$$\begin{cases}\dfrac{dP}{dt}+F(X,P)=-C^T H\left(CX-X_{obs}\right)\\ P(T)=0\end{cases} \qquad (4.7)$$

Then the Gateau derivative $\hat{J}_{k_1}(k_1)$ of the cost function J with respect to k_1 in the directions of $\overline{k}_1$ is: (see formula 4.3):

$$\hat{J}_{k_1}(k_1)=-\int_0^T\left(\hat{X},\frac{dP}{dt}+F(X,P)\right)_{\Im}dt+\left(k_1-k_{1,0}\right)\overline{k}_1=\overline{k}_1\left(-\int_0^T B\cdot P^T dt+\left(k_1-k_{1,0}\right)\right)=\overline{k}_1 J'_{k_1}$$

Therefore, the function $J'_{k_1}(k_1)$ is calculated by the following formula:

$$J'_{k_1}=-\int_0^T\left(B_1P_1+B_2P_2+B_3P_3\right)dt+\left(k_1-k_{1,0}\right) \qquad (4.8)$$

4.2. Algorithm to solve the optimal control problem

The optimal method is based on inverse BFGS update [23 - 26]. The algorithm schema is written as follows:

a. Let I = 0: Get the initial value $k_{1,i}=k_{1,0}$; $H_i=1$; Solve equations 3.3 with the parameter $k_{1,i}$; and the adjoin equations 4.7; Get the function $J'_{k_1}(k_{1,i})$ by the formula 4.8

b. Calculate

$$d_i=-H_i J'_{k_1}(k_{1,i})$$

c. Calculate α_i so that is satisfied the Armijo-Wolfe conditions ([25, 26]):

$$J(k_{1,i}+\alpha_i d_i)\le J(k_{1,i})+\alpha_i\beta J'_{k_1}\left(k_{1,i}\right)d_i$$

where $\beta\in(0,1)$. Typically β ranges from 10^{-4} to 0.1

This α_i can be found by the following schema steps ([27]):

c.1 $\alpha_{initial}=1$;

c.2 Given $\tau\in(0,1)$. Typically $\tau=0.5$;

c.3 Let l=0 then $\alpha^l=\alpha_{initial}$;

c.4 Check:

While not $J(k_{1,i}+\alpha^l d_i) \le J(k_{1,i}) + \alpha^l \beta J'_{k_1}\left(k_{1,i}\right) d_i$

Set $\alpha^{l+1} = \tau\alpha^l$

Increase l by 1

End while

c.5 Set $\alpha_i = \alpha^{(l)}$;

d. Calculate: $\Delta k_{1,i} = s_i = -\alpha_i H_i J'_{k_1}(k_{1,i})$;

e. Calculate: $k_{1.i+1} = k_{1,i} + \Delta k_{1,i}$;

f. Solve equations 3.3 with the parameter $k_{1,i+1}$ and the adjoin equations 4.7.

g. Get the function $J'_{k_1}(k_{1,i+1})$ by the formula 4.8.

h. Calculate $y_i = J'_{k_1}(k_{1,i+1}) - J'_{k_1}(k_{1,i})$

i. Calculate $H_{i+1} = \left(1 - \dfrac{s_i y_i}{y_i\, s_i}\right) H_i \left(1 - \dfrac{s_i y_i}{y_i\, s_i}\right) + \dfrac{s_i s_i}{y_i\, s_i}$;

j. Let i = i + 1

k. Go to step b if $J'_{k_1}(k_{1,i}) \ge \varepsilon$ ($\varepsilon \succ 0$ is given).

If $J'_{k_1}(k_{1,i}) \approx 0$ the optimal process is stopped. Then, we have $k_1 = k_1^*$.

