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THE PARTITIONING METHOD BASED ON HEDGE ALGEBRAS FOR FUZZY TIME SERIES FORECASTING

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ABSTRACT

In recent years, many partitioning methods have been proposed for fuzzy time series, because they strongly affect to forecasting results. In this paper, we present a novel partitioning method based on hedge algebras (HA). The experimental results show that the proposed method is better than the others on the accuracy of forecasting. It is simple and flexible in applying this method because we can determine the parameters of HA for reasonable intervals.

Keywords: fuzzy time series, forecasting time series, reasonable intervals, hedge algebras.

1. INTRODUCTION

In the first research on the fuzzy time series in 1993, Song and Chissom [1] proposed a method (S&C) that used fuzzy time series to forecast time series. According to that, *C(t)* is the conventional time series that needs to be forecasted, this one can be forecasted by converting into fuzzy time series *F(t)*. After that, the forecasting result on *F(t)* is defuzzified to become the forecasting result on *C(t)*. So, *F(t)* is a qualitative view about *C(t)*. Because of this, we offer a convention by giving the collection of all historical values of *F(t)* to be *C(t)* and the values of *F(t)* to be the linguistic terms that are used to qualitatively describe the values of *C(t)*. The method S&C can be summarized in seven steps: (1) Determining *U* which is the universe of discourse of *F(t)*, (2) Partitioning *U* into a collection of intervals, (3) Determining the collection of linguistic terms used to quanlitatively describe the historical values of *F(t)*, (4) Quantifying linguistic terms by means of fuzzy sets, (5) Mining fuzzy relationships, $A_n \rightarrow A_m \circ R_i$ where i = 1, 2, …, A_n, A_m and R_i respectively are fuzzy sets used to quantify the values of *F(t)* at point *t-1*, *t* and fuzzy relation between these values, (6) Calculating forecasting values by the formula $A_a = A_b \circ R$ (*), in which A_a and A_b, the values of *F(t)*, are quantified by fuzzy sets at point *t, t-1*, and $R = \cup R_i$; (7) Defuzzifying forecasting values on *F(t)* to find forecasting values of *C(t)*. Song and Chissom, in [2, 3], used S&C to forecast enrollments at University of Alabama.

We can see that step (2), in the method of Song and Chissom, plays a pivotal role because this step significatly impacts remaining steps and forecasting accuracy. Indeed, if we increase the number of intervals, then we have to get larger computation overhead for performing steps (6), (7) and these steps directly affect to forecasting result. So, how to partition the universe of discourse (how to partition *U*) has become a basic problem in the field of using fuzzy time series to forecast time series.

In 1996, Chen proposed an improved method for using fuzzy time series to forecast enrollments at University of Alabama [4]. This research is remarkable because one used simple arithmetic operations on intervals to compute forecasting values and to significantly reduce calculation time. The most impressive thing is that it has spread a new idea in studying fuzzy time series, in which, researchers just focus on finding reasonable intervals.

Up to now, based on the studies, we can distinguish between two types of partitioning *U*: partitioning *U* into equal or not equal intervals. The studies in [2 - 8] are typical for the first type. The papers [9 -15] are delegated for the second type. Generally, the studies follow the second type of partitioning that are newer ones and usually yield better forecasting result than the others. There are rather many ways to partition *U* following second type. For instance, in [9] Chen et al. based on statistical distribution of historical values in each interval, in [10] Huarng et al. based on ratio between two consecutive historical values, Chen and Kao in [11] employed particle swarm optimization, Wang et al. in [12, 13] used information granules, Bas in [14] exploited modified genetic algorithm, and Lu et al. in [15] also used information granules to partition *U*.

As already mentioned, the second type of partitioning gives more accuracy forecasting result than the others, but, it is quite difficult to find intervals following the second type based on the approaches same as [9-15]. At the same time, the quality of forecasting result is not good enough. In this paper, we present a novel method of partitioning the universe of discourse based on hedge algebras (HA). We can get reasonable intervals with the proposed method.

