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ESTABLISHING FORMULAS FOR DESIGN OF ROOTS PUMP GEOMETRICAL PARAMETERS WITH GIVEN SPECIFIC FLOW RATE

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ABSTRACT

Roots pumps are quite common in industry as well as in modern life. They can be applied in the lubrication system of the diesel engines, or in the automatic production lines etc. One of the most important specifications of the pump is flow rate, but this parameter itself depends on the speed and the volume of the pump chambers. The speed can be changed through transmission units, and the volume of the pump chambers depends on the profile form of the impellers. Finally, the profile form of the impeller depends on the geometrical parameters. That means in order to calculate the geometrical parameters of the pump, we must establish the relationship between the geometrical parameters of the impeller profile and the specific flow rate. Based on the theoretical results, the authors carried out programming software module to calculate the geometrical parameters to form profiles of the impellers, as well as the specific flow rate. The module is built by embedding AutoLISP language into AutoCad environment, in order to automatically design this type of pumps.

Keywords: Roots pump, lobe pump, hypocycloid, epicycloid.

1. INTRODUCTION

The Roots pump were introduced and presented in text books and other scientific publications in Vietnam as far as in 1970 [1], and recently in 2006 [2]. However, most of those works focusing only on setting working principle and designing profile of pump impeller in order to secure continuous engaged state between the lobes. Many industrial companies have manufactured the Roots pumps aiming for different applications [3 - 5]. Air compressors, air blower or lubrication systems of the diesel engines (Figure 1) can be listed as some of the most common industrial application of this kind of pump. In general, the Roots pump consists of gear-train with transmission ratio 1 : 1 driving the pump lobes to rotate inside the pump housing. In the Figure 2, we can see the suction and working chambers are generated between the lobes.

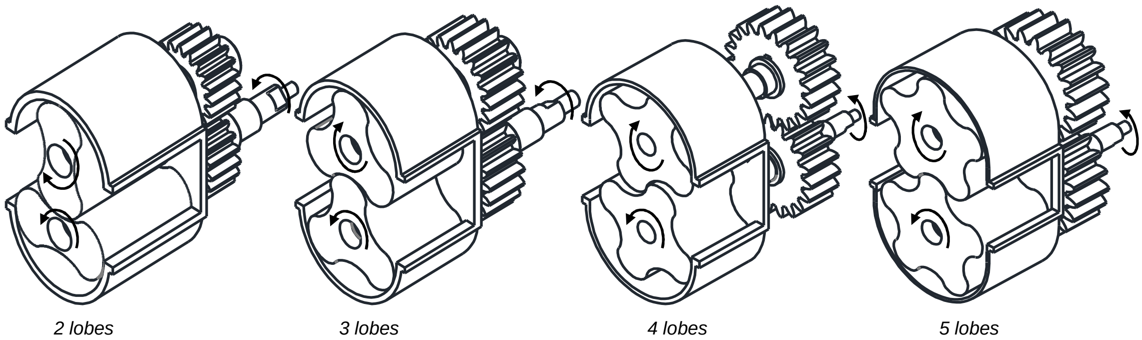

Figure 1. Some common Roots pumps.

Despite already mentioned results, a number of researches on the Roots pumps are still carrying out in order to improve design, manufacturing process, as well as working quality and performance of the pump. In [6, 7], the authors tried to get higher efficiency by improving contour of the lobes. In [8], by using screw vane as a replacement for classic gear vane, higher efficiency and lower noise of the three-lobe air compressor are achieved.

When studying about pump delivery, we can define a specific flow rate or volumetric displacement of the pump (Q_r) as a flow after each rotation of a driving shaft. Therefore, Q_r depends on cross sections *(S)* of the chambers and the impeller thickness *(d)* (Figure 2).

