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An exact rigid/plastic solution for a thick-walled tube subject to internal pressure and axial load considering a general isotropic hardening law and its application

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Abstract. The paper presents a new exact rigid/plastic solution that describes the combined elongation (or shortening) and expansion of a tube. The von Mises yield criterion and its associated flow rule are adopted. No restriction is imposed on the isotropic hardening behavior of the material. The solution is facilitated using a Lagrangian coordinate. A numerical technique is only required for evaluating ordinary integrals. The solution applies to the preliminary design of tube hydroforming. In particular, the variation of the inner pressure with the current tube's length that ensures a prescribed change in the tube's radii is determined. Moreover, the modified Cockroft-Latham fracture criterion applies to predict ductile fracture initiation.

Keywords: combined elongation (or shortening) and expansion, rigid plastic material, general hardening law, tube hydroforming, analytical solution.

Classification numbers: 5.1.1, 5.9.3.

1. INTRODUCTION

Only few analytical solutions for models of strain hardening plasticity are available in the literature. Moreover, some of these solutions are approximate since they do not satisfy all boundary conditions. One of the classical rigid perfectly plastic solutions describes the compression of a layer between two parallel rough plates (Prandtl's problem). This solution can be found in any monograph on plasticity theory, for example [1]. This solution does not satisfy the boundary conditions on the traction-free edges and the symmetry axis. Its extension to strain hardening material models has been proposed in [2]. The flow of plastic mass through an infinite wedge-shaped channel is another classical problem in the theory of rigid perfectly plastic solids (Nadai's problem). Its solution can also be found in any monograph on plasticity theory, for example [1]. This solution has been generalized on a linear hardening material in [3]. The latter has been extended to the flow of plastic mass through an infinite conical channel in [4]. Several solutions are available for expanding a hollow cylinder with applications to describe the autofrettage process. A review of these solutions has been provided in [5]. Solutions for expanding a hollow sphere at large strains have been proposed in [6, 7]. Many solutions for elastic/plastic disks subject to various loading conditions have been found, assuming various hardening laws. Typically, these solutions are based on Tresca's yield criterion, which significantly simplifies the formulation of the boundary value problem. For example, such solutions can be found in [8, 9]. An analytic method for analyzing bending under tension at large strains has been developed in [10]. This work has applied the method to elastic/plastic strain hardening models. A review of this method has been provided in [11]. Elastic/plastic solutions for a circular cylindrical bar subject to simultaneous extension and twist have been found in [12, 13]. Paper [13] has considered linear hardening, and paper [12] has assumed a power law of hardening.

The present paper provides an exact analytical rigid/plastic solution for a thick-walled tube subject to internal pressure and axial compression or tension at large strains. Plastic yielding obeys the von Mises yield criterion. No restriction is imposed on hardening behavior. Note that closed tubes subject to combined torsion and internal pressure have been considered in [14] assuming a rigid perfectly plastic material model at infinitesimal strains.

The solution found can be used for an approximate analysis of tube hydroforming. A review of recent achievements in this modern technology has been provided in [15].

1. STATEMENT OF THE PROBLEM

A tube is subject to expansion and axial deformation (Figure 1). The initial length, inner and outer radii of the tube are denoted as $2h_0$, a_0, and b_0, respectively. It is natural to use a cylindrical coordinate system (r, θ, z) whose $z-$ axis coincides with the tube's symmetry axis. Also, the plane $z = 0$ is the process' symmetry plane.

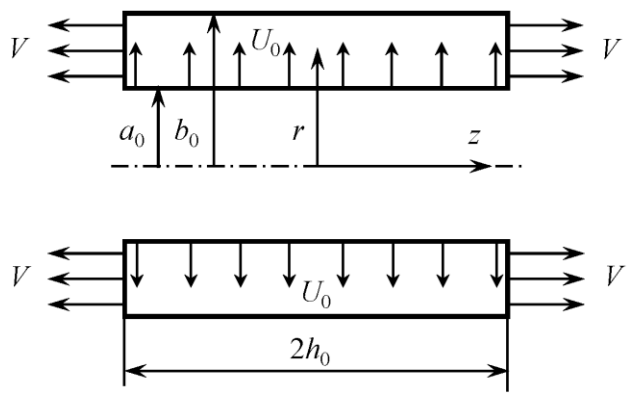

Figure 1. Initial geometry of a tube subject to expansion and axial deformation.