4.3. Simulation experiment on correcting on correcting parameter k_1 so that U is closed to measurement

Let the body with m = 0.025091315 kg, L_1 = 2.5 cm, L_2 = 11.5 cm d = 0.57 cm, d_c = 0.12 cm, U_0 = 240 m/s, W_0 = 0, Q_0 = 1 rad./s, h_0 = 7 m, ϑ_0 = 0, I_y = 1.81.10-4 kgm^2, x_{cm} = 10.01 cm. We will test the problem by considering the following experiments:

- By the same way as [16, 28] we can have the observation data $X_{obs} = \left(U_{obs},\ W_{obs}, Q_{obs}, h_{obs}, \vartheta_{obs}\right)$ as follows:

Let model run in 0.5s with values **k_1=1** simulating the true velocity $X = \left(U,\ W, Q, h, \vartheta\right)$ by solving the equations (3.1)-(3.2).

This velocity X is used as a reference X_{obs}.

The measurement $Xobs$ is obtained by the values of X in all the time period.

Then we have X_{obs} in every time step.

- In the testing the model is running in the time period 0.5s with values k_1=2. k_1. Then, the vector function $X = \left(U,\ W, Q, h, \vartheta\right)$ is obtained by solving equations (3.1)-(3.2).

The equations (3.1)-(3.2) are solved by Runge Kutta method.

- Using the formula of function $J_k^{'}$ (4.8) the optimal control problem (4.2) is solved by the algorithm schema in subsection 4.2. Then the minimum of $J\left(k_1\right)$ is found by the formula (4.1) with k_1^* value.

- The process finding the coefficient is shown in Figure 3. By this process the error of obtain coefficient in the end optimal process is less than 0.00001 percentage. In the Figure 4 the obtain cost function J in the end of optimal process is nearly zero (less than 0.00001). The error percentages of velocities U by X_1 direction with reference $Uobs$ with and without correction coefficient k_1 are shown in Figure 5. With the correction coefficient the percentage errors of velocities are less than 0.00016 %.

- We have done real experimental of projectile running underwater. The cavity is presented in the Picture 1. In the real measurement we have 96 measured points of velocities U by X_1 direction with the initial velocity U_0 = 271.2 m/s. The other initial conditions are chosen approximately W_0 = 0, Q_0 = 1 rad. /s, h_0 = 1 m, $\square_0$ = 0.

- Let the model run with the beginning coefficient k_1 = 2.5 then the optimal coefficient k_1^* = 0.909999046325684 is found by the optimal program.

- The comparison between velocity measurement and the other ones of calculation with k_1 = 2.5 or optimal coefficient k_1^* = 0.909999046325684 is presented in the figure 6.

- By this figure it is easy to see that with optimal coefficient k_1^* = 0.909999046325684 the model is closer to measurement than the other one without correction.

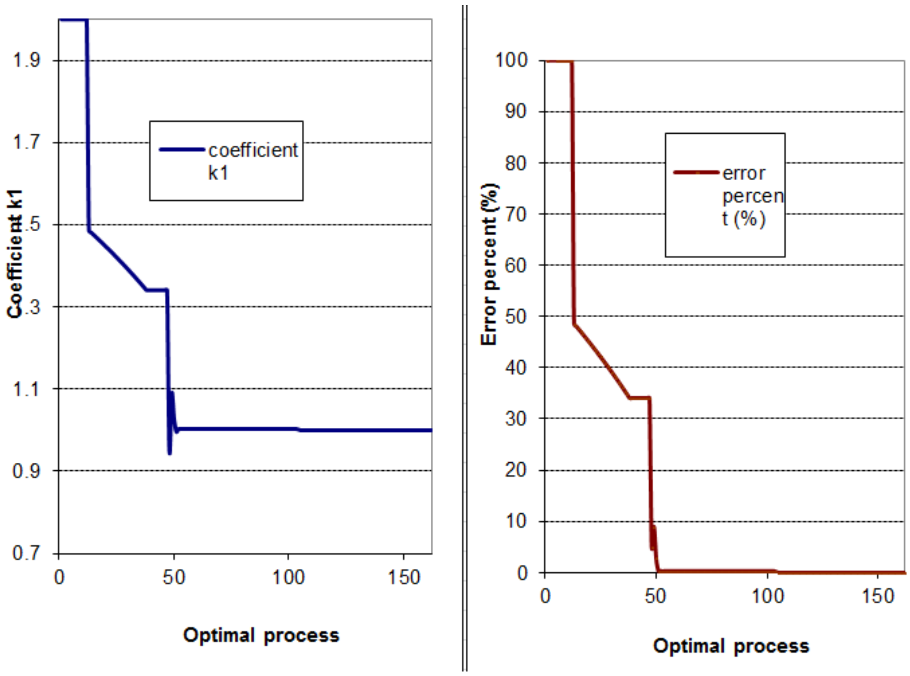


Figure 3. Correcting coefficient k_1 in optimal process (Left); Coefficient error percent in optimal process correcting k_1 (Right).