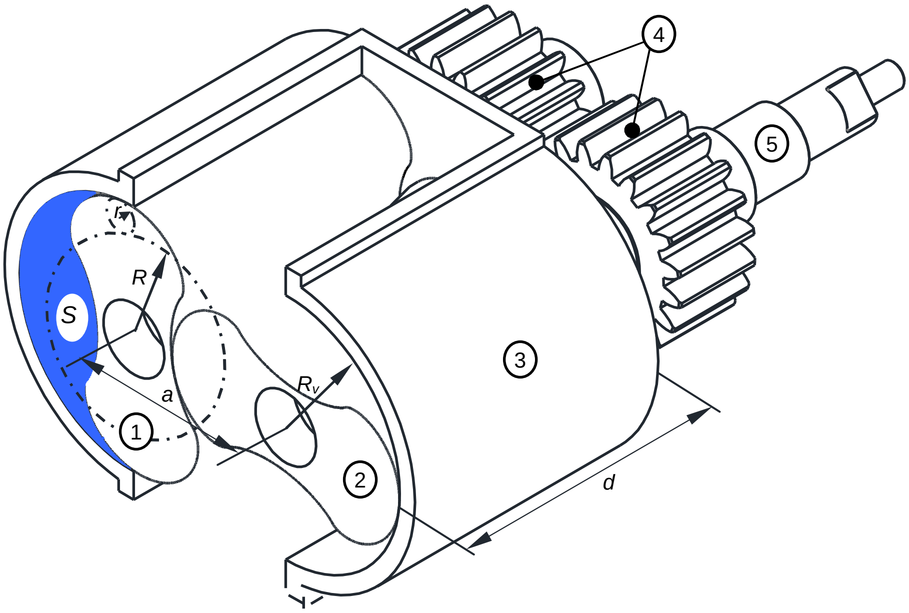

Figure 2. Geometrical parameters of the Roots pump:

S – cross section of the suction chamber; *d* – impeller thickness; *R* - radius of the rotor reference circle; *r* – radius of the lobe-profile generating circle; *a* – distance between shafts; R_v – radius of the pump housing.

According to [2], two radii *R*, *r* are the most important parameters for generating the lobe profile, distance *a* makes two lobe-rotors matched correctly following the gearing theory of the cycloidal gear-trains when the lobes are driven by the gear pair 4 with transmission ratio 1:1.

Therefore, when designing the Roots pump following given specific flow rate Q_r, it is necessary to establish formulas to calculate parameters a, r, R, R_v (Figure 2) in relation with Q_r, which is the content of the next part 2 of this work.

2. ESTABLISHING FORMULAS FOR DETERMINING PUMP GEOMETRICAL PARAMETERS FOLLOWING THE SPECIFIC FLOW RATE

2.1. Equation of the pump lobe profile

According to [1, 2, 4, 7 - 9], the lobe profile consists of two cycloidal curves, i.e. Epicycloid (*for addendum, given by equation 1*), and Hypocycloid (for dedendum, *given by equation 2*). In Figure 3, there are some types of the lobe, which are used in the industrial application. There are a number of publications proposing different methods for establishing epicycloidal and hypocycloidal equations, such as analytical method [9], instantaneous rotation center method [10], transfomation matrix method [11], or algebraic transfomation matrix method [12]. Therefore, we are not going too deeply into this topic.

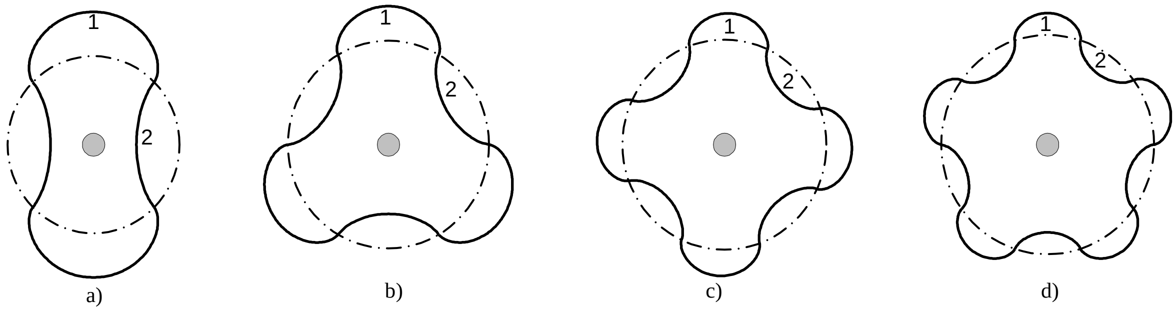

Figure 3. Common profiles of the lobe-rotor in the Roots pump:
a - 2 lobes; b - 3 lobes; c - 4 lobes; d - 5 lobes; 1 –Epycycloid; 2 –Hypocycloid.