Therefore, it is sufficient to derive the solution in the region $z \geq 0$. The solution is independent of θ; the stress and strain rate fields are also independent of *z*. Let u_r and u_z be the radial and axial velocities, respectively. The circumferential velocity vanishes. The boundary conditions imposed on the axial velocity are

$$u_z = 0 \tag{1}$$

for $z = 0$ and

$$u_z = V \tag{2}$$

for $z = h$, where *h* is half the current tube's length and *V* is constant. The latter may be positive or negative. The radial, circumferential, and axial stresses are denoted as σ_r, σ_θ, and σ_z, respectively. These stresses are the principal stresses. The outer surface of the tube is traction free. Therefore,

$$\sigma_r = 0 \tag{3}$$

for $r = b$, where *b* is the current outer radius of the tube. The speed of the tube's expansion is prescribed at its inner radius:

$$u_r = U_0 u \tag{4}$$

for $r = a$, where *u* is a function of *h*, U_0 is constant and *a* is the current inner radius of the tube. It is assumed that $U_0 > 0$ and $u > 0$. It is also possible to assume without loss of generality that $u = 1$ at the initial instant.

The only stress equilibrium equation that is not identically satisfied is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{5}$$

The constitutive equations are the von Mises yield criterion and its associated flow rule. The elastic portion of strain is neglected. In the case under consideration, the von Mises yield criterion can be represented as

$$\sqrt{\frac{3}{2}\left(s_r^2 + s_\theta^2 + s_z^2\right)} = \sigma_0 \Phi\left(\varepsilon_{eq}\right). \tag{6}$$

Here σ_0 is the initial tensile yield stress, $s_r = \sigma_r - \sigma$, $s_\theta = \sigma_\theta - \sigma$, $s_z = \sigma_z - \sigma$, σ is the hydrostatic stress, ε_{eq} is the equivalent strain, and $\Phi\left(\varepsilon_{eq}\right)$ is an arbitrary function of its argument. This function must satisfy the conditions: $\Phi(0) = 1$ and $d\Phi\left(\varepsilon_{eq}\right)/d\varepsilon_{eq} \ge 0$ for all ε_{eq}. The following equation defines the equivalent strain:

$$\frac{d\varepsilon_{eq}}{dt} = \xi_{eq}, \tag{7}$$

where *t* is the time, d/dt denotes the convected derivative, and ξ_{eq} is the equivalent strain rate. In the case under consideration, the latter is defined as

$$\xi_{eq} = \sqrt{\frac{2}{3}\left(\xi_r^2 + \xi_\theta^2 + \xi_z^2\right)}, \tag{8}$$

where ξ_r, ξ_θ, and ξ_z are the radial, circumferential, and axial strain rates, respectively. The plastic flow rule associated with (6) is

$$\xi_r = \lambda s_r, \quad \xi_\theta = \lambda s_\theta, \quad \text{and} \quad \xi_z = \lambda s_z, \tag{9}$$

where λ is a non-negative multiplier. It follows from (9) that the material is incompressible, i.e.,

$$\xi_r + \xi_\theta + \xi_z = 0. \tag{10}$$

The strain rate components are expressed in terms of the velocity components as

$$\xi_r = \frac{\partial u_r}{\partial r}, \quad \xi_\theta = \frac{u_r}{r}, \quad \text{and} \quad \xi_z = \frac{\partial u_z}{\partial z}. \tag{11}$$

Using the following dimensionless quantities is convenient:

$$\alpha = \frac{a}{a_0}, \quad \alpha_0 = \frac{a_0}{h_0}, \quad \beta_0 = \frac{b_0}{a_0}, \quad \beta = \frac{b}{a_0}, \quad \rho = \frac{r}{a_0}, \quad \zeta = \frac{h}{h_0}, \quad \text{and} \quad s = \frac{Va_0}{U_0 h_0}. \tag{12}$$