According to this method, the number of intervals, partitioned on *U*, are equal to the number of linguistic terms used to qualitatively describe the historical values of fuzzy time series and fuzziness interval of each linguistic term is assigned to size of each interval. As a result, intervals can have not equal size. The experimental results show that proposed method has better forecasting performance, on regular time series, than the others published recently.

The rest of this paper is organized as follows. In Section 2, we briefly introduce some basic concepts in HA. The main content of the paper, novel method of partitioning the universe of discourse based on HA, is presented in Section 3. Section 4 presents experimental result and some discussions for applying the proposed method to forecast on some regular time series. Section 5, the last section, is the conclusion of the paper.

2. SOME BASIC CONCEPTS IN HEDGE ALGEBRAS

In this section, we refer to paper [16, 17] to briefly review some basic concepts in HA, these concepts are exploited to form the proposed method.

The HA are denoted by $AX = (X, G, C, H, \leq)$, where, $G = \{c^+, c^-\}$is the collection of primary generators, in which c^+ and c^- are, respectively, the negative primary term and the positive one of a linguistic variable X, $C = \{0, 1, W\}$ is a set of constants, which are distinguished with elements in X, H is the set of hedges, "$\leq$" is a *semantically ordering relation* on X. For each $x \in X$ in HA, $H(x)$ is the set of terms $u \in X$ generated from x by applying the hedges of H and $u = h_n \dots h_1 x$, with $h_n, \dots, h_1 \in H$. $H = H^+ \cup H^-$, in which H^- is the set of all negative hedges and H^+ is the set of all positive ones of X. The possitive hedges increase semantic tendency and vise versa with negative hedges. Without loss of generality, it can be assumed that $H^- = \{h_{-1} < h_{-2} < \dots < h_{-q}\}$ and $H^+ = \{h_1 < h_2 < \dots < h_p\}$.

If X and H are linearly ordered sets, then $AX = (X, G, C, H, \leq)$ is called *linear hedge algebras*, further more, if AX is equipped with additional operations Σ and Φ that are, respectively, infimum and supremum of $H(x)$, then it is called *complete linear hedge algebras* (ClinHA) and denoted $AX = (X, G, C, H, \Sigma, \Phi, \leq)$.

Fuzziness of vague terms and fuzziness measure are two concepts that are difficult to define in fuzzy set theory. However, HA can reasonably define these ones. Concretely, elements of $H(x)$ still express a certain meaning stemming from x, so we can interpret the set $H(x)$ as a model of the fuzziness of the term x. With regard to fuzziness measure, it can be formally defined by following difinitions.

Definition 2.1. Let $AX = (X, G, C, H, \leq)$ be a ClinHA. An $fm: X \rightarrow [0,1]$ is said to be a fuzziness measure of terms in X if:

(1). $fm(c^-) + fm(c^+) = 1$ and $\sum_{h \in H} fm(hu) = fm(u)$, for $\forall u \in X$; in this case fm is called complete;

(2). For the constants $\mathbf{0}$, $\mathbf{W}$ and $\mathbf{1}$, $fm(\mathbf{0}) = fm(\mathbf{W}) = fm(\mathbf{1}) = 0$;

(3). For $\forall x, y \in X$, $\forall h \in H$, $\dfrac{fm(hx)}{fm(x)} = \dfrac{fm(hy)}{fm(y)}$, that is this proportion does not depend on specific elements and, hence, it is called *fuzziness measure of the hedge h* and denoted by $\mu(h)$.

The condition (1) means that the primary terms and hedges under consideration are complete for modelling the semantics of the whole real interval of a physical variable. That is, except the primary terms and hedges under consideration, there are no more primary terms and hedges. (2) is intuitively evident. (3) seems also to be natural in the sense that applying a hedge h to different vague concepts, the relative modification effect of h is the same, i.e. this proportion does not depend on terms that they apply to.

The properties of fuzziness measure are determined clearly through the following proposition.

