A COMBINE WAVELET SEMIPARAMETERIC REGRESSION-SINGULAR IDENTIFICATION DENOISING METHOD

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ABSTRACT

Wavelet theory-based algorithms have been successfully developed in the signal processing with the use of compactly supported functions in time domain for multiresolution analysis [1 - 3]. However, like other linear filtering methods, when applying filtering algorithms in order to remove noise from signals, discontinuous part of the signal cannot be recovered [4 - 6]. In present paper, a class of signal containing singular points is considered to propose a combine process for overcomming the mentioned drawback. The proposed process consists of three steps: In the first one, the compact support property of wavelet transform is employed to identify singular points. In the next step, the discontinuous part of signal is characterized by a method with regression converging to singular points from both sides by the use of generalized inverse estimator. In the last one, a combine Wavelet regression (non-parametric) with local regression is performed for recovering the signal with optimally estimated discontinuities. It is expected that the proposed algorithm for recovering singular parts of the signal would take part to improve overall SNR of the process.

Keywords: *Wavelet regression, denoising, singular identification*

1. INTRODUCTION

Denoising has become an essential process in signal analysis that removes the noise from corrupted signal. Let us state the problem of denoising through out this paper in which an original deterministic piecewise signal $x(n)$ presenting sharp discontinuities has been corrupted by additive noise as $y(n) = x(n) + e(n)$, where distributed zero mean Gaussian with variance σ^2, $e(n)$ are independent. The denoising algorithms task is to estimate $\hat{x}$ on minimizing a defined risk function, usually mean square errors $E\left\{\|x-\hat{x}\|^2\right\}$, so that one can recover the original smooth signal while the discontinuities are being preseved.

To the above mentioned problem, there exists many Fourier transform based methods and wavelet-based methods seems to be useful and powerful tool (wavelet transform has capability of analyzing both transient and stationary behaviors of a signal; are capable of grasping subtle changes and discontinuities in signals). Traditional wavelet based denoising algorithm proposed by Donoho and Johnstone [7] basically shrinks the wavelet coefficients on adopting an universal threshold with dimension *N*, $\lambda = \sigma\sqrt{2\ln N}$ and adopting also hard-soft shrink wavelet (detail) coefficients [8]:

$$\begin{cases} T_\lambda^{hard}(x) = x.1(|x| > \lambda) \\ T_\lambda^{soft}(x) = \mathrm{sgn}(x).(|x| - \lambda)_+ \end{cases} \quad (1)$$

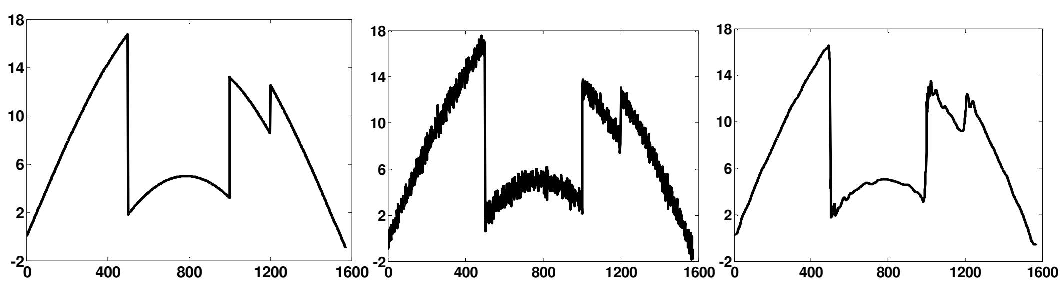
where $1(x \in A)$ is the identity function of *A*, and where x_+ is x for $x > 0$ and zero elsewhere.

The algorithm proposed by Donoho and Johnstone [7] consists of the following steps:

1. *Decomposition:* Apply wavelet transform with *J* levels to noisy signal to get wavelet coefficients $d_i^j = \langle y, \psi_i^j \rangle$ and scaling function coefficients $c_i^J = \langle y, \varphi_i^J \rangle$.
2. *Shrinkage of detail coefficients*: Shrink wavelet (detail) coefficients at the *j* finest scales to get new detail coefficients $T_\lambda(d_i^j) = \eta_\lambda(\langle y, \psi_i^j \rangle)$ with a threshold parameter λ.
3. *Reconstruction:* Reconstruct the denoised version $\hat{x}$ of x from the shrunken wavelet coefficients for an orthogonal basis of wavelet and scaling functions $\left\{\{\psi_i^j\}_{(j,i\in Z)}, \{\varphi_i^J\}_{(i\in Z)}\right\}$:

$$\hat{x} = \sum_{i\in Z} \langle y, \varphi_i^J \rangle \varphi_i^J + \sum_{j=1}^{J} \sum_{i\in Z} T_\lambda\left(\langle y, \psi_i^j \rangle\right) \psi_i^j . \quad (2)$$

This algorithm has been widely used (simplicity, performance closed to an ideal coefficient selection and attenuation for nonlinear thresholding estimator in orthogonal bases [1, 9]). However, the authors' opinions are that wavelet thresholding denoise process is still a regularization one as measurements for parameter estimating are not sufficient and that can not preserve discontinuities in signals showing in figure 1 below.