Since the material is incompressible, $h_0\left(b_0^2 - a_0^2\right) = h\left(b^2 - a^2\right)$. Using (12), one can rewrite this equation as

$$\beta^2 = \alpha^2 + \frac{\left(\beta_0^2 - 1\right)}{\zeta}. \tag{13}$$

2. GENERAL SOLUTION

Taking into account the boundary conditions (1) and (2), one can reasonably put

$$u_z = V\frac{z}{h}. \tag{14}$$

Substituting (11) into (10) and using (14) yields

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{V}{h} = 0. \tag{15}$$

This equation can be immediately integrated to give

$$u_r = \frac{CU_0 a_0}{r} - \frac{Vr}{2h}, \tag{16}$$

where C is constant. Using the first equation in (4) and (12), one can transform (16) to

$$\frac{u_r}{U_0} = \frac{u\alpha}{\rho} + \frac{s\rho}{2\zeta}\left(\frac{\alpha^2}{\rho^2} - 1\right). \tag{17}$$

By definition, $dr/dt = u_r$ and $dh/dt = V$, where t is the time. Employing (12) and (17), one combines these equations to arrive at

$$\frac{d\rho}{d\zeta} = \frac{u\alpha}{\rho s} + \frac{\rho}{2\zeta}\left(\frac{\alpha^2}{\rho^2} - 1\right). \tag{18}$$

It is convenient to use the following substitution:

$$\eta = \rho^2. \tag{19}$$

Then, equation (18) becomes

$$\frac{d\eta}{d\zeta} + \frac{\eta}{\zeta} = \frac{2u\alpha}{s} + \frac{\alpha^2}{\zeta}.$$

Since it is a linear ordinary differential equation, its general solution can be found using standard methods. In particular,

$$\eta = \frac{2}{s\zeta}\int_1^{\zeta} \gamma u(\gamma)\alpha(\gamma)\,d\gamma + \frac{1}{\zeta}\int_1^{\zeta}\alpha^2(\gamma)\,d\gamma + \frac{R^2}{\zeta}, \tag{20}$$

where R is the Lagrangian coordinate such that $\rho = R$ at the initial instant.

By definition, $da/dt = U_0 u$. This equation can be transformed to

$$\frac{d\alpha}{d\zeta} = \frac{u}{s}. \tag{21}$$

using (12). Substituting this equation in the first term on the right-hand side of (19) and integrating by parts, one gets

$$\frac{2}{s\zeta}\int_1^{\zeta}\gamma u(\gamma)\alpha(\gamma)\,d\gamma = \alpha^2(\zeta) - \frac{1}{\zeta} - \frac{1}{\zeta}\int_1^{\zeta}\alpha^2(\gamma)\,d\gamma. \tag{22}$$

It has been taken into account here that $\alpha = 1$ at $\zeta = 1$. Equations (20) and (22) combine to give the following equation:

$$\eta = \alpha^2(\zeta) + \frac{R^2 - 1}{\zeta}. \tag{23}$$

Employing (11), (12), (14), (17), and (20), one can calculate the normal strain rate components in the cylindrical coordinate system as

$$\xi_z = \frac{V}{h_0\zeta}, \qquad \xi_\theta = \frac{V}{h_0 s}\left[\frac{u\alpha}{\eta} + \frac{s}{2\zeta}\left(\frac{\alpha^2}{\eta} - 1\right)\right], \qquad \xi_r = -\frac{V}{h_0 s}\left[\frac{u\alpha}{\eta} + \frac{s}{2\zeta}\left(\frac{\alpha^2}{\eta} + 1\right)\right]. \tag{24}$$

Substituting (24) into (8) and using (23) leads to

$$\xi_{eq} = \frac{V}{\sqrt{3}h_0 s\left(\zeta\alpha^2 + R^2 - 1\right)}\sqrt{\left[\alpha^4 + \frac{3\left(\zeta\alpha^2 + R^2 - 1\right)^2}{\zeta^2}\right]s^2 + 4\alpha^2 u\zeta\left(u\zeta + \alpha s\right)}. \tag{25}$$

Replacing differentiation with respect to t with differentiation with respect to ζ and employing (25), one can rewrite (7) as

$$\varepsilon_{eq} = \frac{1}{\sqrt{3}s}\int_1^{\zeta}\frac{1}{\left(\gamma\alpha^2 + R^2 - 1\right)}\sqrt{\left[\alpha^4 + \frac{3\left(\gamma\alpha^2 + R^2 - 1\right)^2}{\gamma^2}\right]s^2 + 4\alpha^2 u\gamma\left(u\gamma + \alpha s\right)}\,d\gamma. \tag{26}$$

It is understood here that α and *u* are functions of γ. Numerical integration in (26) allows the equivalent strain to be calculated as a function of *R* at any process stage. Therefore, the right-hand side of (6) is also determined as a function of *R*.