Proposition 2.1. *For each fuzziness measure fm on X the following statements hold*:

(1). $fm(hx) = \mu(h) fm(x)$, *for every* $x \in X$;

(2). $fm(c^-) + fm(c^+) = 1$;

(3). $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i c) = fm(c)$, $c \in \{c^-, c^+\}$;

(4). $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i x) = fm(x)$;

(5). $\sum_{-q \leq i \leq -1} \mu(h_i) = \alpha$ *and* $\sum_{1 \leq i \leq p} \mu(h_i) = \beta$, *where* $\alpha, \beta > 0$ *and* $\alpha + \beta = 1$.

HA build the method of quantifying the semantic of linguistic terms based on the fuzziness measures and hedges through υ mapping that fit to the conditions in the following definitions.

Definition 2.2. Let $AX = (X, G, C, H, \Sigma, \Phi, \leq)$ be a CLinHA. A mapping $\upsilon : X \rightarrow [0,1]$ is said to be an semantically quantifying mapping of AX, provided that the following conditions hold: (1). υ is a one-to-one mapping from X into [0,1] and preserves the order on X, i.e. for all $x, y \in X$, $x < y \Rightarrow \upsilon(x) < \upsilon(y)$ and $\upsilon(\mathbf{0}) = 0$, $\upsilon(\mathbf{1}) = 1$, where $\mathbf{0}, \mathbf{1} \in C$;

(2). $\forall x \in X$, $\upsilon(\Phi x) = infimum\ \upsilon(H(x))$ and $\upsilon(\Sigma x) = supremum\ \upsilon(H(x))$.

Semantically quantifying mapping υ is determined concretely as follows.

Definition 2.3. Let fm be a fuzziness measure on X. A mapping $\upsilon : X \rightarrow [0,1]$, which is induced by fm on X, is defined as follows:

(1). $\upsilon(\boldsymbol{W}) = \theta = fm(c^-)$, $\upsilon(c^-) = \theta - \alpha fm(c^-) = \beta fm(c^-)$, $\upsilon(c^+) = \theta + \alpha fm(c^+)$;

(2). $\upsilon(h_j x) = \upsilon(x) + Sign(h_j x)\{\sum_{i=Sign(j)}^{j} fm(h_i x) - \omega(h_j x)\, fm(h_j x)\}$,

where $j \in \{j: -q \leq j \leq p\ \&\ j \neq 0\} = [-q \wedge p]$

and $\omega(h_j x = \frac{1}{2}[1 + Sign(h_j x) Sign(h_p h_j x)(\beta - \alpha)] \in \{\alpha, \beta\}$;

(3). $\upsilon(\Phi c^-) = 0$, $\upsilon(\Sigma c^-) = \theta = \upsilon(\Phi c^+)$, $\upsilon(\Sigma c^+) = 1$, and for $j \in [-q \wedge p]$,

$$\upsilon(\Phi h_j x) = \upsilon(x) + Sign(h_j x)\{\sum_{i=sign(j)}^{j-sign(j)} \mu(h_i)\, fm(x)\} - \frac{1}{2}(1 - Sign(h_j x))\mu(h_j) fm(x),$$

$$\upsilon(\Sigma h_j x) = \varphi(x) + Sign(h_j x)\{\sum_{i=sign(j)}^{j-sign(j)} \mu(h_i)\, fm(x)\} + \frac{1}{2}(1 + Sign(h_j x))\mu(h_j) fm(x).$$

The *Sign* function and fuzziness interval are determined in the following difinitions.

Definition 2.4. A function *Sign*: $X \rightarrow \{-1, 0, 1\}$ is a mapping which is defined recursively as follows, for $h, h' \in H$ and $c \in \{c^-, c^+\}$:

(1). $Sign(c^-) = -1$, $Sign(c^+) = +1$;

(2). $Sign(hc) = -Sign(c)$, if h is negative w.r.t. c; $Sign(hc) = +Sign(c)$, if h is positive w.r.t. c;

(3). $Sign(h'hx) = -Sign(hx)$, if $h'hx \neq hx$ and h' is negative w.r.t. h; $Sign(h'hx) = +Sign(hx)$, if $h'hx \neq hx$ and h' is positive w.r.t. h.