(a) (b) (c)

Figure 1. Wavelet denoising of heavisine

(a): Original signal; (b): Noisy signal, and (c): Denoised version by wavelet shrink

A class of signal containing singular points constitutes an important part in real, particularly in biomedical signals, images engineering ... So, for the denoise of containing singular points signal purpose, in this paper, a process is proposed to overcome the above mentioned demerits consisting of three main steps: In the first one, the compact support property of wavelet transform is used to identify singular points. In the next step, discontinuous part of the signal is characterized by novel method proposed hereby on adopting a regression from both sides, converging to the singular points with the use of generalized inverse estimator concept. In the last one, a combine of wavelet regression (non-parametric) with local regression is made in recovering the signal. The proposed process is justified by a simulation example.

The paper is organised as follows. In the next paragraph, a brief on preliminaries is made consisting also of continuous and discrete wavelet transform, discontinuity analysis. In paragraph III, the proposed algorithms is described. Different results obtained from experiments is reported in paragraph IV and in paragraph V is for different discussions and direction for further researches.

2. PRELIMINARIES

2.1. A brief on continuous and discrete wavelet transform

Wavelet transform is a linear transform with varying time and frequency resolutions and non-fixed *mother waveforms* [3]. That permits one to choose basic functions in the transformation to separate unintended from intended components in the analytic signal purpose [3, 11].

A function $\psi \in L^1(\Re) \cap L^\infty(\Re)$ with $\int_{-\infty}^{\infty} \psi(t)dt = 0$ is called a *wavelet*. For every $s \in L^p(\Re), 1 \le p \le \infty$, a called *mother wavelet* ψ defines *Continuous wavelet transform with the dilation a* and *translation* τ as:

$$W_\psi s(a,\tau) = \int_{-\infty}^{\infty} s(t)\frac{1}{\sqrt{a}}\psi^*\left(\frac{t-\tau}{a}\right)dt = \left\langle s, \psi_{a,\tau}\right\rangle_{L^2},\ \psi_{a,\tau} = \tfrac{1}{\sqrt{a}}\psi\left(\tfrac{t-\tau}{a}\right), \text{ for all } a,\tau \in \Re_+ \times \Re \qquad (3)$$

Set $\{\psi_{a,\tau}\}$ forms an orthonormal basis and can generate any function in $L^2(\Re)$ provided $\psi(t)$ satisfying the admissibility condition [5]:

$$\int_{-\infty}^{\infty} \left|\Psi(\omega)\right|^2 \left|\omega\right|^{-1} d\omega < \infty \qquad (4)$$

To analyze discrete signal, the scale and shift parameters are to be discredited with integer numbers n and m ($a = 2^m$ and $\tau = n.2^m$). In $L^2(\Re)$ the set of dilated and shifted versions of wavelets forms a basis of *Discrete Wavelet Transform*:

$$\left\{\psi_{m,n}(t)\right\}_{m,n\in Z} = \left\{2^{-m/2}\psi(2^{-m}t - n)\right\}. \qquad (5)$$

A finite energy signal $x(t)$ can always be uniquely, stably decomposed in terms of discrete wavelet transform $\{\psi_{m,n}(t)\}_{m,n\in Z}$ and wavelet coefficients $d_{\mathrm{m,n}}$ [6]:

$$x(t)=\sum_{m-\infty}^{\infty}\sum_{n=-\infty}^{\infty}d_{m,n}\psi_{m,n}(t);\ d_{m,n}=\left\langle x(t),\psi_{m,n}(t)\right\rangle \ . \tag{6}$$

In general, an exact representation of $x(t)$ requires a discrete wavelet family $\{\psi_{m,n}(t)\}$ of infinite number of functions. However, from the practical (approximation) point of view, a low-pass-natured complementary scaling function $\varphi(t)$ is employed playing a key role in multi-resolution analysis theory. Wavelet function has zero average, each $d_{m,n}$ measures a local variation of $x(t)$ at resolution 2^m and the partial sum represents an approximation of $x(t)$ at resolution 2^{J+1} [11]:

$$x_{J+1}(t)=\sum_{m=J+1}^{\infty}\sum_{n=-\infty}^{\infty}d_{m,n}\psi_{m,n}(t) \tag{7}$$