Equation (6) is satisfied by the following substitution:

$$s_r = -\frac{2}{3}\sigma_0 \Phi\left(\varepsilon_{eq}\right)\sin\psi, \quad s_\theta = \frac{1}{3}\sigma_0\Phi\left(\varepsilon_{eq}\right)\left(\sin\psi + \sqrt{3}\cos\psi\right),$$
$$s_z = \frac{1}{3}\sigma_0\Phi\left(\varepsilon_{eq}\right)\left(\sin\psi - \sqrt{3}\cos\psi\right). \tag{27}$$

Eliminating λ between the first and third equations in (9), one gets

$$\xi_r s_z = \xi_z s_r. \tag{28}$$

Equations (24), (27), and (28) combine to give

$$\sqrt{3}\cot\psi = 1 - \frac{4\eta s}{2u\alpha\zeta + s\left(\alpha^2 + \eta\right)}. \tag{29}$$

It is assumed that the tube is expanding. In this case, $\xi_\theta > 0$, and the second equation in (9) demands $s_\theta > 0$. Then, it follows from the second equation in (27) that

$$\sin\psi + \sqrt{3}\cos\psi > 0. \tag{30}$$

If $V > 0$, then $\xi_z > 0$, and the third equation in (9) demands $s_z > 0$. Then, it follows from the third equation in (27) that

$$\sin\psi - \sqrt{3}\cos\psi > 0. \tag{31}$$

Equations (30) and (31) combine to yield

$$\frac{\pi}{3} < \psi < \frac{2\pi}{3}, \tag{32}$$

if $V > 0$. Repeating the line of reasoning above for $V < 0$ leads to

$$-\frac{\pi}{3} < \psi < \frac{\pi}{3}. \tag{33}$$

The value of ψ can be uniquely determined from (29) using (32) or (33).

Using (12), (20), and (23), one can rewrite (5) as

$$\frac{\partial\sigma_r}{\partial R} + \frac{\left(s_r - s_\theta\right)R}{\zeta\alpha^2 + R^2 - 1} = 0. \tag{34}$$

This equation can be further transformed by employing (27) as

$$\frac{\partial\sigma_r}{\sigma_0\partial\mu} - \frac{\Phi\left(\varepsilon_{eq}\right)\left(\sqrt{3}\sin\psi + \cos\psi\right)}{2\sqrt{3}\left(\zeta\alpha^2 + \mu - 1\right)} = 0, \tag{35}$$

where

$$\mu = R^2. \tag{36}$$

Since *R* is a Lagrangian coordinate, the boundary condition (3) becomes $\sigma_r = 0$ for $R = \beta_0$ or, using (36), for $\mu = \beta_0^2$. Then, the solution of equation (35) can be represented as

$$\frac{\sigma_r}{\sigma_0} = \frac{1}{2\sqrt{3}} \int_{\beta_0^2}^{\mu} \frac{\Phi\left(\varepsilon_{eq}\right)\left(\sqrt{3}\sin\psi + \cos\psi\right)}{\left(\zeta\alpha^2 + \gamma - 1\right)} d\gamma. \tag{37}$$

It is understood here that ε_{eq} and ψ are functions of ζ and γ to be determined from (26) and (29). The pressure over the inner radius is determined from (37) as

$$P = \frac{\sigma_0}{2\sqrt{3}} \int_{1}^{\beta_0^2} \frac{\Phi\left(\varepsilon_{eq}\right)\left(\sqrt{3}\sin\psi + \cos\psi\right)}{\left(\zeta\alpha^2 + \gamma - 1\right)} d\gamma. \tag{38}$$

2. APPLICATION TO TUBE HYDROFORMING DESIGN

Tube hydroforming is an important metal forming process for modern industry [15, 16]. The inner pressure and axial displacement greatly affect the appearance of various defects during this process [17]. Various methods are used to design the tube hydroforming process [18, 19]. The solution provided in the previous section can be adopted for the preliminary design of this process. In particular, even simpler solutions based on the assumption that $V = 0$ have led to reasonable results confirmed by experimental data [20, 21]. However, these solutions cannot predict the effect of axial displacement.