(4). $Sign(h'hx) = 0$ if $h'hx = hx$.

Definition 2.5. The fuzziness interval of the linguistic terms $x \in \boldsymbol{X}$, denoted by $\Im(x)$, is a subinterval of [0,1], if $|\Im(x)| = fm(x)$ where $|\Im(x)|$ is the length of $\Im(x)$, and recursively determnied by the length of x as follows:

(1). If length of x is equal to 1 ($l(x)$=1), that mean $x \in \{c^-, c^+\}$, then $|\Im(c^-)| = fm(c^-)$, $|\Im(c+)| = fm(c^+)$ and $\Im(\boldsymbol{c}^-) \leq \Im(c^+)$;

(2). Suppose that n is the length of x ($l(x)=n$) and fuzziness interval $\Im(x)$ has been definied with $|\Im(x)| = fm(x)$. The set $\{\mathfrak{I}(h_jx)| j \in [-q^p]\}$, where $[-q^p] = \{j \mid -q \leq j \leq -1 \text{ or } 1 \leq j \leq p\}$, is a partition of $\mathfrak{I}(x)$ and we have: for $Sgn(h_px) = -1$, $\mathfrak{I}(h_px) \leq \mathfrak{I}(h_{p-1}x) \leq \ldots \leq \mathfrak{I}(h_1x) \leq \mathfrak{I}(h_{-1}x) \leq \ldots \leq \mathfrak{I}(h_{-q}x)$; for $Sgn(h_px) = +1$, $\mathfrak{I}(h_{-q}x) \leq \mathfrak{I}(h_{-q+1}x) \leq \ldots \leq \mathfrak{I}(h_{-1}x) \leq \mathfrak{I}(h_1x) \leq \ldots \leq \mathfrak{I}(h_px)$.

3. THE PARTITIONING METHOD BASED ON HA

Following fuzzy set approach, the linguistic terms used to qualitatively describe historical values of fuzzy time series, $X_i(t)$ (i = 1, 2, …), are quantified by mean of fuzzy sets. In the HA approach, $X_i(t)$ are quantified by mean of the semantically quantifying mapping and fuzziness measure. So we need to adjust the definition of fuzzy time series for meeting with HA approach. This adjustment does not change the nature of fuzzy time series.

Definition 3.1. The definition of fuzzy time series based on HA

Let $X(t)$ (t = …, 0, 1, 2, …) a subset of R^1, be the universe of discourse of linguistic terms $X_i(t)$ (t = 1, 2, …), $F(t)$ is the collection of $X_i(t)$. Then $F(t)$ is called a fuzzy time series on $X(t)$.

The proposed method is expressed in the following:

Considering linguistic variable l, from domain of l we can organize a hedge algebra $AX = (X, G, H, \leq)$. $F(t)$ is the fuzzy time series containing a collection of linguistic terms of l, so $F(t) \subseteq X$ and the values of $F(t)$ are generated from c^- and c^+. The number of intervals on U of $F(t)$ are equal to linguistic terms that are used to qualitatively describe historical values of $F(t)$. Each value of $F(t)$, a linguistic term, determines an interval which is the length of it's fuzziness interval. Formally, this method, called DI, comprises following steps:

Step 1: Determining the linguistic terms used to qualitatively describe the historical values of $F(t)$.

Step 2: Normalizing the linguistic terms so that they simultaneously generate from c^-, c^+ and belong to HA $AX = (X, G, H, \leq)$. If we need to generate more linguistic term to match with the number of linguistic terms in Step 1, then if H has more than two hedges, then we use two hedge h_g, $h_e \in H'$ (H' just contain h_g and h_e, $H' \neq H$) where h_g is a nagative hedge, h_e is a positive one and $fm(h_g) + fm(h_e) = 1$. Next, choosing a linguistic term that has fuzziness interval containing the maximum amount of historical values, called y. From this one we can generate h_gy and h_ey. Otherwise, if H has only two hedges, then use these hedges to generate more hedges from y.

Step 3: Determining the number of intervals. These are equal to the number of linguistic terms in Step 2.