Developed and transformed equation (4) in [11], we have:

Equation of the addendum curve (*Epycycloid*):

$$\begin{cases} x_e = -r\cos\left(\left(\dfrac{R+r}{r}\right)\varphi\right) + (R+r)\cos\varphi \\ y_e = -r\sin\left(\left(\dfrac{R+r}{r}\right)\varphi\right) + (R+r)\sin\varphi \end{cases} \tag{1}$$

Equation of the dedendum curve (*Hypocycloid*):

$$\begin{cases} x_h = r\cos\left(\left(\dfrac{R-r}{r}\right)\varphi\right) + (R-r)\cos\varphi \\ y_h = -r\sin\left(\left(\dfrac{R-r}{r}\right)\varphi\right) + (R-r)\sin\varphi \end{cases} \tag{2}$$

where: R – is the radius of the rotor reference circle, r – is the radius of the lobe-profile generating circle, $\varphi \in [0 \div 2\pi]$.

According to [2], condition for forming the lobe profile of R and r is:

$$\frac{R}{r} = 2z \quad \text{with } z \geq 2 \tag{3}$$

where: z – is the number of the pump lobes.

Appying (3) into (1 and 2), and rearranging, we have:

$$\begin{cases} x = \mp r\cos((2z \pm 1)\varphi) + r(2z \pm 1)\cos\varphi \\ y = -r\sin((2z \pm 1)\varphi) + r(2z \pm 1)\sin\varphi \end{cases} \tag{4}$$

in equation (4):

+ The signs ($\mp$ và $\pm$) are chosen following the upper sign in the Epicycloid case, and lower sign in the Hypocycloid case.

+ $\varphi \in \left[i\frac{\pi}{z}, (i+1)\frac{\pi}{z} \right]$ with $i = 0 \div (2z-1)$. i is even in the Epicycloid case, and i is odd in the Hypocycloid case.

Conclusion: from the equation (4), we can see that the equation of the lobe profile depends only on the number of the lobes z and the radius of the lobe-profile generating circle r. Therefore, in order to design kinematical pump dimension following given specific flow rate Q_r, we need to establish relation between z, r and Q_r as it is presented in the next section 2.2.

2.2. Specific flow rate Q_r calculation

The lobe number 1 and 2 are driven by the gear-train 4 with transmission ratio 1:1 (Figure 2) as it was described in the Introduction section. After one revolution of the driving shaft, the suction volume of the pump is V. Therefore, the specific flow rate Q_r can be calculated as:

$$Q_r = Vn = 2zSd \quad (\text{mm}^3/\text{rev.}) \tag{5}$$

where: S is the cross section of the pump chamber (Figure 4).

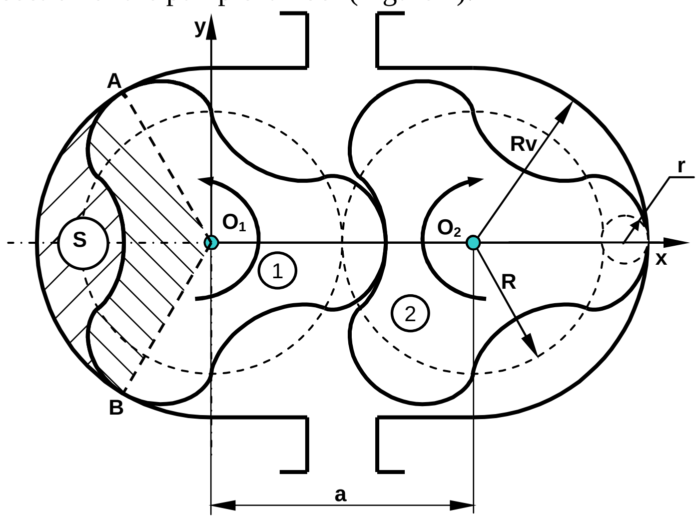

Figure 4. Cross section perpendicular to the pump shaft.

If: S_1 – is the area of the circle sector AO_1B (A, B are the instantaneuos contact points between the lobe profile and the pump housing).
S_2 – is the area generated by the lobe profiles at the moment when the contact points are A and B.