The calculations below have been carried out for stainless steel tubes whose hardening behavior is described by the following equation (in our nomenclature) [22]:

$$\Phi\left(\varepsilon_{eq}\right) = \left(1 + \frac{\varepsilon_{eq}}{\varepsilon_0}\right)^m, \tag{39}$$

where $\varepsilon_0 = 0.06$ and $m = 0.624$ are the constitutive parameters found experimentally. Moreover, it has been assumed that $\beta_0 = 0.2$ and $u = 1$. Then, equation (21) supplies

$$\alpha = \frac{\zeta - 1 + s}{s}. \tag{40}$$

It has been taken into account here that $\alpha = 1$ at $\zeta = 1$.

In the case of hydroforming, the range of *s* important for engineering application is $s < 0$. The variation of the dimensionless pressure, $p = P/\sigma_0$, with ζ found from (38) is depicted in Figure 2 for several *s* – values. The solutions resulting in negative *p*-values are mathematically correct. However, these solutions are not feasible for engineering applications. Therefore, the range of applicability of the solutions illustrated in Figure 2 is $\zeta_m \le \zeta \le 1$. The dependence of the value of ζ_m on *s* is provided in Table 1.

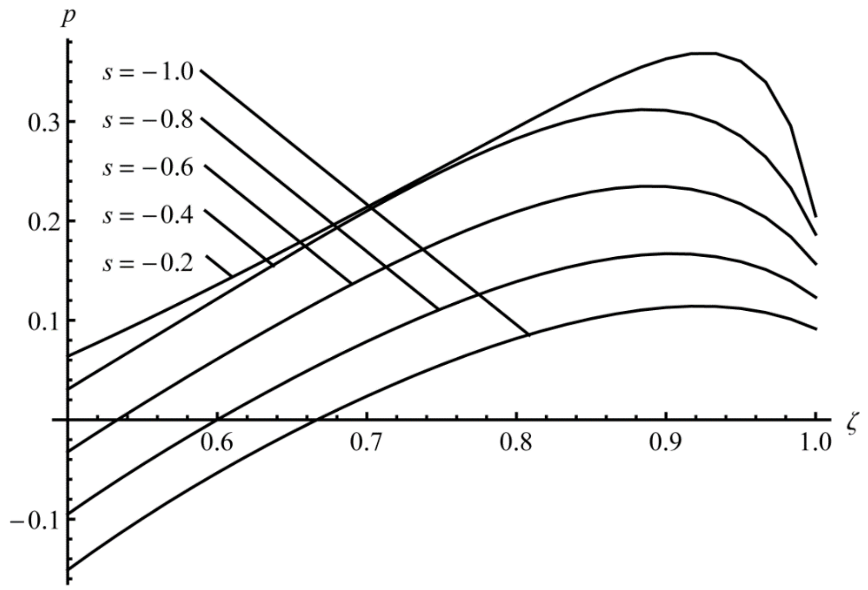

Figure 2. Effect of s-value on the inner pressure.

Table 1. Range of validity of the engineering solution.

s	-1	-0.8	-0.6	-0.4	-0.2
ζ_m	0.67	0.6	0.53	< 0.5	< 0.5

The wall's thickness is an essential technological parameter [23, 24]. This thickness is determined as $W = b - a$. Using (12) and (13), one can find the dimensionless thickness as

$$w = \frac{W}{a_0} = \beta - \alpha = \sqrt{\alpha^2 - \frac{\left(\beta_0^2 - 1\right)}{\zeta}} - \alpha. \tag{41}$$

It is understood here that α should be eliminated employing (40). The variation of w with ζ is depicted in the range $\zeta_m \le \zeta \le 1$ in Figure 3 for the same s-values for which the dimensionless pressure has been calculated.

Empirical fracture criteria are often used for predicting ductile fracture initiation in metal forming processes [25]. The modified Cockroft-Latham criterion proposed in [26] is one of the most often used. An advantage of this criterion is that its form contains no stress in the case of free surface fracture. In particular, the modified Cockroft-Latham criterion reduces to [27]

$$2\varepsilon_1 + \varepsilon_2 = C. \tag{42}$$

if the fracture occurs at a traction free surface. In equation (42), C is a constitutive parameter, and ε_1 and ε_2 are the in-surface principal strains satisfying the inequality $\varepsilon_1 \ge \varepsilon_2$. In the case of the tube hydroforming process under consideration, ε_1 is the circumferential strain, and ε_2 is the axial strain. Using (12) and (24), one can derive the equations for calculating the axial and circumferential strains as

$$\frac{\partial \varepsilon_z}{\partial \zeta} = \frac{1}{\zeta} \quad \text{and} \quad \frac{\partial \varepsilon_\theta}{\partial \zeta} = \frac{u\alpha}{s\eta} + \frac{1}{2\zeta}\left(\frac{\alpha^2}{\eta} - 1\right). \tag{43}$$