Approximation function $x_{J+1}(t)$ can be expressed in terms of shifted versions of a function called the scaling function $\varphi_{J,n}(t)=\frac{1}{2^{J/2}}\varphi(t/2^J-n)$ as:

$$x_{J+1}(t)=\sum_{n=-\infty}^{\infty}y_{J,n}\varphi_{J,n}(t) \tag{8}$$

Hence, an any function in $L^2(\Re)$ can be completely represented by using J-finite resolution of wavelet and scaling function $y_{J,n}=\left\langle x(t),\varphi_{J,n}(t)\right\rangle$ (measure of the local regularity of $x(t)$ at scale 2^J) as:

$$x(t)=\sum_{n=-\infty}^{\infty}y_{J,n}\varphi_{J,n}(t)+\sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty}d_{m,n}\psi_{m,n}(t) \tag{9}$$

Therefore, $\sum_{n=-\infty}^{\infty}y_{J,n}\varphi_{J,n}(t)$ represents a coarse version of $x(t)$ as opposed to presented by $\sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty}d_{m,n}\psi_{m,n}(t)$.

Wavelet function and scaling one are related but relationship between them does not reduce to the expansion in (9) which is used for describing the multi-resolution structure of the wavelet transform [7].

2.2. Lipschitz exponents

Irregular, singular structures usually carry important informations of signals, hence efficient charaterization of these structures is crucial. However, wavelet transform is capability of characterizing the regularity of functions and Lipschitz exponents can be used for characterizing discontinuous functions [3]. With respect to the Lipschitz exponents, some important informations are listed as:

1. Function $f(t)$ is called Lipschitz degree $\alpha \geq 0$ at v if there exists a constant $K > 0$ and polynomial $p_v(t)$ degree $m = \lfloor \alpha \rfloor$ as to $\forall t \in R, \left| f(t) - p_v(t) \right| \leq K \mid t - v \mid^{\alpha}$.

2. Function $f(t)$ is uniformly Lipschitz α over $[a,b]$ if it satisfies the above mentioned inequality with every $v \in [a,b]$ for a constant K independent with v.

3. Lipschitz regularity of $f(t)$ at v or over $[a,b]$ is superior bound (supremum) of α in order to $f(t)$ be Lipschitz.

4. A function is singular at v if it is not Lipschitz 1 at v.

3. PROPOSED ALGORITHM

3.1. Singularity analysis and identification

For a given function $x(t)$, the Taylor expansion of function $x(t)$ around vicinity $t = t_0$ is:

$$x(t) = x(t_0) + a_1(t - t_0) + a_2(t - t_0)^2 + ... + a_n(t - t_0)^n + ... \tag{10}$$

Denote $\tau = t - t_0$, then one has:

$$x(t) = x(t_0) + a_1\tau + a_2\tau^2 + ... + a_n\tau^n + R_n(\tau^{n+1}) = P_n(t) + R_n(\tau^{n+1}) \tag{11}$$

where $P_n(t)$ is a polynomial degree of n and $R_n = \frac{\tau^{n+1}}{(n+1)!} x^{(n+1)} t^*$, $t^* \in [t_0, t]$ is the remainder.

When the vicinity of t is small enough, one can find a non-negative constant A such that:

$$\left| x(t_0 + \tau) - P_n(t_0 + \tau) \right| \square R_n(\tau^{n+1}) \leq A\left|\tau\right|^{\alpha},\ n < \alpha \leq n+1 \tag{12}$$

Lipschitz regularity of $x(t)$ at t_0 is defined as the superior bound of all values α. For the wavelet $\psi(t)$ with at least n vanishing moments (wavelet orthogonal to polynominals up to degree of n), one has:

$$\int_{-\infty}^{+\infty} t^m \varphi(t)\, dt = 0 \qquad m \in [0, n] \tag{13}$$

Wavelet transform is then expressed as the n order derivative of the signal $x(t)$ smoothed by a smoothing function $\theta(x)$ of the form [9]:

$$W_{\psi}x(s,t) = x(t) * \psi_s(x) = s^n \frac{d^n}{dt^n}\left(x * \bar{\theta}_s \right) t,\ \ \bar{\theta}_s(t) = \frac{1}{s}\theta(t/s) \tag{14}$$

Therefore, one can estimate the rate of change in the signal by choosing suitable wavelet function at a scale. With a wavelet having n vanished moments, there exists a constant A such that for a point in vicinity t_0 and at a scale s [10, 11]

$$\left|W_\psi x(t,s)\right| \le A\left(s^\alpha + \left|t-t_0\right|^\alpha\right) \tag{15}$$

If $\left|W_\varphi x(t_0,s_0)\right|$ is a local maxima at $t = t_0$, then modulus maxima at the point (t_0,s_0) an be used to estimate the singularity. Consider a signal formed by a single Dirac at t_0, $x(t) = a\delta(t-t_0)$, J level wavelet decomposition of this signal as:

$$x(t) = \sum_{n=-\infty}^{\infty} y_{J,n}\varphi_{J,n}(t) + \sum_{m=-\infty}^{J}\sum_{n=-\infty}^{\infty} d_{m,n}\psi_{m,n}(t) \tag{16}$$