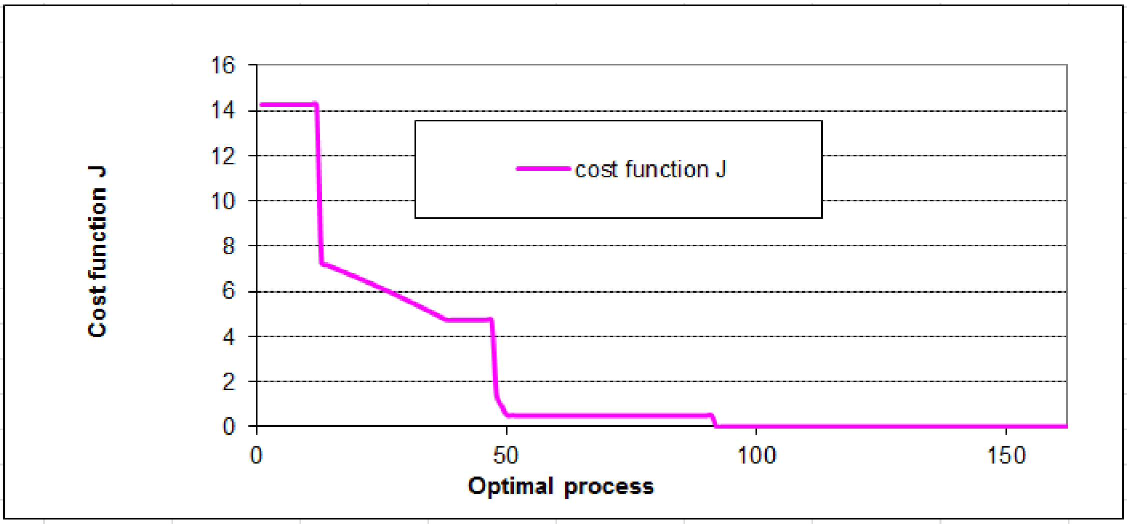
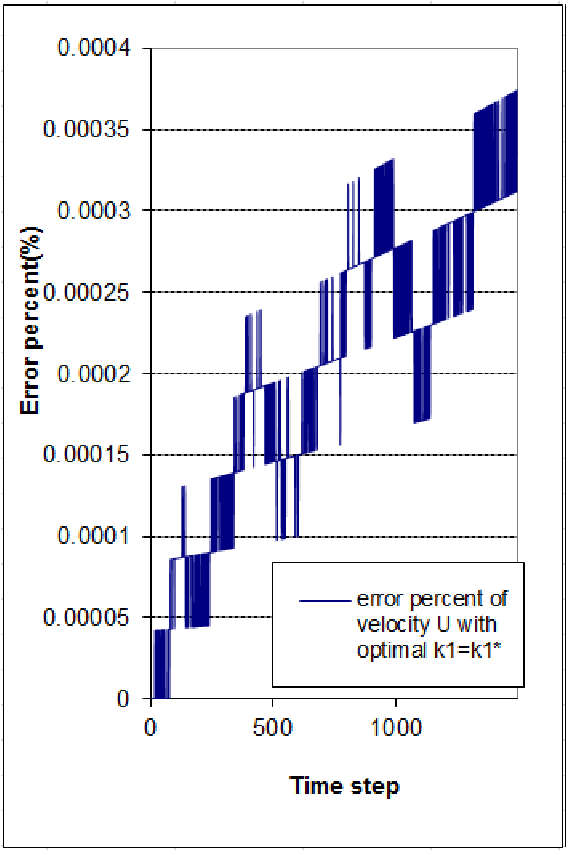


Figure 4. Cost function J in optimal process correcting k_1.