The first of these equations can be immediately integrated using the initial condition $\varepsilon_z = 0$ at $\zeta = 1$ to give

$$\varepsilon_z = \ln \zeta. \tag{44}$$

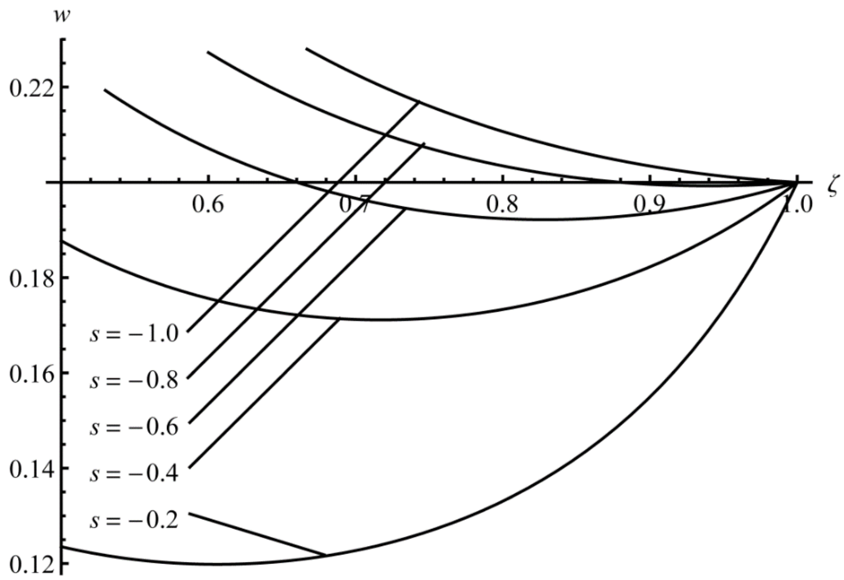

Figure 3. Effect of s-value on the wall's thickness.

Using (23) and (40) and taking into account that $u = 1$ and $R = \beta_0$ at the outer radius, one can transform the second equation in (43) to

$$\varepsilon_\theta = \frac{1}{2}\int_1^\zeta \frac{\left[2\gamma^3 + 2(s-1)\gamma^2 + s^2\left(1-\beta_0^2\right)\right]}{\gamma\left[2s\gamma(\gamma-1) + \gamma(1-\gamma)^2 + s^2\left(\beta_0^2 + \gamma - 1\right)\right]} d\gamma. \tag{45}$$

Equations (42), (44), and (45) combine to give

$$\Lambda(\zeta) = \int_1^\zeta \frac{\left[2\gamma^3 + 2(s-1)\gamma^2 + s^2\left(1-\beta_0^2\right)\right]}{\gamma\left[2s\gamma(\gamma-1) + \gamma(1-\gamma)^2 + s^2\left(\beta_0^2 + \gamma - 1\right)\right]} d\gamma + \ln\zeta = C. \tag{46}$$

The function $\Lambda(\zeta)$ is depicted in the range $\zeta_m \le \zeta \le 1$ in Figure 4 for the same *s*-values for which the dimensionless pressure has been calculated. The value of *C* should be determined from experiments. Once this value has been found, equation (46) can be used to predict fracture initiation.

3. CONCLUSIONS

A new analytical rigid/plastic solution has been found. The solution describes a thick-walled tube's combined expansion and axial deformation at large strains. The specific calculations illustrated in Figures 2 to 4 have been carried out to the maximum value of the axial strain $|\varepsilon_z| \approx 0.7$. The material obeys the von Mises yield criterion and its associated flow rule. The general isotropic hardening law has been adopted. The solution has been facilitated by using the Lagrangian coordinate *R*. A numerical technique is only necessary to evaluate the ordinary integrals in (26), (37), and (38). The solution supplies the dependence of the inner pressure on the axial displacement. The loading path is controlled by the parameter *s*.