By moving from discontinuous- to discrete-time signal $x[n]$, $n = 0,\ldots,N-1$ with Haar wavelets, one has:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l\varphi_{J,l}[n] + \sum_{j=1}^{J}\sum_{l=0}^{N/2^J-1} d_{j,l}\psi_{j,l}[n] \tag{17}$$

Haar wavelet has one vanishing moment and finite support. Hence, only a limited number of wavelets which overlap the location t_0 is influenced by this Dirac. Set of points such that t_0 is included in support of wavelet is known as *cone of influence* [3], illustrating in Fig 3. The only non-zero wavelet coefficients of (11) are the coefficients in this cone of influence. Thus (17) becomes:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l\varphi_{J,l}[n] + \sum_{j=1}^{J} d_{j,k_j}\psi_{j,k_j}[n], \text{ where } k_j = \left\lfloor k/2^j \right\rfloor \tag{18}$$

Therefore, for a signal with only one Dirac at position k, a scale-space vector $f_k[n]$ is obtained by gathering all the wavelet coefficients together in the cone of influence of k then imposing its norm equal to 1:

$$f_k[n] = \sum_{j=1}^{J} c_{j,k_j}\psi_{j,k_j}[n], \text{ where } c_{j,k_j} = d_{j,k_j} \Big/ \sqrt{\sum\nolimits_{j=1}^{J} d^2_{j,k_j}} \tag{19}$$

One can express a step discontinuity at k signal in terms of the scaling functions and $f_k[n]$ as:

$$x[n] = \sum_{l=0}^{N/2^J-1} y_l\varphi_{J,l}[n] + \alpha f_k[n],\ \alpha = \left\langle x, f_k[n]\right\rangle = \sum\nolimits_{j=1}^{J} c_{j,k_j} d_{j,k_j} \tag{20}$$

For a signal with one discontinuity at location t, $p(t) = a_0^{(0)} 1_{[0,t_1[}(t) + a_1^{(0)} 1_{[t_1,T[}(t)$, wavelet coefficient at m,n is $\left\langle p(t), \psi_{m,n}(t)\right\rangle$. If wavelet has k degrees of freedom and fast decay, it can be rewritten as the k^{th} order of derivative of a function θ (also fast decay) [6]:

$$\psi(t)=(-1)^k\frac{d^k\theta(t)}{dt^k} \text{ and } \psi_{m,n}(t)=(-1)^k\frac{2^{km}d^k\theta_{m,n}(t)}{dt^k}, \text{ where } \theta_{m,n}(t)=\frac{1}{2^{m/2}}\theta\left(2^{-m}t-n\right) \quad (21)$$

Since the k^{th} derivative of a function is well defined distribution and $\left\langle p(t),\frac{d\theta(t)}{dt}\right\rangle=-\left\langle\frac{dp(t)}{dt},\theta(t)\right\rangle$, then:

$$\left\langle p(t),\psi_{m,n}(t)\right\rangle=-2^m\int_{-\infty}^{\infty}\frac{dp(t)}{dt}\theta_{m,n}(t)dt=-2^m\int_{-\infty}^{\infty}(a_1^{(0)}-a_0^{(0)})\delta(t-t_1)\theta_{m,n}(t)dt \quad (22)$$

From (22), it is seen that the wavelet coefficients depend on the difference of $(a_1^{(0)}-a_0^{(0)})$ which is argument of the discontinuity.

Identification algorithm: Procedure of identify the discontinuities is described as follows:

1. Compute discrete wavelet transform coefficients within a period signal using Haar wavelet as:

   Set $c_i^0 = x[i]$; $i = 1 \ldots N-1$;

   Compute scaling function coefficients by $c_i^j=\frac{c_{2i}^{j-1}+c_{2i+1}^{j-1}}{\sqrt{2}}$;

   Compute wavelet function coefficients by $d_i^j=\frac{c_{2i}^{j-1}-c_{2i+1}^{j-1}}{\sqrt{2}}$.

2. Set universal threshold equals to $\lambda=\sigma\sqrt{2\ln N}$ due to noise with variance σ.

3. Compute $\sum_j d_{k_j}^j$ where $k_j=\lfloor k/2^j\rfloor$;

   Compare to threshold, if it is greater than threshold, there exits a discontinuity at k.