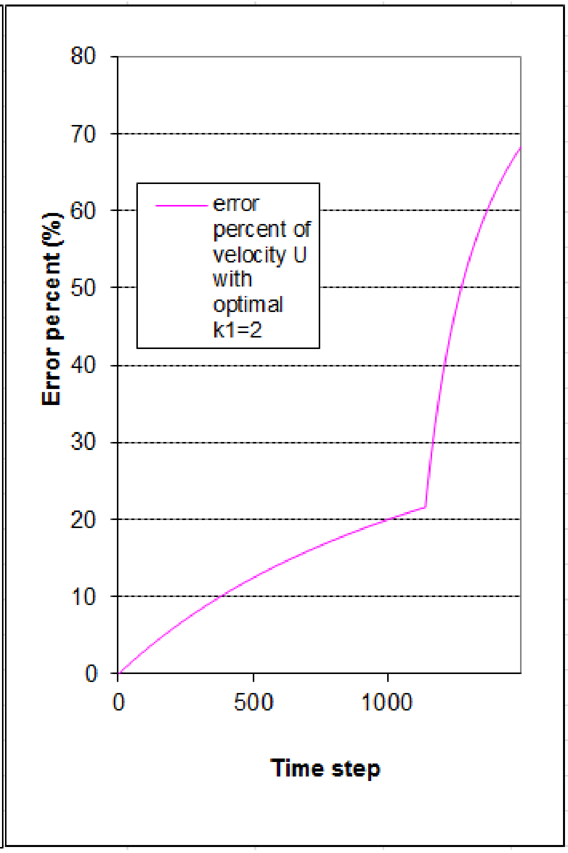
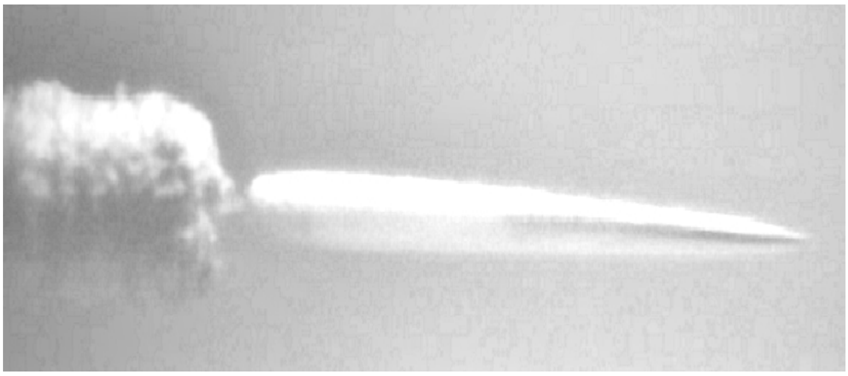


Figure 5. Percent error of velocity $U(t)$ with optimal correction of coefficient $k_1 = \mathbf{k}_1^*$ (left); Percent error of velocity $U(t)$ with coefficient $k_1 = 2$ (Right).



Picture 1. The full cavity arising in very fast motion of projectile under water.