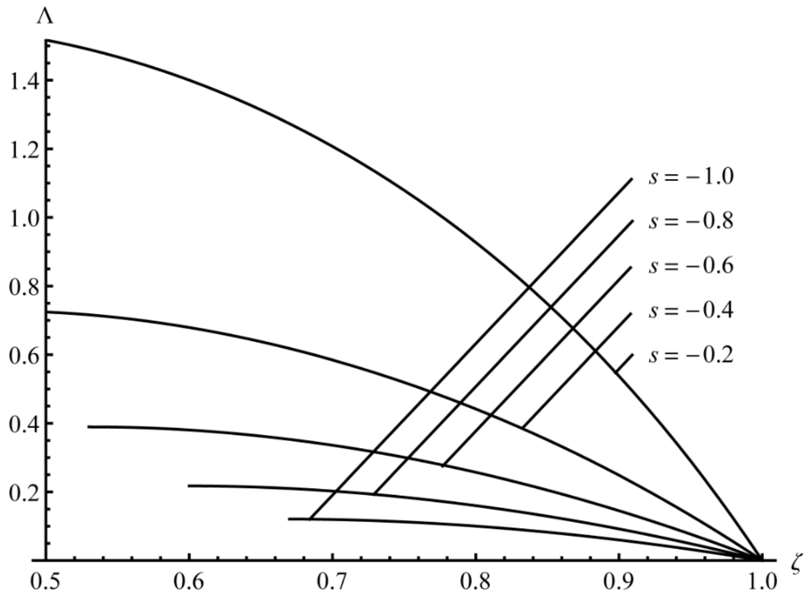

Figure 4. Geometric interpretation of the modified Cockroft-Latham criterion at the outer radius.

The solution has been adopted for the preliminary design of tube hydroforming processes (Figures 2 to 4). In particular, these figures illustrate the effect of the loading path on the inner pressure, the wall's thickness, and the occurrence of ductile fracture according to the modified Cockroft-Latham criterion.

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REFERENCES

1. Hill R. - The mathematical theory of plasticity, Oxford, Clarendon Press, 1950.
2. Collins I. F., Meguid S. A. - On the influence of hardening and anisotropy on the plane-strain compression of thin metal strip, Transactions ASME is J. Appl. Mech. Trans. **44** (1977) 271-278.
3. Durban D., Budiansky B. - Plane-strain radial flow of plastic materials, J. Mech. Phys. Solids **26** (5 - 6) (1978) 303-324. doi.org/10.1016/0022-5096(78)90002-9.

4. Durban D. - Axially symmetric radial flow of rigid/linear-hardening materials, ASME. J. Appl. Mech. **46** (2) (1979) 322-328. doi.org/10.1115/1.3424549.
5. Shufen R., Dixit U. S. - A review of theoretical and experimental research on various autofrettage processes, Transactions of the ASME is J. Press. Vessel Technol. **140** (5) (2018) 050802. doi.org/10.1115/1.4039206.
6. Durban D., Baruch M. - Analysis of an elasto-plastic thick walled sphere loaded by internal and external pressure, Int. J. Non-Linear Mech. **12** (1) (1977) 9-22. doi.org/10.1016/0020-7462(77)90012-9.
7. Alexandrov S., Pirumov A., Jeng Y. R. - Expansion/contraction of a spherical elastic/plastic shell revisited, Continuum Mech. Thermodyn. **27** (2015) 483-494. doi.org/10.1007/s00161-014-0365-6.
8. Gamer U. - The elastic-plastic shrink fit with supercritical interference, Acta Mechanica **61** (1986) 1-14. doi.org/10.1007/BF01176358.
9. Parmaksizogˇlu C., Güven U. - Plastic stress distribution in a rotating disk with rigid inclusion under a radial temperature gradient, Mech. Based Des. Struct. Mach. **26** (1) (1998) 9-20. doi.org/10.1080/08905459808945417.
10. Alexandrov S., Manabe Ki., Furushima T. - A general analytic solution for plane strain bending under tension for strain-hardening material at large strains, Arch. Appl. Mech. **81** (2011) 1935-1952. doi.org/10.1007/s00419-011-0529-9.
11. Alexandrov S., Lyamina E., Hwang Y.-M. - Plastic bending at large strain: A review, Processes **9** (3) (2021) 406. doi.org/10.3390/pr9030406.
12. Brooks D.S. - The elasto-plastic behaviour of a circular bar loaded by axial force and torque in the strain hardening range. Int. J. Mech. Sci. **11**(1) (1969) 75-85. doi.org/10.1016/0020-7403(69)90081-2.
13. Hünlich R. - On simultaneous torsion and tension of a circular cylindrical bar consisting of an elastoplastic material with linear hardening, Z. Angew. Math. Mech. **59** (1979) 509-516. doi.org/10.1002/zamm.19790591003.
14. Crossland B., Hill R. - On the plastic behaviour of thick tubes under combined torsion and internal pressure, J. Mech. Phys. Solids **2** (1) (1953) 27-38. doi.org/10.1016/0022-5096(53)90024-6.
15. Hwang Y. M., Manabe K. I. - Latest hydroforming technology of metallic tubes and sheets, Metals **11** (2021) 1360. doi.org/10.3390/met11091360.
16. Lin C., Chu G., Sun L. - An investigation into the effect of pre-bending on the tube hydro-forging technology, Int. J. Adv. Manuf. Technol. **121** (2022) 2343-2363. doi.org/ 10.1007/s00170-022-09483-8.
17. Yasui H., Miyagawa T., Yoshihara S., Furushima T., Yamada R., Ito Y. - Influence of internal pressure and axial compressive displacement on the formability of small-diameter ZM21 magnesium alloy tubes in warm tube hydroforming, Metals **10** (2020) 674. doi.org/10.3390/met10050674.
18. Ghorbani-Menghari H., Kahhal P., Jung J., Mohammadhosseinzadeh M., Hoon Moon Y., Hoon Kim J. - Multi-objective evolutionary neural network optimization of process parameters for double-stepped tube hydroforming, Int. J. Precis. Eng. Manuf. **24** (2023) 915–929. doi.org/10.1007/s12541-023-00802-x.