3.2. Wavelet semiparametric regression combined with generalized inverse estimator approach

As wavelet transform is linear and orthogonal one. Then it can be expressed by an isometry transformation **W** (matrix with condition $\mathbf{W}^{-1}=\mathbf{W}^{\mathrm{T}}$). Denote $\boldsymbol{y}$ for input vector of dimension n where $n=2^J$, one has:

$$\mathbf{Y}=\mathbf{W}\mathbf{y},\ \mathbf{W}^{-1}=\mathbf{W}^{\mathrm{T}} \quad (23)$$

Practically, wavelet transform are carried out in fast algorithms by quadrature mirror filters and a wavelet decomposition can be written in terms of quadrature mirror filters G (high) and H (low) as:

$$\mathbf{Y}=\left(H^l\mathbf{y},GH^{l-1}\mathbf{y},\ldots,GH^2\mathbf{y},GH\mathbf{y},G\mathbf{y}\right) \quad (24)$$

where $l = 1 \ldots J = \log_2 w$, with $\left(Ha\right)_K = \sum_{m\in Z} h_{m-2k} am$ and $\left(Ga\right)_K = \sum_{m\in Z} g_{m-2k} am, k \in Z$, g and h are high and low pass filters.

Wavelet semiparametric regression takes the form:

$$\hat{y}_{wsr}\left(t\right) = \sum_{r=1}^{d} \boldsymbol{\beta} t^2 + \mathbf{W}^{-1} T_\lambda \left(\mathbf{W}\mathbf{y}\right) \tag{25}$$

Second part of the above expression is wavelet regression, getting of the form in practice as:

$$\mathbf{W}^{-1} T_\lambda \left(\mathbf{W}\mathbf{y}\right) = \sum_{l=1}^{e^J - 1} \hat{c}_{0,l} \varphi_l \left(t\right) + \sum_{j=0}^{J-1} \sum_{l=0}^{2^j - 1} \hat{d}_{j,l}^{\,s} \psi_{j,l}\left(t\right) \tag{26}$$

Local regression model expressed by the first term of (26) can be used to characterize singularity part of the signal. By least square errors criterion to estimate the parametric vector $\boldsymbol{\beta}$, one gets:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^T \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{y} \tag{27}$$

By fitting locally-weighted $\omega_i\left(t, t_i; \lambda_i\right) = K\left(\dfrac{t - t_i}{\lambda_i}\right)$ of local bandwidth λ_i at each point t_i with Kernel function K for local polynomial smoother, the model to be minimized becomes:

$$\sum_{i=1}^{n} \left(Y_i - \beta x_i\right)^2 K\left(\frac{t - t_i}{\lambda_i}\right) \tag{28}$$

One has

$$\left(\mathbf{X}^T \boldsymbol{\Omega} \mathbf{X}\right)\boldsymbol{\beta} = \mathbf{X}^T \boldsymbol{\Omega} \mathbf{Y}, \text{ where } \boldsymbol{\Omega} = diag\left(\omega_1, \omega_2, \ldots, \omega_n\right) \tag{29}$$

Mean square errors of $\hat{\boldsymbol{\beta}}$ in terms of variance σ of Gaussion noise and of ranked eigenvalues λ_i of $\mathbf{X}^T \boldsymbol{\Omega} \mathbf{X}$ · ($\lambda_1 > \lambda_2 > \ldots > \lambda_p > 0$) becomes:

$$MSE(\hat{\beta}) = \sigma^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \tag{30}$$

If either $\lambda_p << 1$ or rank of $\mathbf{X}^T \boldsymbol{\Omega} \mathbf{X}$ less than the degree of that product matrix, an ill-conditioned problem arises then. In such a case, generalized inverse estimator is used in an iteration procedure:

$$\hat{\boldsymbol{\beta}}^{(n)} = \left(\mathbf{I} - h\mathbf{G}\mathbf{X}\right)\hat{\boldsymbol{\beta}}^{(n-1)} + h\mathbf{G}\mathbf{y} \tag{31}$$

where,

$$0 < h < \frac{2}{\delta_{\max}};\ \delta i = \sum_{j=1}^{a} c_j \lambda_i^j > 0,\ i = 1, \ldots, p;\ \mathbf{G} = \left[c_1 \mathbf{I}_\beta + c_2 \mathbf{X}^T \boldsymbol{\Omega} \mathbf{X} + \ldots + c_q \left(\mathbf{X}^T \boldsymbol{\Omega} \mathbf{X}\right)^{q-1}\right] \mathbf{X}^T.$$