We have:

$$S = S_1 - S_2 = 4\frac{\pi}{z}r^2(z+1)^2 - 2\pi r^2\left(2z+\frac{1}{z}\right) = 2\pi r^2\left(4+\frac{1}{z}\right) \quad (6)$$

Applied (6) into (5), we have:

$$r = \sqrt{\frac{Q_r}{4\pi zd\left(4+\frac{1}{z}\right)}} \quad (7)$$

From [2], and also from Figure 2 and Figure 4, a and R_v can be further expressed as:

$$a = 4zr \quad (8)$$

$$R_v = 3zr \quad (9)$$

Finally, from equations (3, 7, 8, 9), we can calculate:

$$\begin{cases} r = \sqrt{\dfrac{Q_r}{4\pi zd\left(4+\frac{1}{z}\right)}} \\ a = 4zr \\ R_v = 3zr \\ R = 2zr \end{cases} \quad (10)$$

Applied the set of equation (10) in some of the most common industrial Roots pumps, the following Table 1 can be presented as below:

Table 1. Kinematic parameters of some common industrial Roots pumps.

STT	z	r (mm)	a (mm)	R (mm)	R_v (mm)
1	2	$\frac{1}{6\sqrt{2}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{2\sqrt{2}}{3}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{\sqrt{2}}{3}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{Q_r}{\pi d}}$
2	3	$\frac{1}{2\sqrt{13}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{6}{\sqrt{13}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{3}{\sqrt{13}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{27}{\sqrt{13}}\sqrt{\frac{Q_r}{\pi d}}$
3	4	$\frac{1}{2\sqrt{17}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{8}{\sqrt{17}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{4}{\sqrt{17}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{48}{\sqrt{17}}\sqrt{\frac{Q_r}{\pi d}}$
4	5	$\frac{1}{2\sqrt{21}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{10}{\sqrt{21}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{5}{\sqrt{21}}\sqrt{\frac{Q_r}{\pi d}}$	$\frac{25\sqrt{21}}{7}\sqrt{\frac{Q_r}{\pi d}}$

Conclusion: The equation (10) can provide the set of parameters determining the lobe profile and parameters of the pump geometrical dimension, which can be used for designing the pump with the specific flow rate Q_r and thickness d in the shaft asix.

3. SOFTWARE FOR SIMULATION OF CALCULATION AND DESIGN PROCESS

3.1. Software module

Based on the theoretical calculation in section 2, for automatic design of this type of the Roots pumps, the authors have built software **Cycloidpumps V1.1** in $C^{\#}$ with VisualStudio 2010. This software module, whose the interface is presented in Figure 5, can provide to design options. The first one is forward design (*in module ① given are the number of lobes and the radius of the lobe-profile generating circle r, the impeller thickness d, need to calculate the specific flow rate Q_r and other pump dimension*). The second one is reverse design (*in module ②, given are Q_r , d and z, need to determine other dimensional parameters of the pump*). After this calculating and simulating process, designers can use **Export to file CAD** ③ to automatically generate CAD drawings (*this function is written in Autolisp following an algorithm in Figure 6*). Basically, the function uses the results of the module **Cycloidpumps V1.1** for automatic design in CAD environment. Figure 5 shows steps of the designing process.