Step 4: Determining the size of intervals through determining fuzziness intervals of the linguistic terms by Proposition 2.1.

The values of $F(t)$ may not simultaneously generate from certain generators, so Step 2 need to be performed. We can replace a linguistic term by a different linguistic term so that all of them belong to one HA.

Method DI is served as one step in the method of forecasting fuzzy time series. This method refers to the some ideas in [4] and [14]. We name this method FL.

Denoting $co\mathfrak{I}(x)$ and $co\vartheta(x)$, respectively, are fuzziness intervals and semantically quantifying values of x that are mapped from [0, 1] to the universe of discourse, U, of $F(t)$. From here, when we mention "fuzziness interval" and "semantically quantifying value" of *x* that means we are mentioning to $co\mathfrak{I}(x)$ and $co\vartheta(x)$.

Method FL, forecasting fuzzy time series:

Step 1: Applying DI to determine intervals on the universe of discourse of *F(t).*

Step 2: Calculating the semantically quantifying values of linguistic terms that are used to qualitatively describe historical values of *F(t).*

Step 3: Mining the fuzzy relationships among the linguistic terms.

To facilitate calculating, each linguistic terms, obtaining from Step 2, are denoted by *Ai* where $I = \overline{1,k}$. The fuzzy relationships are denoted: $At \rightarrow Au$ (*p*) … *Av* (*q*), where *At*, *Au*,…*Av* are linguistic terms; *p*, *q* are positive integers that refer to the number of iteration of *Au*, and *Av* in the fuzzy relationships that have left side *At*.

Step 4: Calculating forecasting values

Forecasting value of fuzzy time series at point *t+1* is computed as follows:

Considering historical value of fuzzy time series at point *t*, denoted *f(t)*, if *f(t)* belong to *cofm*(*At*), then compute the forecasting value at point *t+1* by following formula:

$$\frac{p*co\vartheta(Au(h(Au))+...+q*co\vartheta(Av(h(Av))}{p+...+q}$$

where $co\vartheta(Ai(hAi))$ is the semantically quantifying value of *Ai* or *hAi* which is chosen, *h* is the negative or positive hedge mentioned in Step 2.

Let θ be average of values falling into *Ai*'s fuzziness interval, θ describes the density of historical values of *F(t)* and tend to lean left, right or evenly distribute in this interval. $co\vartheta(Ai)$ or $co\vartheta(hAi)$ are chosen depending on the distance from them to θi, *dij* where *j* = 1, 2, 3. This distance is reflective of the suitability between semantics of linguistic term and distribution rule of historical values of fuzzy time series on intervals, so if any semantically quantifying value has minimum distance to θi, then that value will be chosen.

4. RESULTS AND DISCUSSIONS

In this section, method DI and FL are applied on regular time series used in some previous researchs. These time series are enrollments at University of Alabama, TAIEX index [15] and Unemployment rates [15]. From here, for short, these time series are briefly called Alabama, TAIEX and UEP. This paper takes the forecasting results of different methods used in paper [15] to compare with forecasting results of the proposed method.

Annually, it can use the linguistic terms such as [2, 3] to qualitatively describe the enrollments at University of Alabama. However, we use the following linguistic terms to facilitate for applying the proposed method: *very very low* (*A1*), *little very low* (*A2*), *very little low* (*A3*), *little little low* (*A4*), *little little hight* (*A5*), *very little hight* (*A6*) and *very hight* (*A7*). These linguistic terms completely cover semantic description of the enrollments (from minimum enrollments to maximum enrollments). It can be seen that the linguistic terms belonging to domain of linguistic variable "enrollment" forming HA: $AX = (X, G, H, \leq)$, where *G* = {*low*, *hight*}, *H* = {*very*, *little*}, *X* = *H*(*G*).

Applying FL to forecast enrollments at University of Alabama as follows:

Step 1: Applying DI to determine the intervals: Let D_{min} and D_{max}, respectively, be minimum and maximum enrollment from 1971 to 1991. Based upon D_{min} and D_{max} we define U as $[D_{min} – D_1, D_{max} + D_2]$ where $D_1 = 55$, $D_2 = 663$, the same as [2-3], so U=[13000, 20000]. The length of U, denote LU, LU = 20000 – 13000 = 7000.