The iterative process is started from:

$$\hat{\boldsymbol{\beta}}^{(0)} = h\mathbf{G}\mathbf{y} \tag{32}$$

By **UDV** decomposition (**U**, **V** are of (*nxn*) orthogonal matrices, $\mathbf{D}^* = \left[\boldsymbol{\Lambda}^{1/2}, 0\right]$ with $\boldsymbol{\Lambda}^{1/2} = diag\left\{\lambda_i^{1/2}\right\}_{i=1}^{p}$) then $\mathbf{X} = \mathbf{UDV}^*$ and canonical model (27) becomes:

$$\mathbf{Y} = \mathbf{Zd} + \mathbf{e},\ \text{with } \mathbf{Z} = \mathbf{UD} = \mathbf{XV} \text{ and } a = \mathbf{V}^*\boldsymbol{\beta} \tag{33}$$

$$\hat{\alpha} = \left(\mathbf{Z}^*\mathbf{Z}\right)^{-1}\mathbf{Z}^*\mathbf{y} = \boldsymbol{\Lambda}^{-1}\mathbf{Z}^*\mathbf{y} \tag{34}$$

In present case, $(\mathbf{Z}^*\mathbf{Z})^{-1}$ is known as a *group or Drazin inverse* if the rank of this product matrix is equal to the order of that product matrix and $(\mathbf{Z}^*\mathbf{Z})^{-1}\mathbf{Z}^*$ is known as *left generalised inverse* of **Z**. In the case, where $(\mathbf{Z}^*\mathbf{Z})^{-1}\mathbf{Z}^*$ is of full rank, then the left generalised inverse (uniquely determined) becomes a *pseudo inverse* or *Moore Penrose inverse* denoted by $\mathbf{Z}^+$. Generally, $(\mathbf{Z}^*\mathbf{Z})$ is ranked deficit (rank of this product matrix is less than the order of the said), Drazin inverse does not exist. In this case, the concept of *generalized Drazin inverse* is required. Thus:

$$\mathbf{G} = \mathbf{V}\left[c_1\mathbf{I}_p + c_2\boldsymbol{\Lambda} + c_3\boldsymbol{\Lambda}^2 + \ldots + c_q\boldsymbol{\Lambda}^{(q-1)}\right]\mathbf{D}^*\mathbf{U}^* \tag{35}$$

$$\hat{\alpha}^{(n)} = \mathbf{V}^*\hat{\boldsymbol{\beta}}^{(n)} = \left(\mathbf{I} - h\mathbf{H}\boldsymbol{\Lambda}\right)\hat{\alpha}^{(n-1)} + h\mathbf{H}\boldsymbol{\Lambda}\hat{\alpha},\ \text{with } \mathbf{H} = \left[c_1\mathbf{I}_p + c_2\boldsymbol{\Lambda} + c_3\boldsymbol{\Lambda}^2 + \ldots + c_q\boldsymbol{\Lambda}^{(q-1)}\right] \tag{36}$$

The above proposed method is verified by simulating different typical wave forms describing here by.

4. SIMULATIONS

Two results on assessing the performance of the proposed algorithm are reported in this paragraph. The first one is for comparing the present method to some other ones in term of signal to noise ratio (SNR) and of recovered mutations. In second experiment, two typical signals considered by other authors (*heavisine* with two discontinuities, *blocks* with more dicontinuities) are retaken hereby for the comparison purpose.

Signals used in simulation are of 15 and 10 mutation amplitudes at the time of 1/3 and 2/3 cycle. Signal including non-linear part contains AWGN with S/N = 18 dB, with length of 1024. The SNR depends on the size (length *N*) of the signal [11]. So, length N of the signal must be large enough to ensure the experiment to be properly carried out.

Table 1. Denoising of signal with two break points in Fig. 2

N (length of signal)	**256**	**512**	**1024**	**2048**
Hard thresholding with db4 Wavelet	20.2 dB	20.7 dB	21.6 dB	21.8 dB
Soft thresholding with db4 Wavelet	19.7 dB	20.7 dB	21.1 dB	21.7 dB
Proposed method	20.6 dB	21.9 dB	22.0 dB	22.0 dB

Table 1 shows numerical results of different denoising methods according to different length of signals with two break points. It is shown that at each length N of the signals, the presently proposed denoising method gives a better result but not so much with respect to that obtained by the others [8 - 11].

Figure 2 illustrate the results of denoising by the different methods and variable length of the signal of sine form with two discontinuities (a), signal corrupted by noise (b), signal denoised by soft thresholding (c), signal obtained by hard thresholding (d) and signal obtained by proposed method of denoising.

By soft thresholding method with db4 wavelet, SNR of recovered signal is 19.7dB with lenghth of 256; 20.7dB for 512 and 21.7dB for 2048, respectively. As the original signal contains the discontinuities, the hard thresholding methods gives better results, respectively 20.2, 20.7 and 21.8dB for different values of signal length *N*. It is also found out that the proposed methods didn't improve SNR so much but in Figure 2, the mutations recovery and evaluation ability has improved evidently.