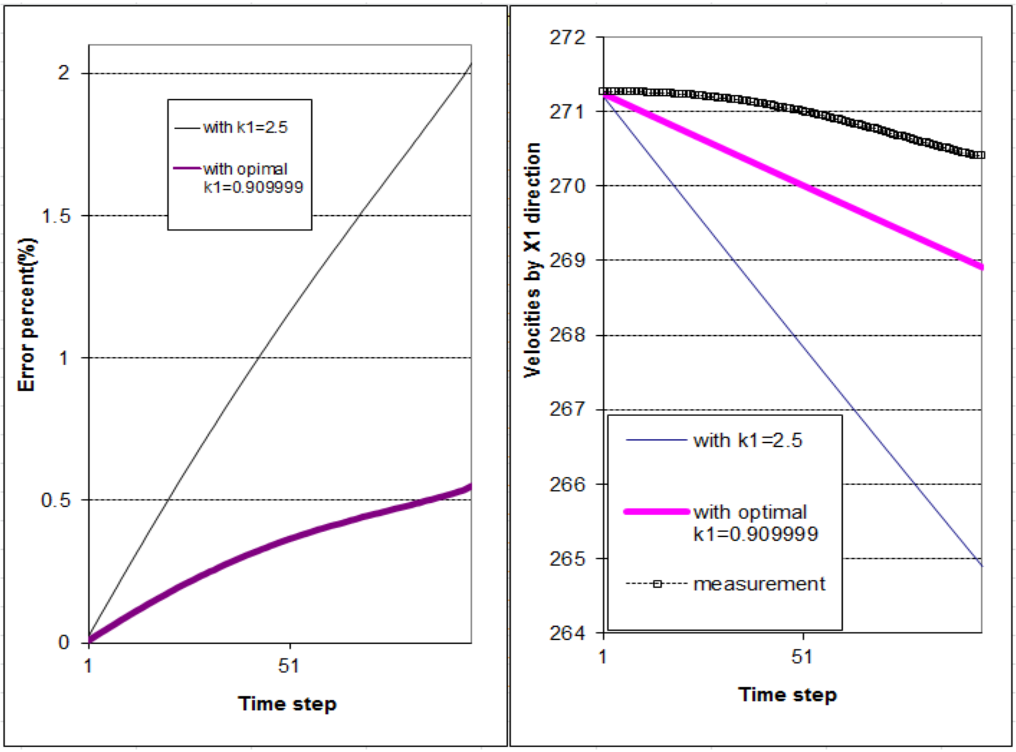


Figure 6. Percent error of velocities U by X_1 direction with and without optimal correction of coefficient k_1 comparing with measurement (left); Comparison of velocities U by X_1 direction with or without correction and measurement.

4. CONCLUSIONS

In the model of slender body running very fast under water the coefficient k_1 strongly effects to the simulation results (the right of Figure 5). By the results presented in Figures 3,4 it is easy to see that by the data assimilation method the corrected coefficient k_1^* can be nearly equal to the reference coefficient k_1 . It follows that the velocity $U(t)$ is closed to the one in reference model (the left of the Figure 5 or Figure 6). Then the data assimilation method can be used as the good tool to correct coefficient in the model of body running fast under water.

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TÓM TẮT

ỨNG DỤNG PHƯƠNG PHÁP ĐỒNG HÓA SỐ LIỆU ĐỂ HIỆU CHỈNH THAM SỐ TRONG MÔ HÌNH SIÊU XÂM THỰC

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Trong môi trường nước, khi một vật thể có hình dạng mảnh di chuyển với vận tốc nhanh hướng về phía trước sẽ tự quay trong một khe rỗng (còn gọi là khoang hơi hay túi hơi xâm thực).

Trong mô hình khe rỗng hệ số cản của vật thể đóng vai trò rất quan trọng trong quá trình di chuyển. Theo Salis, Garabedian, Kiceniukm hệ số cản này được chọn bới các giá trị thích hợp trong khoảng từ 0,8 đến 1. Theo Rand, Kirschner thì hệ số cản này được viết bởi công thức $k = C_{D0}(1+\sigma)\cos^2\alpha$ với σ là số cavitation (số xâm thực), α là góc giữa trục của vật thể mảnh và hướng của di chuyển. C_{D0} là tham số thường được chọn trong khoảng từ 0.6 đến 0,85. Trong bài báo này hệ số cản được viết dưới dạng $k = k_1 C_{D0}(1+\sigma)\cos^2\alpha$, trong tính toán hệ số C_{D0} được lấy bằng 0,82 và bằng phương pháp toán học hệ số chưa biết k_1 sẽ được hiệu chỉnh sao cho các vận tốc di chuyển trong mô hình gần với các số liệu quan sát được. Phương pháp toán học được áp dụng để tìm hệ số chưa biết k_1 là phương pháp đồng hóa số liệu. Trong phương pháp này các số liệu quan sát được sử dụng trong hàm mục tiêu. Đây chính là một trong những phương pháp hữu hiệu để giải các bài toán tối ưu bằng cách giải bài toán liên hợp rồi tính gradient của hàm mục tiêu.

Từ khóa: đồng hóa số liệu, tối ưu, phương pháp Runge-Kutta.