Table 1. The fuzzified historical enrollments.

Year	Actual enrollment	Fuzzified enrollment	Year	Actual enrollment	Fuzzifield enrollment
1971	13055	*A1*	1982	15433	*A3*
1972	13563	*A1*	1983	15497	*A3*
1973	13867	*A1*	1984	15145	*A3*
1974	14696	*A2*	1985	15163	*A3*
1975	15460	*A3*	1986	15984	*A4*
1976	15311	*A3*	1987	16859	*A6*
1977	15603	*A4*	1988	18150	*A7*
1978	15861	*A4*	1989	18970	*A7*
1979	16807	*A6*	1990	19328	*A7*
1980	16919	*A6*	1991	19337	*A7*
1981	16388	*A5*			

The number of linguistic terms used to qualitatively describe the historical values of Alabama are 7, so U is partitioned into 7 intervals. Specificially, the intervals are determined as follows:

Domain U is mapped into [0, 1]. If we suppose that 16000 is low, then it can set the parameters: $fm(low) = \dfrac{16000-13000}{20000-13000}$ = 0.428, so $fm(hight)$ = 0.572. Reversely mapping these values into U, we respectively have $co\mathfrak{I}(low)$ and $co\mathfrak{I}(hight)$: $fm(low)$ x LU = 0.428 x 7000 = 2996, $fm(hight)$ x LU = 0.572 x 7000 = 4004.

It can choose: $\mu(Little) = 0.4$, $\mu(Very) = 0.6$. Based on these parameters we determined the fuzziness intervals of the linguistic terms that are also intervals on U:

$co\mathfrak{I}(A1) = \mu(Very) \times \mu(Very) \times co\mathfrak{I}(low)$ = 0.6 x 0.6 x 2996= 1079. The interval corresponding to $A1$ is [13000, 14079). Similarly, we have the rest intervals: [14079, 14798), [14798, 15517), [15517, 15996), [15996, 16637), [16637, 17598), [17598, 20000).

Step 2: The semantically quantifying values of Ai and hAi (i =1, ...,7) are calculated by difinition 2.3 as follows:

$co\vartheta(A1) = \beta \times co\mathfrak{I}(low) - co\mathfrak{I}(A2) - \alpha \times co\mathfrak{I}(A1)$ = 0.6 x 2996 – 719 – 0.4 x 1079 = 13647. Similarly, we have semantically quantifying values of the rest linguistic terms. Based on historical values of Alabama, we computed θi and dij (i =1, ..., 7, j = 1, 2, 3). All of the values are shown in Table 2 in the following:

Table 2. The values of $co\vartheta(Ai)$, $co\vartheta(hAi)$, θi and dij.

$co\vartheta(A1) = 13647$ $d11 = 169$	$co\vartheta(VeryA1) = 13388$ $d12 = 90$	$co\vartheta(LittleA1) = 13906$ $d13 = 428$	$\theta 1 = 13478$
$co\vartheta(A2) = 14510$ $d21 = 186$	$co\vartheta(VeryA2) = 14338$ $d22 = 358$	$co\vartheta(LittleA2) = 14683$ $d23 = 13$	$\theta 2 = 14696$
$co\vartheta(A3) = 15229$ $d31 = 106$	$co\vartheta(VeryA3) = 15056$ $d32 = 278$	$co\vartheta(LittleA3) = 15402$ $d33 = 67$	$\theta 3 = 15335$
$co\vartheta(A4) = 15804$ $d41 = 12$	$co\vartheta(VeryA4) = 15689$ $d42 = 127$	$co\vartheta(LittleA4) = 15919$ $d43 = 103$	$\theta 4 = 15816$
$co\vartheta(A5) = 16252$ $d51 = 136$	$co\vartheta(LittleA5) = 16099$ $d52 = 289$	$co\vartheta(VeryA5) = 16406$ $d53 = 18$	$\theta 5 = 16388$
$co\vartheta(A6) = 17021$ $d61 = 159$	$co\vartheta(LittleA6) = 16790$ $d62 = 71$	$co\vartheta(VeryA6) = 17252$ $d63 = 390$	$\theta 6 = 16862$
$co\vartheta(A7) = 18559$ $d71 = 374$	$co\vartheta(LittleA7) = 17982$ $d72 = 950$	$co\vartheta(VeryA7) = 19135$ $d73 = 203$	$\theta 7 = 18932$