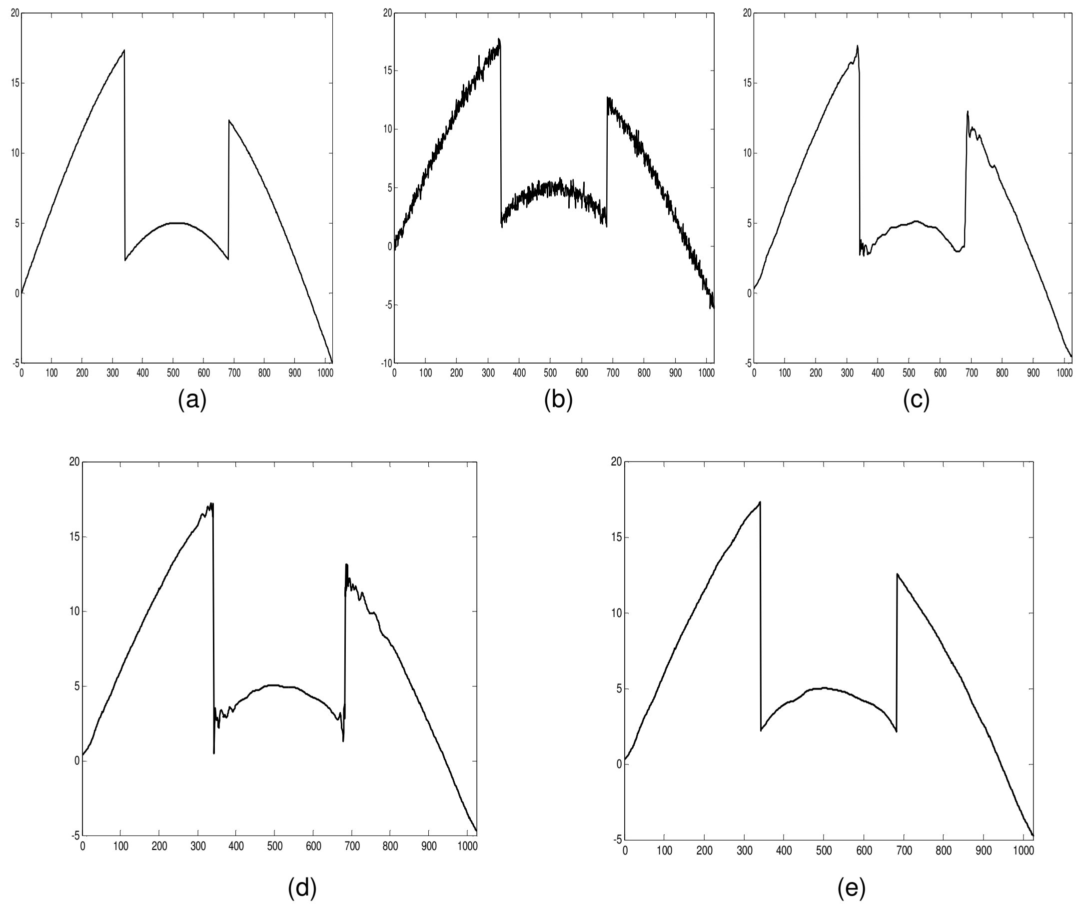
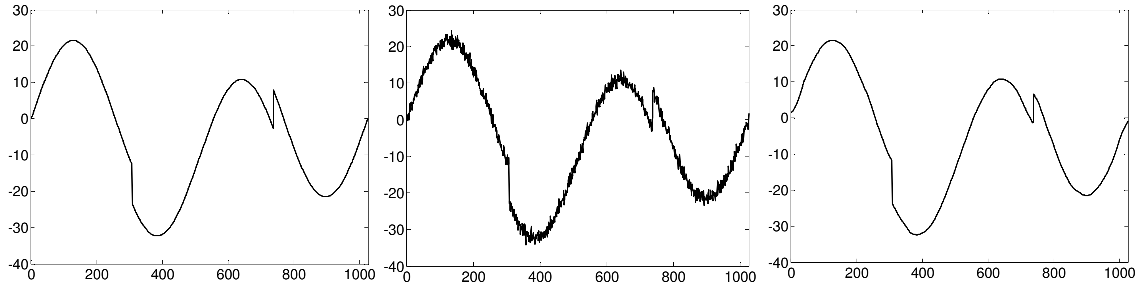


Figure 2. SNR results for denoising

(a): Original signal (sine with two discontinuities); (b):Noisy signal (18 dB);

(c):Soft thresholding (21.6 dB); (d): Hard thresholding (21.1 dB); (e): Proposed method (22.0 dB)