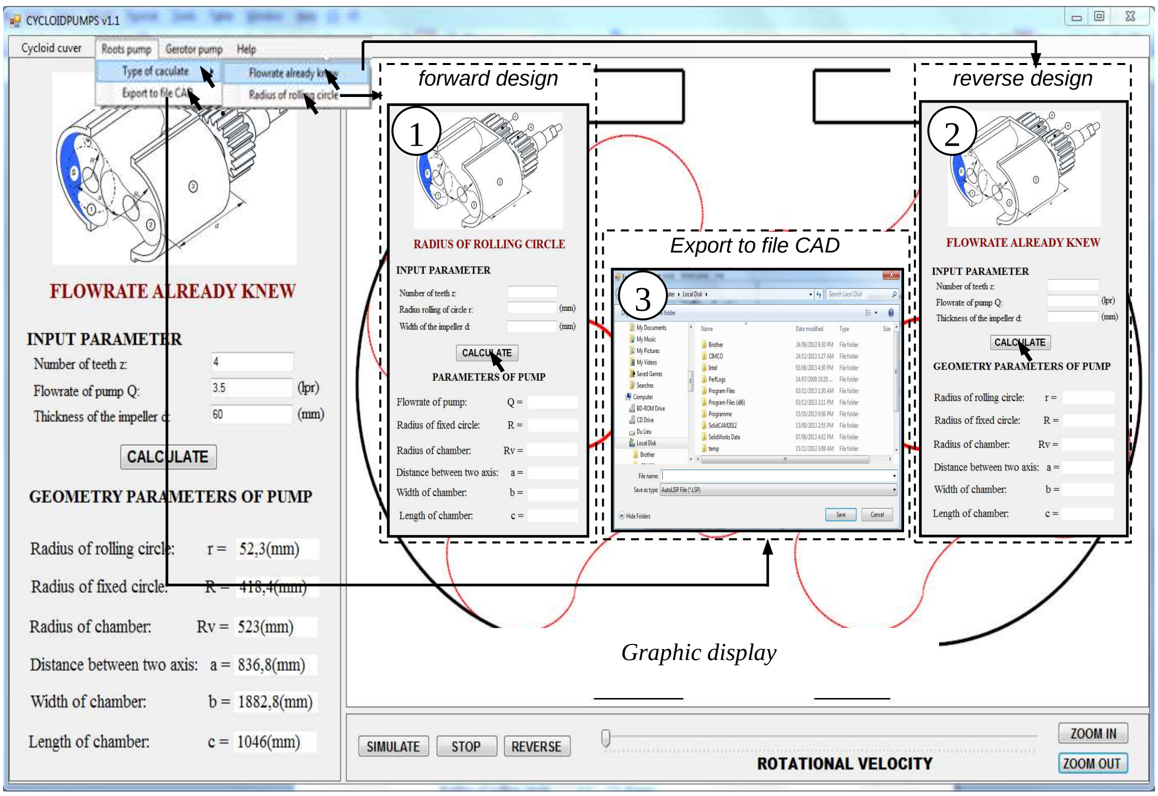

Figure 5. Interface of the calculation and simulation software.