19. Vu Q. D., Nguyen T. D., Dang H. V., Phan D. T. - Effect of loading paths on hydroforming ability of stepped hollow shaft components from double layer pipes, EUREKA Phys. Eng. **4** (2023) 143-154. doi.org/10.21303/2461-4262.2023.002797.
20. Alexandrov S., Lyamina E., Lang L. - Description of the expansion of a two-layer tube: an analytic plane-strain solution for arbitrary pressure-independent yield criterion and hardening law, Metals **11** (5) (2021) 793. doi.org/10.3390/met11050793.
21. Strashnov S., Alexandrov S., Lang L. - A new semi-analytical solution for an arbitrary hardening law and its application to tube hydroforming, Materials **15** (17) (2022) 5888. doi.org/10.3390/ma15175888.
22. Luege M., Luccioni B.M. - Numerical simulation of the lubricant performance in tube hydroforming. J. Mater. Process. Technol. **198** (1 - 3) (2008) 372-380. doi.org/10.1016/j.jmatprotec.2007.07.040.
23. Zhu H. L., Xu Y., Chen W. J., Zhang S. H., Banabic D., Lăzărescu L., Pokrovsky A. I. - Research on hydroforming through combination of internal and external pressures for manufacturing the structure of double-layer tube with gap, Int. J. Mater. Form. **15** (2022) 55. doi.org/10.1007/s12289-022-01699-z.
24. Marlapalle B. G., Hingole R. S. - Numerical and experimental predictions of formability parameters in tube hydroforming process, Aust. J. Mech. Eng. **21** (3) (2023) 991-1007. 10.1080/14484846.2021.1938952.
25. Ribas L. M., Gipiela M. L., Lajarin S. F., Filho R. A. C., Marcondes P. V. P. - Comparative study of six failure criteria via numerical simulation of stamped DP600 steel, Int. J. Adv Manuf. Technol. **121** (2022) 2427-2435. doi.org/10.1007/s00170-022-09440-5.
26. Oh S. I., Chen C. C., Kobayashi S. - Ductile fracture in axisymmetric extrusion and drawing, Part 2: Workability in extrusion and drawing, ASME. J. Eng. Ind. **101** (1) (1979) 36-44. doi.org/10.1115/1.3439471.
27. Alexandrov S., Vilotic D. - A theoretical - experimental method for the identification of the modified Cockroft-Latham ductile fracture criterion, Proc. Inst. Mech. Eng. C: J. Mech. Eng. Sci. **222** (9) (2008) 1869-1872. doi:10.1243/09544062JMES1055