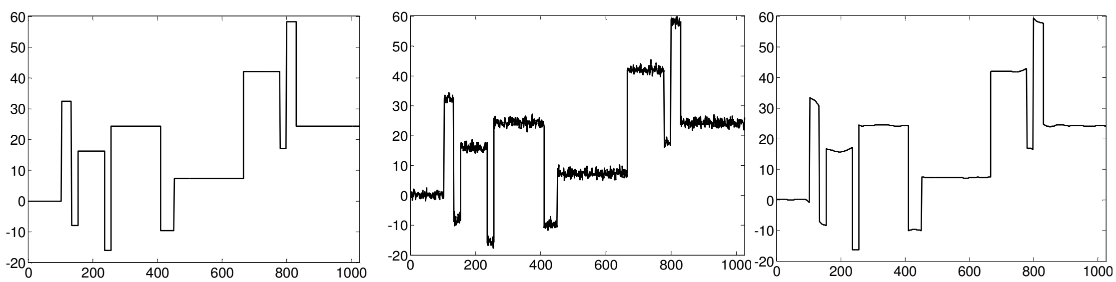
In Fig. 3 and Fig. 4 show the graphs resulting SNR for denoising of heavisine and of blocks signals obtained by presently proposed method. In which, original signal (a) is either heavisine in Fig. 3 or blocks signal in Fig.4, corresponding corrupted noisy signals (b) for both figures and denoised signals (c) respectively also for both. It is found that mutations recovery and evaluation ability has proved evidently.



(a) (b) (c)

Figure 3. SNR results for denoising of heavisine signal.

(a): Original signal; (b): Noisy signal; (c): Denoised signal



(a) (b) (c)

Figure 4. SNR results for denoising of blocks signal.

(a): Original signal; (b): Noisy signal; (c): Denoised signal

4. CONCLUSION

A denoising method on the basics of combining wavelet semiparametric regression with generalized inverse estimator approach has been proposed. On adopting exponential power series for nonlinear dynamical presentation, an optimization-based algorithm has been suggested which allows one to model more accurate different types of discontinuities in signals. The method has been verified by a simulation process with different wave forms and shown that the suggested algorithm not only improves the signal to noise ration, but allows measuring accurately the mutations of signals also.

However, with the idea of using the concept of generalized inverse in proposed method, different research topics may be carried with respect to the algorithm point of view. First one is related with dynamical linear (LD) for measurement data of first order derivative of signals with respect to time; in present case is for the measurement of kernel in exponential power series. Second is related with representation of signal in multi-dimension so that different spectrum can be obtained giving rise to different algorithms that may be constructed from the generalization concept.

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TÓM TẮT

VỀ MỘT PHƯƠNG PHÁP LOẠI TRỪ NHIỄU DỰA TRÊN HỒI QUY WAVELET BÁN THAM SỐ KẾT HỢP NHẬN DẠNG ĐIỂM KỲ DỊ

Biến đổi wavelet là công cụ mạnh trong lĩnh vực xử lí tín hiệu, có khả năng thực hiện các phương pháp phân tích đa phân giải khi sử dụng các hàm compact support. Nhưng, khi áp dụng các thuật toán lọc để loại trừ nhiễu thì phần đột biến của tín hiệu không khôi phục được. Bài báo này đề cập đến phương pháp loại trừ nhiễu cho một lớp tín hiệu có các đoạn giao nhau bởi các điểm kì dị áp dụng hồi quy wavelet bán tham số kết hợp nhận dạng điểm kì dị gồm các bước như sau. Đầu tiên, sử dụng tính chất compact support của biến đổi wavelet để nhận dạng các đột biến; sau đó mô hình hóa và đánh giá đột biến bằng phương pháp hoàn toàn mới (hồi quy hai phía theo thời gian về điểm kì dị bằng bộ đánh giá nghịch đảo tổng quát) và cuối cùng kết hợp

hồi quy tích phân wavelet và hồi quy cục bộ (bán tham số) để tái tạo tín hiệu trong miền thời gian. Thuật toán gồm các bước sau:

Bước 1: Xác định tập các điểm kì dị theo thuật toán nhận dạng điểm kì dị ở trên.

Bước 2: Với mỗi điểm kì dị:

(1) xác định tập các mẫu nằm phía phải và phía trái của điểm kì dị;

(2) xác định vùng kì dị dùng hồi quy wavelet không tham số về hai phía của điểm kì dị;

(3) Xác định tập các giá trị mẫu và giá trị biên giới hạn phải và giới hạn trái của miền kì dị.

Bước 3: Thực hiện hồi quy có tham số nghịch đảo tổng quát dùng phương pháp lặp trên tập các giá trị từ bước 2.3; lặp đến khi hội tụ.

Bước 4: Khôi phục các mẫu từ khâu hồi quy wavelet và hồi quy nghịch đảo tổng quát.

Phương pháp này được kiểm chứng qua mô phỏng và thấy rằng phương pháp cho kết quả đánh giá khá tốt những đột biến, khôi phục được phần kì dị của tín hiệu và cải thiện được tỷ số SNR.

Từ khóa: Hồi quy sóng con, loại trừ nhiễu, nhận dạng kỳ dị

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