In Table 2, the grey cells have $co\vartheta(Ai)$ or $co\vartheta(hAi)$ which is chosen.

Step 3: Based on Table 1 we mined the fuzzy relationships as follows:

Table 3. Group of fuzzy relationships.

Group 1	$A1\rightarrow A1$ (2), $A1\rightarrow A2$
Group 2	$A2\rightarrow A3$
Group 3	$A3\rightarrow A3$ (4), $A3\rightarrow A4$ (2)
Group 4	$A4\rightarrow A4$, $A4\rightarrow A6$ (2)
Group 5	$A5\rightarrow A3$
Group 6	$A6\rightarrow A5A6A7$
Group 7	$A7\rightarrow A7$(4)

Step 4: Based on the data from Table 1 and Table 3, the forecasting values of the years from 1972 to 1992 are calculated by method of FL as follows:

[1972]: The linguistic term used to qualitatively describe the historical value of 1971 is *A1* and from Table 3 we can see that the fuzzy relationships have left side *A1* belonging to Group 1: $A1\rightarrow A1$, $A1\rightarrow A2$. The picked semantically quantifying values correspond to *A1* and *A2* respectively are $co\vartheta(VeryA1) = 13388$, $co\vartheta(LittleA2) = 14683$. So the forecasting value of 1972 is $\frac{1}{3}$ x (13388 x 2 + 14683) = 13820. Similar to that, we have the forecasting values of the rest years. The forecasting result is shown as well as different forecasting results (belonging to some recently other methods) in the following Table 4.

Table 4. Comparing forecasting result on Alabama.

Year	Actual value	Wang 2013	Wang 2014	Chen 2013	Lu 2015	Proposed method
1972	13563	13486	13944	14347	14279	13820
1973	13867	14156	13944	14347	14279	13820
1974	14696	15215	13944	14347	14279	13820
1975	15460	15906	15328	15550	15392	15402
1976	15311	15906	15753	15550	15392	15536
1977	15603	15906	15753	15550	15392	15536
1978	15861	15906	15753	15550	16467	16461
1979	16807	16559	16279	16290	16467	16461
1980	16919	16559	17270	17169	17161	17444
1981	16388	16559	17270	17169	17161	17444
1982	15433	16559	16279	16209	14916	15402
1983	15497	15906	15753	15550	15392	15536
1984	15145	15906	15753	15550	15392	15536
1985	15163	15906	15753	15550	15392	15536
1986	15984	15906	15753	15550	15470	15536
1987	16859	16559	16279	16290	16467	16461
1988	18150	16559	17270	17169	17161	17444
1989	18970	19451	19466	18907	19257	19135
1990	19328	18808	18933	18907	19257	19135
1991	19337	18808	18933	18907	19257	19135
1992	18876	18808	18933	18907	19257	19135
RMSE		578.3	506.0	486.3	445.2	441.3

The root mean square error (RMSE) criteria is usually used to estimate forecasting perfomance in the literature: RMSE = $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x'_i - x_i)^2}$, where x_i' is the forecasting value, x_i is historical value and n is the number of forecasting values. Applying RMSE for the forecasting result of the proposed method we have: RMSE = 441.3.

Similarly, applying FL for TAIEX 1992 [15] with 7 intervals we have Table 5 in the following:

Table 5. Comparing forecasting result on TAIEX.