3.2. Case study

a) Design

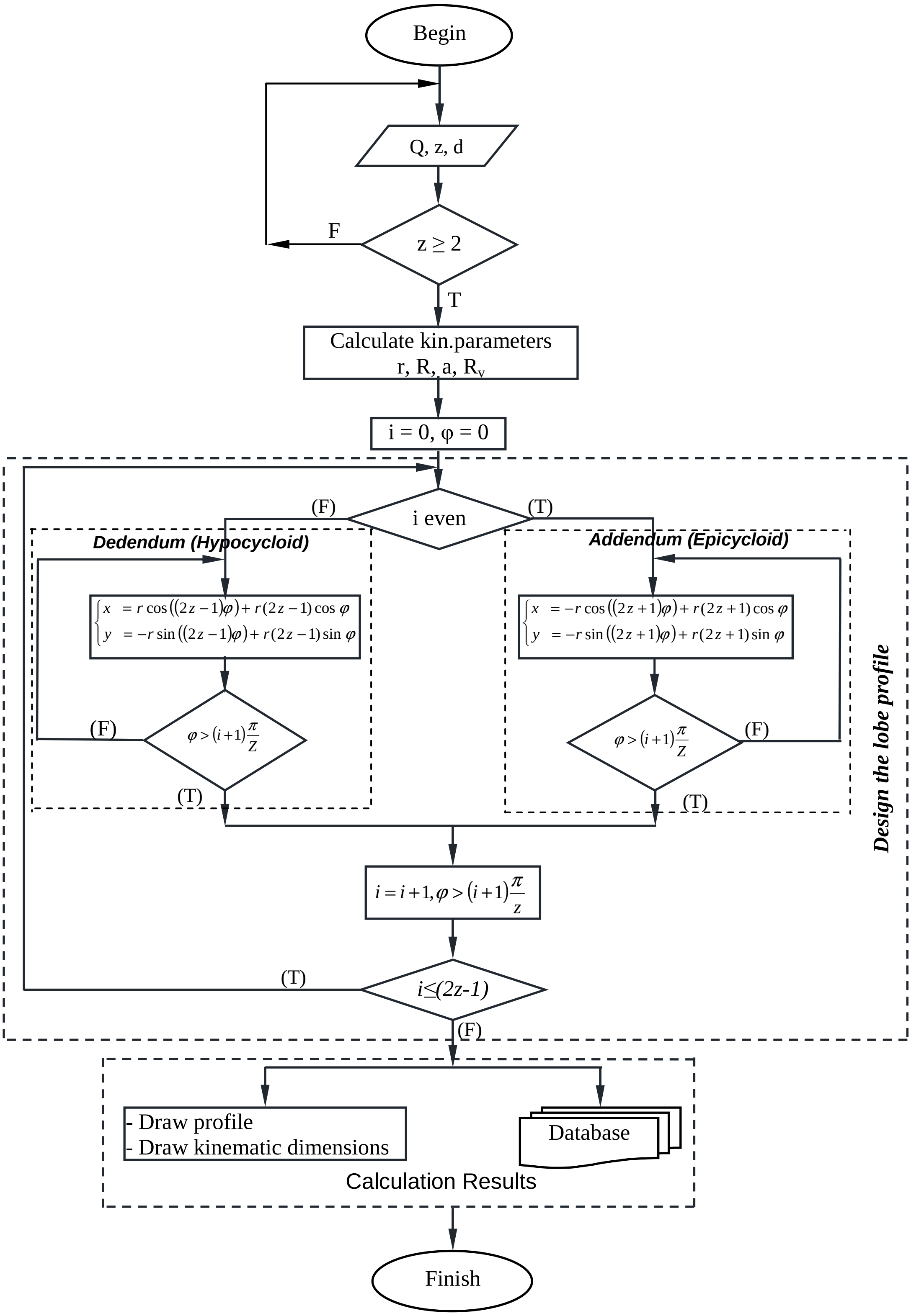

Figure 6. Algorithm flowchart for calculating kinematic parameters and design lobe profile.

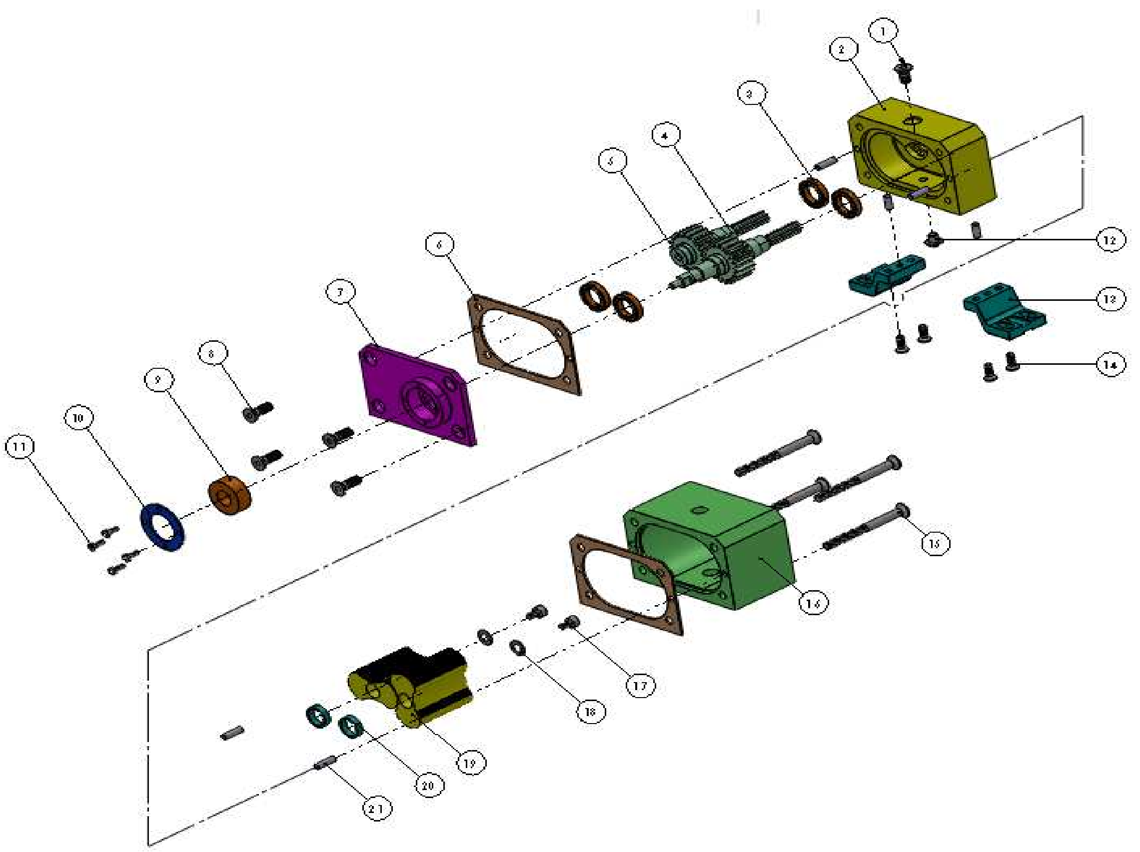

Figure 7. Root pump drawings in SolidWorks.
1 – Oil inlet, 2 – Housing of driving gear, 3 - Bearing, 4 – Driven gear, 5 – Driving gear, 6 – Amiang washer, 7 - Flange, 8 - Bolt M4x0.5x12, 9 – Oil seal, 10 – Seal washer, 11 - Bolt M2x0.25x6, 12 – Oil outlet M4x0.5x4, 13 - Frame, 14 - Bolt M3x0.5x8, 15 - Bolt M4x0.5x40, 16 - Housing, 17 - Bolt M3x0.5z5, 18 – Lock washer, 19 – Lobe rotor, 20 - Lining, 21 – Anchor pin Φ3x10.

a) b) c)

Figure 8. The prototype of the Roots pump.

Referring technical parameters of some common commercial industrial Roots pumps made by Viking Pump, Netzsch, Johnson Pump [3, 4, 5], the authors used **Cycloidpumps V1.1** for design the mini-pump prototype with following specification: Q_r = 0,002 (liter/rev.), pressure P = 1,2 bar. The Solidworks technical drawings are presented in Figure 7 *(calculated results of the shafts, driving gear, housing etc. are not shown in this paper).*

b) Prototype fabrication

Based on the formulas for designing geometrical dimension when Q_r is given, and on the result of designing step, the authors have built the prototype of this pump. Figure 8 shows the prototype. The main elements are shown in Figure 8a, the impeller lobes and driving unit are shown in Figure 8b, and the mini pump is presented in Figure 8c.

4. CONCLUSION

a) Conclusion

Equation (10) and the software module **Cycloidpumps V1.1** firstly provide us a way to determine geometrical parameters, and moreover, provide us a solution for designing Roots pump with given specific flow rate Q_r and impeller thickness d. Another practical application is re-designing after maintenance process, when the pump impeller has been worn and causing drop of pressure and flow. In that case, it is required to repair and restore the lobe profile.

The presented software module can be used in automatic design of the Roots pumps, which help to shorten designing time as well as to provide a database with necessary parameters. Depending on the requirement (*Repair of new design*), we can choose the forward or the inverse design process as it was presented in section 3.

b) Discussion

In the Roots pumps, because of the gap between the flanges of the lobes and of the housing, as well as the gap between the lobes while engaging, there is a loss of pressure and flow. This effect is caused by the manufacturing and assembling errors. Therefore, it requires high degree of precision in the fabrication and assemble process of the parts of the Roots pump. Other setbacks are noise and generated heat. The first one is caused by the geometrical and positional errors of the driving shafts, which leads to mis-engaging between the lobes. Consequently, the second setback occurs due to sliding and discontinuous matching between the impellers. To overcome these problems is the next goal of our researching group.

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TÓM TẮT

THIẾT LẬP BIỂU THỨC THIẾT KẾ KÍCH THƯỚC HÌNH HỌC CỦA BƠM ROOT THEO LƯU LƯỢNG RIÊNG

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Bơm Root đang được ứng dụng khá phổ biến trong công nghiệp cũng như trong cuộc sống hiện đại, trong đó phải kể đến hệ thống bôi trơn của động cơ điêzen hay trong các dây chuyền sản xuất tự động. Một trong hai thông số kĩ thuật quan trọng nhất của bơm là lưu lượng, trong khi đó lưu lượng của bơm lại phụ thuộc vào số vòng quay và thể tích của các khoang bơm. Trong hai thông số trên thì số vòng quay có thể thay đổi thông qua các bộ truyền động, còn thể tích của các khoang bơm thì phụ thuộc vào biên dạng hình thành cánh bơm. Trong khi đó biên dạng cánh bơm lại phụ thuộc vào các thông số hình học, điều đó có nghĩa để tính toán các thông số hình học của bơm theo lưu lượng riêng ta cần phải thiết lập mối quan hệ giữa các thông số hình học hình thành biên dạng cánh bơm với lưu lượng riêng đây chính là nội dung trình bày của bài báo này. Trên cơ sở lí thuyết nghiên cứu, các tác giả tiến hành viết mô đun phần mềm tính toán kích thước động học hình thành biên dạng bơm theo lưu lượng riêng cho trước bằng ngôn ngữ AutoLisp chạy trên môi trường AutoCad, để từ đó tự động hóa thiết kế các loại bơm này.

Từ khóa: bơm Root, bơm lobe, hypocycloid, epicycloid.