Date	Actual data	Wang 2013	Chen 2013	Wang 2014	Lu 2015	Proposed method
02/12/1992	3635.7	3629.3	3740.9	3564.5	3693.1	3709.8
03/12/1992	3614.1	3629.3	3740.9	3564.5	3693.1	3709.8
04/12/1992	3651.4	3629.3	3740.9	3564.5	3693.1	3709.8
05/12/1992	3727.9	3629.3	3740.9	3564.5	3693.1	3709.8
07/12/1992	3755.8	3629.3	3740.9	3859.9	3693.1	3709.8
08/12/1992	3761	3629.3	3740.9	3859.9	3693.1	3709.8
09/12/1992	3776.6	3629.3	3740.9	3859.9	3693.1	3709.8
10/12/1992	3746.8	3629.3	3740.9	3859.9	3693.1	3709.8
11/12/1992	3734.3	3629.3	3740.9	3859.9	3693.1	3709.8
12/12/1992	3742.6	3629.3	3740.9	3859.9	3693.1	3709.8
14/12/1992	3696.8	3629.3	3740.9	3859.9	3693.1	3709.8
15/12/1992	3688.3	3629.3	3740.9	3564.5	3693.1	3709.8
16/12/1992	3674.9	3629.3	3740.9	3564.5	3693.1	3709.8
17/12/1992	3668.7	3629.3	3740.9	3564.5	3693.1	3709.8
18/12/1992	3658	3629.3	3740.9	3564.5	3693.1	3709.8
21/12/1992	3576.1	3629.3	3740.9	3564.5	3693.1	3709.8
22/12/1992	3578	3629.3	3477.1	3564.5	3519.4	3442.3
23/12/1992	3448.2	3629.3	3477.1	3564.5	3519.4	3442.3
24/12/1992	3456	3629.3	3477.1	3413.3	3519.4	3442.3
28/12/1992	3327.7	3629.3	3477.1	3413.3	3519.4	3442.3
29/12/1992	3377.1	3629.3	3368.1	3413.3	3519.4	3491.4
RMSE		114.2	85.7	107.2	75.7	68.9

Also Applying FL for UNE [15] with 9 intervals, the forecasting result is presented in the following Table 6:

Table 6. Comparing forecasting result on UNE.

Date	Actual data	Wang 2013	Chen 2013	Wang 2014	Lu 2015	The proposed method
02/01/2013	7.7	7.39	7.60	7.62	7.58	7.51
03/01/2013	7.5	7.39	7.60	7.62	7.58	7.51
04/01/2013	7.5	7.39	7.60	7.62	7.58	7.51
05/01/2013	7.5	7.39	7.60	7.62	7.58	7.51
06/01/2013	7.5	7.39	7.60	7.62	7.58	7.51
07/01/2013	7.3	7.39	7.60	7.62	7.58	7.51
08/01/2013	7.2	7.39	7.12	7.13	7.07	6.99
09/01/2013	7.2	6.89	7.12	7.13	7.07	6.99
10/01/2013	7.2	6.89	7.12	7.13	7.07	6.99
11/01/2013	7.0	6.89	7.12	7.13	7.07	6.99
12/01/2013	6.7	6.89	7.12	7.13	7.07	6.99
RMSE		0.20	0.18	0.19	0.17	0.16

Comparing forecasting results of the proposed method with some forecasting result of recently different methods on regular time series such as Alabama, TAIEX, UNE in Table 4, Table 5 and Table 6 show that the proposed method gives better forecasting performance. Besides, the proposed method only use arithmetic operations with simple way to calculate forecasting result.

5. CONCLUSION

This paper presented a novel method of partitioning the universe of discourse, and used this method in the method of using fuzzy time series to forecast time series, to improve forecasting performance. The proposed method is formed by mean of the linguistic terms that are used to qualitatively describe the historical values of fuzzy time series. Based on the linguistic terms, the number of intervals, corresponding to the number of linguistic terms, and length of intervals, corresponding to the fuzziness intervals, are determined.

From the experimental results on the regular time series, compare to forecasting result of different methods, we can see that when using the proposed method to model fuzzy time series gives better forecasting accuracy. The proposed method also shows that it is rather simple because of using only arithmetic operations and simple way to calculate forecasting values.

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